

A Practical Guide to Decibels

A Companion to The Art of Electronics Student Manual Lab 2

UCSD Physics 120A

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Simply put, a **decibel** is a way to express a ratio.

§1. Power Ratios

We will first consider a ratio of powers. The formula to memorize and take to your deathbed is:

$$\frac{P_2}{P_1} [dB] = 10 \log_{10} \left(\frac{P_2}{P_1} \right)$$

Here, the square brackets mean “units of dB”. Notice that this formula is easy to remember because it has lots of tens, hence the “deci-” in “decibels”.

Example 1-1:

An attenuator is anything that takes in a signal and spits out a weaker one. Consider the attenuator below. How many dB’s of attenuation does this attenuator provide?

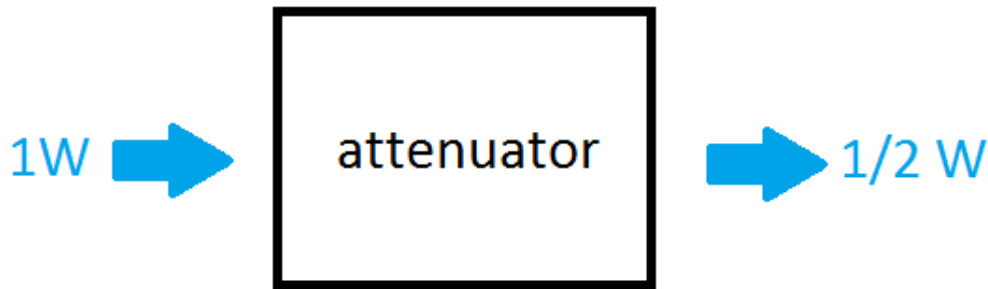


Figure 1

In this problem, our power ratio is output power over input power, often simply known as **power gain**.

$$\begin{aligned} \frac{P_{out}}{P_{in}} [dB] &= 10 \log_{10} \left(\frac{P_{out}}{P_{in}} \right) \\ &= 10 \log_{10} \left(\frac{1/2}{1} \right) \\ &\approx -3 dB \end{aligned}$$

We have learned something important (memorize this!) from the above example:

$$\boxed{-3 \text{ dB} \longleftrightarrow \text{half power}}$$

Question 1-1:

An amplifier has a power gain of 2 (if you put in one watt, you will get out two watts). How many dB's of power gain does this amp provide?

ANSWER: +3 dB

Question 1-2:

You have a -10 dB attenuator. What is P_{out}/P_{in} ?

ANSWER: 1/10

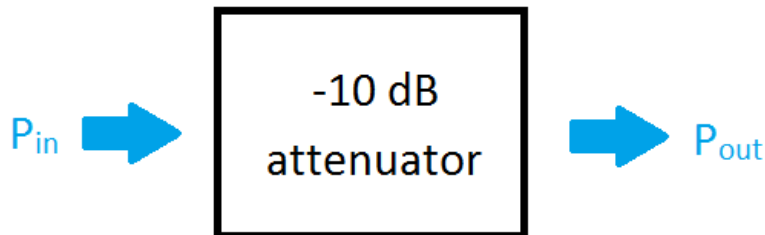


Figure 2

§2. Voltage Ratios

In low frequency electronics, we are typically more concerned with voltages than power. However, the decibel is defined with respect to power. How can we use decibels for voltage ratios? Recalling that $P = V^2/R$,

$$\begin{aligned} \frac{V_{out}}{V_{in}} [dB] &= 10 \log_{10} \left(\frac{V_{out}^2/R}{V_{in}^2/R} \right) \\ &= 10 \log_{10} \left(\left(\frac{V_{out}}{V_{in}} \right)^2 \right) \end{aligned}$$

$$\boxed{\frac{V_{out}}{V_{in}} [dB] = 20 \log_{10} \left(\frac{V_{out}}{V_{in}} \right)}$$

Example 2-1:

An RC filter provides -3 dB of voltage attenuation at a particular frequency. What is the ratio of V_{out}/V_{in} ?

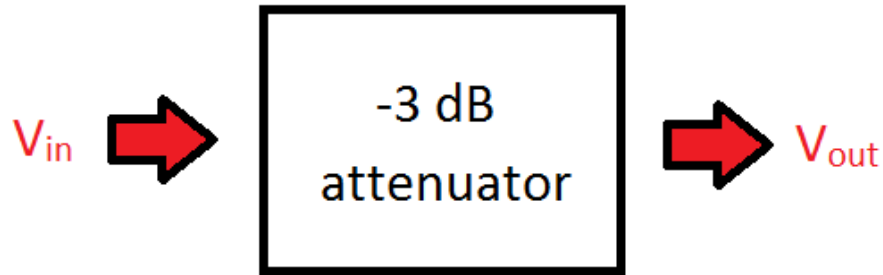


Figure 3

$$-3 \text{ dB} = 20 \log_{10}\left(\frac{V_{out}}{V_{in}}\right)$$

$$-\frac{3}{20} = \log_{10}\left(\frac{V_{out}}{V_{in}}\right)$$

$$10^{-\frac{3}{20}} = \frac{V_{out}}{V_{in}}$$

$$\frac{V_{out}}{V_{in}} = 0.707$$

Notice that this is the same number which is quoted without explanation on Hayes & Horowitz page 56. This is significant for an RC filter because at f_{3dB} , the filter has a $V_{out}/V_{in} = 1/(2)^{1/2} = 0.707$ (see Horowitz & Hill 2 ed. page 36).

Question 2-1:

If you have a component in your system that passes through signal voltage without attenuation or gain, how many dB's does it provide?

ANSWER: 0 dB

Question 2-2:

If you have a component in your system that completely attenuates your signal, how many dB's does it provide?

ANSWER: $-\infty$ dB

Question 2-3:

Without using a calculator, how many decibels correspond to half voltage? Hint: half power is -3 dB.

ANSWER: -6 dB

As you have seen in Example 2-1, converting an arbitrary ratio to decibels is often an arduous task that requires a calculator (unless you can do logarithms in your head). The following chart summarizes important ratio to decibel conversions.

Ratio	Power Ratio in dB	Voltage Ratio in dB
0	$-\infty$ dB	$-\infty$ dB
1/10	-10 dB	-20 dB
1/2	-3 dB	-6 dB
1	0 dB	0 dB
2	3 dB	6 dB
10	10 dB	20 dB
100	20 dB	40 dB

I personally memorize the power conversions, and then simply multiply by two for voltages; however, you should do whatever suits your fancy!

Once you know these, you can quickly express almost any ratio as decibels in your head because **multiplication of ratios means addition of decibels**, as demonstrated in the example below.

Example 2-2:

Without using a calculator, how many decibels correspond to a voltage ratio of 0.05?

Notice that

$$0.05 = (0.1)\left(\frac{1}{2}\right)$$

But we have already memorized that $1/10$ means -20 dB in voltage and $1/2$ means -6 dB! This means we can just add decibels to get -26 dB. If you don't believe me, here is the rigorous math:

$$\begin{aligned} 20 \log_{10}(0.05) &= 20 \log_{10}\left(\frac{1}{10} \times \frac{1}{2}\right) \\ &= 20 \log_{10}\left(\frac{1}{10}\right) + 20 \log_{10}\left(\frac{1}{2}\right) \\ &= -20 \text{ dB} - 6 \text{ dB} \\ &= -26 \text{ dB} \end{aligned}$$