4 Notes on Inhibitory Feedback

4.1 Overview

Today we will study inhibitory feedback, a general principle across all of science that began with the work of Harold Black in the 1930s. Black showed that feedback can be used to turn an amplifier with high but irregular gain into a low gain amplifier. stable amplifier. Similarly, feedback will increase the spectral bandwidth of an amplifier at the price of gain.

4.2 Open loop

Suppose we have an amplifier with gain, referred to as open-loop gain, of $\mathbf{A}(\omega)$. Then for a feedforward architecture we have

$$\tilde{V}_{out}(\omega) = \tilde{\mathbf{A}}(\omega)\tilde{V}_{in}(\omega) \tag{4.1}$$

as shown in figure 1. Suppose that the gain fluctuates by an amount $\delta\tilde{\mathbf{A}}(\omega)$. We see that the the output voltage will fluctuate by an equal amount. Noting that

$$\frac{d\tilde{V}_{out}(\omega)}{d\tilde{\mathbf{A}}(\omega)} = \tilde{V}_{in}(\omega). \tag{4.2}$$

where the functional derivative is evaluated at each frequency ω, we have

$$\frac{\delta\tilde{V}_{out}(\omega)}{\tilde{V}_{out}(\omega)} = \frac{\delta\tilde{\mathbf{A}}(\omega)}{\tilde{\mathbf{A}}(\omega)} \tag{4.3}$$

Thus any variation in the gain is turned into a variation in the output.

4.3 Negative Feedback

If we now add a small amount of negative feedback, say an amount F, we have a new circuit (figure 2) and a new expression for the output, *i.e.*,

$$\tilde{V}_{out}(\omega) = \frac{\tilde{\mathbf{A}}(\omega)}{1 + \mathrm{F}\tilde{\mathbf{A}}(\omega)}\tilde{V}_{in}(\omega). \tag{4.4}$$

In this case, the change in output with respect to a change in gain is

$$\frac{d\tilde{V}_{out}(\omega)}{d\tilde{\mathbf{A}}(\omega)} = \frac{\tilde{V}_{in}(\omega)}{\left[1 + \mathrm{F}\tilde{\mathbf{A}}(\omega)\right]^2} \tag{4.5}$$

so that

$$\frac{\delta \tilde{V}_{out}(\omega)}{\tilde{V}_{out}(\omega)} = \frac{\delta \tilde{\mathbf{A}}(\omega)}{\tilde{\mathbf{A}}(\omega)} \frac{1}{1 + \mathrm{F}\tilde{\mathbf{A}}(\omega)} \tag{4.6}$$

and we see that feedback reduces the noise by the same factor, $1 + \mathrm{F}\tilde{\mathbf{A}}(\omega)$, that it reduces the gain! This can be useful when $\tilde{\mathbf{A}}(\omega)$ is very large, but we only require modest overall gain, referred to as the closed loop gain $\tilde{\mathbf{G}}(\omega)$, where

$$\tilde{\mathbf{G}}(\omega) = \frac{\tilde{\mathbf{A}}(\omega)}{1 + \mathrm{F}\tilde{\mathbf{A}}(\omega)}. \tag{4.7}$$

The fluctuations at each frequency, of course, are reduced by the ratio of $\tilde{\mathbf{G}}(\omega)$ to $\tilde{\mathbf{A}}(\omega)$, *i.e.*

$$\frac{\delta \tilde{V}_{out}(\omega)}{\tilde{V}_{out}(\omega)} = \frac{\delta \tilde{\mathbf{A}}(\omega)}{\tilde{\mathbf{A}}(\omega)} \frac{\tilde{\mathbf{G}}(\omega)}{\tilde{\mathbf{A}}(\omega)}. \tag{4.8}$$

Black's great insight is that it is often simple to build very high gain but noisy amplifiers - think of op amps with $\tilde{\mathrm{A}}(0) \approx 10^6$ - and that accurate feedback can used to bring the gain down to more reasonable numbers. Suppose we wanted an overall gain of $\tilde{\mathrm{G}}(0) = 10^2$ Then we can feedback $\mathrm{F} = 10^{-2}$ or 1 % of the output. So long as the product $\mathrm{F}\tilde{\mathbf{A}}(\omega)$ satisfies $\mathrm{F}\tilde{\mathbf{A}}(\omega) >> 1$, we have

$$\begin{aligned} \tilde{V}_{out}(\omega) &= \tilde{\mathbf{G}}(\omega)\tilde{V}_{in}(\omega) \\ &\approx \frac{1}{\mathrm{F}}\tilde{V}_{in}(\omega) \end{aligned} \tag{4.9}$$

and a reduction in the variability of the output, at each frequency, by a factor of $\tilde{\mathbf{G}}(\omega)/\tilde{\mathbf{A}}(\omega)$, for which $\tilde{\mathrm{G}}(0)/\tilde{\mathrm{A}}(0) = 10^{-4}$.

