Let's build circuit to form the second derivative of an input, so that

\[ V_{\text{out}}(t) \propto \frac{d^2 V_{\text{in}}(t)}{dt^2} \]

or

\[ \tilde{V}_{\text{out}}(\omega) \propto \omega^2 \tilde{V}_{\text{in}}(\omega) \]

We include a cut-off frequency, or frequencies, in our design so that

\[ \tilde{V}_{\text{out}}(\omega) \to \text{constant for } \omega \to \infty . \]

(1) With reference to the circuit above, find the closed-loop gain function \( \tilde{G}(\omega) \equiv \frac{\tilde{V}_{\text{out}}(\omega)}{\tilde{V}_{\text{in}}(\omega)} \).

Determine the single cut-off frequency \( \omega_{\text{HP}} \) in terms of \( R \) and \( C \).

Calculate and make Bode plots of the magnitude and phase of \( \tilde{G}(\omega) \).

Note the asymptotic power-law dependence of the magnitude of the gain for \( \omega << \omega_{\text{HP}} \) and for \( \omega >> \omega_{\text{HP}} \).

(2) With reference to the circuit above, find the closed-loop gain function \( \tilde{G}(\omega) \equiv \frac{\tilde{V}_{\text{out}}(\omega)}{\tilde{V}_{\text{in}}(\omega)} \).

Is there a single cut-off frequency? Find the value or values for the frequency breaks. Calculate and make Bode plots of the magnitude and phase of \( \tilde{G}(\omega) \).

(3) Why are the closed-loop gain functions found for the circuits in (1) and (2) not the same?