Because of the buffer, this can be treated as two high pass filters one after another.

\[
\frac{V_{out}}{V_{in}} = \frac{R}{R + \frac{1}{j\omega C}} \cdot \frac{R}{R + \frac{1}{j\omega C}} = \frac{-R^2 \omega^2}{(1 + 2R\omega C)^2}
\]

**Magnitude**

Gain = \[\sqrt{\frac{V_{out}}{V_{in}} \cdot \frac{V_{out}}{V_{in}}} = \frac{R^2 \omega^2}{1 + R^2 \omega^2} \]

\[T = \sqrt{R^2 \omega^2} = RC\]

\[\omega_{HP} = \frac{1}{T}\]

For \(\omega < \omega_{HP}\)

\[\text{Gain} \approx \frac{R^2 \omega^2}{1} = R^2 \omega^2\]

\[20 \log_{10} (\text{Gain}) = 20 \log_{10} (R^2 \omega^2) + 40 \log_{10} (\omega)\]

Goes linearly with \(\log_{10} (\omega)\) w/slope of 40

For \(\omega >> \omega_{HP}\)

\[\text{Gain} \approx \frac{R^2 \omega^2}{R^2 \omega^2} = 1\]

\[20 \log_{10} (\text{Gain}) = 0\]

**Phase**

\[\phi = \tan^{-1} \left( \frac{\text{Im} \left( \frac{V_{out}}{V_{in}} \right)}{\text{Re} \left( \frac{V_{out}}{V_{in}} \right)} \right)\]

\[\phi = \tan^{-1} \left( \frac{2R^2 \omega^2}{R^4 \omega^4 - R^2 \omega^2} \right)\]

\[= \tan^{-1} \left( \frac{2R \omega^2}{2R^2 \omega^2 - 1} \right)\]

Limits: As \(\omega \to \infty\) \(\phi \to 0\)

As \(\omega \to 0\) \(\phi \to 0\)

As \(\omega \to \omega_{HP}\) but \(\omega >> \omega_{HP}\) \(\phi \to \pi\)

As \(\omega \to \omega_{HP}\) but \(\omega << \omega_{HP}\) \(\phi \to -\frac{\pi}{2}\)

To make Bode Plot for Magnitude we need to analyze the gain in the limits \(\omega << \omega_{HP}\) & \(\omega >> \omega_{HP}\).

Extract Real & Imaginary parts by multiplying the conjugate of the denominator into the numerator and denominator.

\[\frac{V_{out}}{V_{in}} = \frac{-R^2 \omega^2}{(1 + 2R\omega C)^2} \left[ \frac{(1 + 2R\omega C)^2}{(1 + 2R\omega C)^2} \right] \]

\[= \frac{R^4 \omega^4 - R^2 \omega^2}{(1 + 2R\omega C)^2(1 - 2R\omega C)^2}\]

\[\text{Imaginary Part}\]

\[\text{Real Part}\]

**Phase Bode Plot**

[Diagram showing phase plot with frequency on log scale]
\[ V_{in} \rightarrow R \rightarrow \text{OpAmp} \rightarrow \frac{V_{out}}{R} \rightarrow V_{out} \]

\[ I_+ = I_- = 0 \]

\[ I_+ = \frac{V_{in} - V_t}{R} = \emptyset \Rightarrow V_+ = V_{in} = V_- \]

**KVL:**

\[ V_{out} - I Z_C - I R - I Z_L = \emptyset \]

Current over capacitor \( I \):

\[ I = \frac{V_{out} - V_{in}}{Z_C} \]

\[ \Rightarrow V_{out} = \frac{Z_C + R + Z_L}{R + Z_L} = \frac{-\omega^2 L C + \omega W R C + 1}{\omega^2 W R C - \omega^2 L C} \]

\[ T_1 = \sqrt{L C} \text{ (invert to find break frequencies)} \]

\[ T_2 = \frac{R C}{} \]

\[ \frac{1}{\sqrt{L C}} = \frac{0.01}{R C} \]

\[ \Rightarrow L = 10,000 \text{ R}^2 \text{ C} \]

\[ \omega_1 = 1000 R C \]

\[ \omega_2 = \frac{1}{RC} \]

**Gain:**

\[ G_{Am} = \sqrt{\frac{\text{Vout}}{\text{Vin}}} \]

**Magnitude**

\[ \frac{G_{Am}}{\omega^4} = \frac{1}{\omega^2 Z_t^2 + \omega Z_t^4} \]

\[ \frac{20 \log_{10} G_{Am}}{\omega^4} \]

**Phase**

\[ \tan^{-1}(\frac{\text{Im(Vout)}}{\text{Re(Vout)}}) = \phi = -\tan^{-1}\left(\frac{-1}{10^8 R C^3 \omega^3 - 999 R C \omega} \right) \]

**Phase Graph:**

**Limits:**

\[ As \ \omega \to 0, \ \phi \to \tan^{-1}(0) = \frac{\pi}{2} \]

\[ As \ \omega \to \infty, \ \phi \to \tan^{-1}(\infty) = \emptyset \]

\[ As \ \omega \to \omega_1 \text{ but } \omega - \omega_1 > 0, \ \phi \to -\frac{\pi}{2} \]

\[ As \ \omega \to \omega_1 \text{ but } \omega - \omega_1 < 0, \ \phi \to \frac{\pi}{2} \]
### OP AMP Handout

#### 3. Gain

\[
\text{Gain} = \frac{\omega \, \tau_c}{\sqrt{(1 + \omega^2 \tau_f^2)(1 + \omega^2 \tau_i^2)}}
\]

\[
\tau_i = \sqrt{\tau_i}
\]

\[
\tau_f = \sqrt{\tau_f}
\]

\[
\tau_i \gg \tau_f
\]

#### 4. Voltage Ratio

\[
\frac{V_{out}}{V_i} = \frac{\text{Im} \left[ \frac{\tau_i}{\tau_f} \right]}{\text{Re} \left[ \frac{\tau_i}{\tau_f} \right]}
\]

\[
\phi = \tan^{-1} \left( \frac{1 - \omega^2 \tau_f \tau_i}{\omega (\tau_f + \tau_i)} \right)
\]

You only need the ratio of Im to Re so prefactor & denominator cancel.

#### 5. As \( \omega \to 0 \)

\[
\phi \to \tan^{-1} \left( \frac{1}{\tau_f + \tau_i} \right)
\]

\[
\phi \to \tan^{-1}(-\infty) = -\frac{\pi}{2}
\]

#### As \( \omega \to \infty \)

\[
\phi \to \tan^{-1} \left( \frac{-\omega^2 \tau_f \tau_i}{\omega (\tau_f + \tau_i)} \right)
\]

\[
\phi \to \tan^{-1}(-\infty) = -\frac{\pi}{2}
\]

For \( \tau_i \gg \tau_f \)

\[
(\tau_i \tau_f) \approx 1
\]

\[
\phi \to \tan^{-1}(0) = 0
\]

For "ideal" differentiator

\( V_{out} \) has no real components

imaginary component is negative so:

\[
\phi = \tan^{-1}(-\infty) = -\frac{\pi}{2}
\]