4.4 Example

Let's recast the non-inverting amplifier (figure 3) in the form of the feedback circuit discussed above (figure 2).

First, recall our equations

$$\frac{\tilde{V}_{-}(\omega)}{R_1} + \frac{\tilde{V}_{-}(\omega) - \tilde{V}_{out}(\omega)}{R_2} = 0 \tag{4.10}$$

and

$$\tilde{V}_{in}(\omega) = \tilde{V}_{+}(\omega) \tag{4.11}$$

and

$$\tilde{\mathbf{A}}(\omega)\left[\tilde{V}_{+}(\omega) - \tilde{V}_{-}(\omega)\right] = \tilde{V}_{out}(\omega) \tag{4.12}$$

from which

$$\tilde{V}_{out}(\omega) = \frac{\tilde{\mathbf{A}}(\omega)}{1 + \frac{R_1}{R_1+R_2}\tilde{\mathbf{A}}(\omega)}\tilde{V}_{in}(\omega). \tag{4.13}$$

Thus the feedback faction F is just the fraction of the output voltage fed back to $\tilde{V}_-(\omega)$, *i.e.*,

$$\mathrm{F} = \frac{R_1}{R_1 + R_2}. \tag{4.14}$$

4.5 Bandwidth

Not only is the variability in the gain reduced, but the closed-loop bandwidth of the amplifier is increased relative to the open loop case.

Let us write the frequency dependence of the gain in an explicit way that makes the point while keeping the algebra simple. We take the frequency dependence as that of a single-pole low-pass filter, so that

$$\begin{aligned}\tilde{\mathbf{A}}(\omega) &= \frac{\tilde{\mathrm{A}}(0)}{1 + i\,\omega/\omega_o} \\ &= \frac{\tilde{\mathrm{A}}(0)}{\sqrt{1 + (\omega/\omega_o)^2}}\, e^{i\,\tan^{-1}(\omega/\omega_o)}\end{aligned} \tag{4.15}$$

where ω_o is the cut-off frequency, effectively the bandwidth. Thus for the open-loop case we have

$$\tilde{V}_{out}(\omega) = \frac{\tilde{\mathrm{A}}(0)}{1 + i\,\omega/\omega_o}\tilde{V}_{in}(\omega) \tag{4.16}$$

while for the closed-loop case we have

$$\begin{aligned}\tilde{V}_{out}(\omega) &= \frac{\tilde{\mathbf{A}}(\omega)}{1 + \mathrm{F}\tilde{\mathbf{A}}(\omega)}\tilde{V}_{in}(\omega) \\ &= \frac{\frac{\tilde{\mathrm{A}}(0)}{1+i\,\omega/\omega_0}}{1 + \frac{\mathrm{F}\tilde{\mathrm{A}}(0)}{1+i\,\omega/\omega_0}}\tilde{V}_{in}(\omega) \\ &= \frac{\tilde{\mathrm{A}}(0)}{1 + i\,\omega/\omega_0 + \mathrm{F}\tilde{\mathrm{A}}(0)}\tilde{V}_{in}(\omega) \\ &= \frac{\frac{\tilde{\mathrm{A}}(0)}{1+\mathrm{F}\tilde{\mathrm{A}}(0)}}{1 + i\,\omega/\left[\omega_o(1 + \mathrm{F}\tilde{\mathrm{A}}(0))\right]}\tilde{V}_{in}(\omega) \\ &= \frac{\tilde{\mathrm{G}}(0)}{1 + i\,\omega/\left[\omega_o\tilde{\mathrm{A}}(0)/\tilde{\mathrm{G}}(0)\right]}\tilde{V}_{in}(\omega)\end{aligned} \tag{4.17}$$

where

$$\tilde{\mathrm{G}}(0) = \frac{\tilde{\mathrm{A}}(0)}{1 + \mathrm{F}\tilde{\mathrm{A}}(0)}. \tag{4.18}$$

The bandwidth has increased from ω_o to $\omega_o\tilde{\mathrm{A}}(0)/\tilde{\mathrm{G}}(0)$. This number can be very large, *i.e.*, 10^4 for the non-inverting amplifier example.

