The basics

\( I_G = 0 \)

In active region, device characteristics are defined by\(^1\):

1. \( V_{GS(\text{off})} \leq V_{GS} \leq 0 \)
2. \( V_{DS} > V_{GS} - V_{GS(\text{off})} \), or equivalently \( V_{DS} > |V_{GS(\text{off})}| - |V_{GS}| \)
3. \( I_D = I_S \)

4. \( I_D \) is function of \( V_{GS} \), with \( I_D = I_{DSS} \left( 1 - \frac{V_{GS}}{V_{GS(\text{off})}} \right)^2 \)

5. \( I_D \) is independent of \( V_{DS} \) (ideal current source)

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\(^1\) The turn-off gate-to-source voltage \( V_{GS(\text{off})} \) has a number of aliases, such as threshold voltage or pinch-off voltage, denoted \( V_{GS(\text{off})} = V_T = V_P = V_{PO} = V_{P0} \). The active region is also called the "saturation region" or "pentode region".
For small changes in gate voltage, we can calculate the changes in source or drain current. The constant of proportionality is referred to as the transconductance, denoted $g_m$, where

$$g_m = \frac{dI_S}{dV_{GS}} = \frac{dI_D}{dV_{GS}} = 2 \frac{I_{DSS}}{-V_{GS\, (off)}} \left(1 - \frac{V_{GS}}{V_{GS\, (off)}}\right)$$

so that $\Delta I_S = g_m \Delta V_{GS}$. The transconductance plays a role analogous to $\beta$ with bipolar junction transistors, but is not a constant, i.e., it depends on $V_{DS}$!
Current Sources

Let's now consider the world's simplest current source.

Here \( V_{GS} = 0 \) so the current is forced to be \( I_{DSS} \). This can be accomplished so long as the load line can accommodate \( V_{DS} > V_{GS} - V_{GS(\text{off})} \), i.e., maintain a value of \( V_{DS} \) in the active region. The load line is given by (ignoring the transconductance \( 1/g_m \)):

\[
0 = -V_{DD} + I_D R_{\text{Load}} + V_{DS}
\]

\[
V_{DD} - I_{DSS} R_{\text{Load}} > -V_{GS(\text{off})}
\]

which implies

\[
R_{\text{Load}} < \frac{V_{DD} + V_{GS(\text{off})}}{I_{DSS}}
\]

Here, we slide along the (flat) line of \( I_D = I_{DSS} \) so long as \( V_{DS} > -V_{GS(\text{off})} \). This simple device suffers only from having a value of \( I_D \) that is no adjustable!

A more sophisticated source uses a resistor between the source and ground to determine \( I_D \).

Here we have \( V_G = 0 \). The loop equation encompassing the gate and source satisfies (ignoring the transconductance \( 1/g_m \)): 
\[ 0 = -V_G + V_{GS} + I_S R_s \]

or

\[ I_D = I_S = -\frac{V_{GS}}{R_s}. \]

The other equation that relates \( I_D \) and \( V_{GS} \) is the constitutive equation

\[ I_D = I_{DSS} \left( 1 - \frac{V_{GS}}{V_{GS}(\text{off})} \right)^2. \]

We are free to pick a desired current, denoted \( I_{DQ} \), with \( I_{DQ} < I_{DSS} \). Then the required value of \( R_s \) is found from

\[ I_{DQ} = I_{DSS} \left( 1 + \frac{I_{DQ} R_s}{V_{GS}(\text{off})} \right)^2, \]

for which

\[ R_s = \frac{-V_{GS}(\text{off})}{I_{DQ}} \left( 1 - \sqrt{\frac{I_{DQ}}{I_{DSS}}} \right). \]

As an example relevant to the laboratory 7 (2N5485), the choice \( I_{DQ} = 0.4 \) mA with \( I_{DSS} = 8 \) mA and \( V_{GS}(\text{off}) = -3 \) V, we find \( R_s = 5.8 \) k\( \Omega \).

The load line for \( I_D \) versus \( V_{DS} \) is found by computing the voltage drops along the loop, \( i.e., \)

\[ 0 = -V_{DD} + V_{DS} + I_D R_L + I_S R_s. \]

Thus

\[ I_D = \frac{V_{DD} - V_{DS}}{R_s + R_L} \]

and we slide along a curve of constant \( I_D \).

These current sources are independent of fluctuations in the power supply voltage and largely independent of \( g_m \).
The analysis is rather similar to that for the BJT emitter follower.

**Left loop**

\[ 0 = -V_{in} + V_{GS} + R_S I_S \]

so \[ I_D = \frac{V_{in} - V_{GS}}{R_S} \] and

\[ V_{out} = I_D R_S = V_{in} - V_{GS}. \]

If we include the nonzero value of \( g_m \), these are modified to:

\[ 0 = -V_{in} + V_{GS} + \left( \frac{1}{g_m} \right) I_S + R_S I_S \]

so \[ I_D = \frac{V_{in} - V_{GS}}{R_S + \frac{1}{g_m}} \] and

\[ V_{out} = I_D R_S = \frac{g_m R_S}{1 + g_m R_S} \left( V_{in} - V_{GS} \right) \]

Recall that \( V_{GS} < 0 \) so the offset is positive.

**Right loop**

\[ 0 = -V_{DD} + V_{DS} + \left( \frac{1}{g_m} \right) I_S + R_S I_D \]

so \[ I_D = \frac{V_{DD} - V_{DS}}{R_S + \frac{1}{g_m}} \]

This defines the load line for \( I_D \) versus \( V_{DS} \). Here, changes in the input cause us to move along the load line as the \( V_{GS} \) changes, with changes in \( V_{in} \).
An improved follower may be built in which the offset voltage VGS(off) is minimized. We use a current source to define the current through Rs, as shown below.

![Diagram of transistor circuit]

Here we may write an expression for the equilibrium current (ignoring $g_m$):

$$0 = -V_{in} + V_{GS} (Q1) + I_{DQ} R_1 + V_{out}$$

But we previously solved for the self limiting current source, where $I_{DQ} = -\frac{V_{GS} (Q2)}{R_2}$.

Then

$$0 = -V_{in} + V_{GS} (Q1) - \frac{V_{GS} (Q2)}{R_2} R_1 + V_{out}$$

For $R_1 = R_2$ the output voltage is exactly the input voltage and we have a perfect follower with a very large input impedance.