As a final comment, this example illustrates that the gain-bandwidth product stays constant between the open-loop and closed-loop cases, *i.e.*,

$$\tilde{\mathrm{A}}(0)\,\omega_o = \tilde{\mathrm{G}}(0)\,\omega_o\,\frac{\tilde{\mathrm{A}}(0)}{\tilde{\mathrm{G}}(0)}. \tag{4.19}$$

The trade-off of gain and bandwidth is illustrated in figure 4. A constant gain-bandwidth product for a system may be derived under very general assumptions and is an important concept in biology as well as electronics!

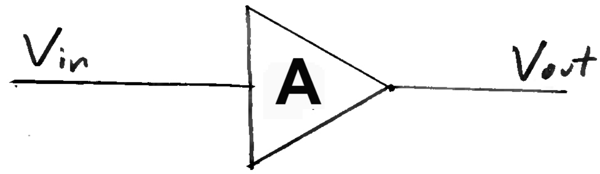

Figure 1

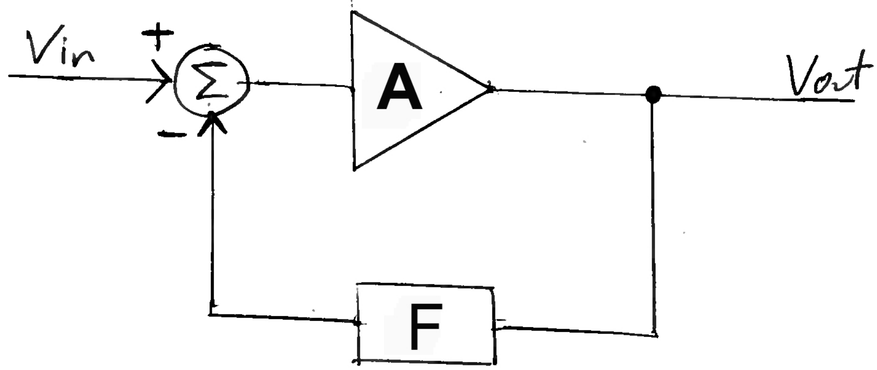

Figure 2

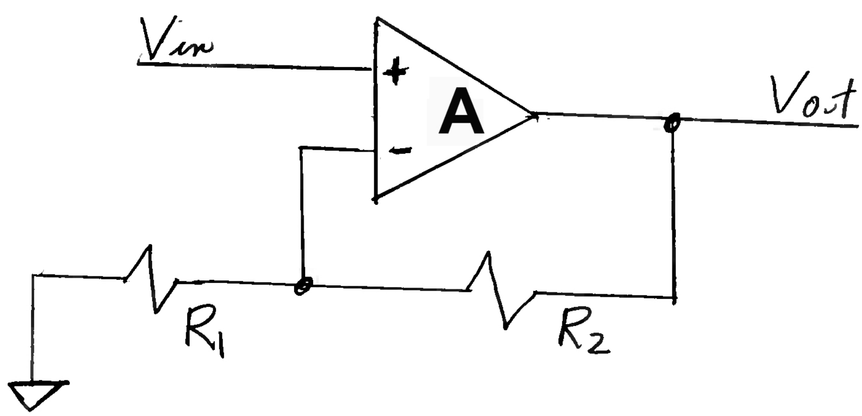

Figure 3

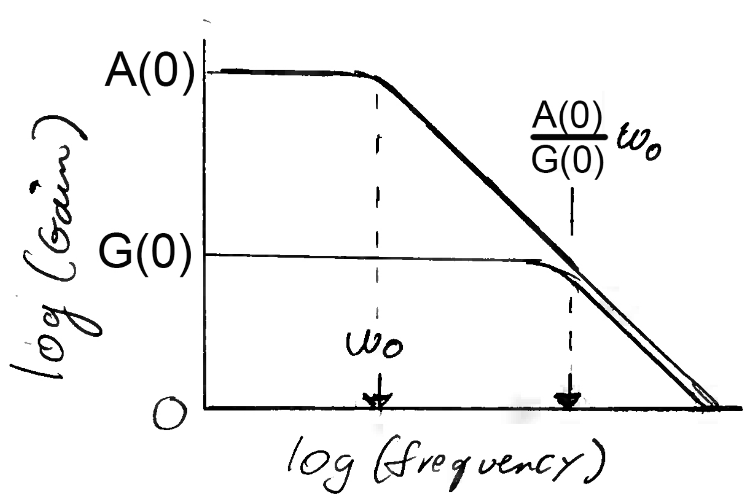

Figure 4