Notes on n-channel JFETs in the Active Region

1. Basics
   1. $I_G = 0$
   2. $I_D = I_S = I_{DS}$

In active region, device characteristics are defined by\(^1\):

1. $V_{GS(\text{off})} \leq V_{GS} \leq 0$
2. $V_{DS} > V_{GS} - V_{GS(\text{off})}$; recall that both $V_{GS}$ and $V_{GS(\text{off})}$ are negative
3. $I_{DS}$ is independent of $V_{DS}$ (ideal current source)
4. $I_D$ is function of $V_{GS}$, with $I_{DS} = \frac{I_{DSS}}{V_{GS}^2(\text{off})} \left[ V_{GS} - V_{GS(\text{off})} \right]^2 = I_{DSS} \left[ 1 - \frac{V_{GS}}{V_{GS(\text{off})}} \right]^2$

This implies $I_{DS} = I_{DSS}$ for $V_{GS} = 0$.

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\(^1\) The turn-off gate-to-source voltage $V_{GS(\text{off})}$ has a number of aliases, such as threshold voltage or pinch-off voltage, denoted $V_{GS(\text{off})} = V_T = V_P = V_{PO} = V_{P0}$. The active region is also called the "saturation region" or "pentode region", while the Ohmic region is also called the "linear region" or the "triode region".
Transconductance: The small signal limit

For small changes in gate voltage, we can calculate the changes in source or drain current. The constant of proportionality is referred to as the transconductance, denoted $g_m$, where

$$g_m = \frac{dl_{DS}}{dV_{GS}} = 2 \frac{i_{DSS}}{V_{GS(\text{off})}} \left[ 1 - \frac{V_{GS}}{V_{GS(\text{off})}} \right]$$

so that $\Delta I_S = g_m \Delta V_{GS}$. We will see later that the transconductance plays a role analogous to $\beta$ with bipolar junction transistors, but is not a constant, i.e., it depends on $V_{DS}$!

2. Fixed current source

Let's now consider the world's simplest current source.
Here $V_{GS} = 0$, so the current is forced to be $I_{DSS}$. This current is maintained so long as the load line can accommodate the condition to maintain a value of $V_{DS}$ in the active region, i.e., $V_{DS} > V_{GS} - V_{GS(\text{off})}$, which reduces to $V_{DS} > -V_{GS(\text{off})}$.

For example, the 2N5485 has $I_{DSS} = 8$ mA, enough to drive a typical LED.

The load line is given by writing Kirchoff's rule for voltage drops and ignoring the transconductance $1/g_m$:

$$0 = -V_{DD} + I_{DS} R_{Load} + V_{DS}$$

to yield the load line:

$$I_{DS} = \frac{V_{DD} - V_{DS}}{R_{Load}}$$

This must intercept $I_D = I_{DSS}$ in the active region. Noting that $V_{DS} > -V_{GS(\text{off})}$, this implies

$$R_{Load} = \frac{V_{DD} - V_{DS}}{I_{DSS}} < \frac{V_{DD} + V_{GS(\text{off})}}{I_{DSS}}$$

Here, we slide along the (flat) line of $I_{DS} = I_{DSS}$ so long as $V_{DS} > -V_{GS(\text{off})}$.

This source suffers from having a value of $I_D$ that is not adjustable and that may vary with manufacturing!

3. Improved current source

A more sophisticated source uses a resistor between the source and ground to determine $I_{DS}$.

Here we have $V_G = 0$ since the gate is grounded but $V_{GS} < 0$. The loop equation encompassing the gate and source satisfies (ignoring the transconductance term $1/g_m$) is:
\[ 0 = -V_G + V_{GS} + I_{DS} R_S \]

or

\[ I_{DS} = -\frac{V_{GS}}{R_S}. \]

We need to choose a value of \( R_S \) to fix \( I_{DS} \). The second equation that relates \( I_{DS} \) and \( V_{GS} \) is the constitutive equation

\[ I_{DS} = I_{DSS} \left( 1 - \frac{V_{GS}}{V_{GS\text{(off)}}} \right)^2. \]

We are free to pick a desired set-point, or so called "quiescent current", denoted \( I_{DS,Q} \), with \( I_{DS,Q} < I_{DSS} \). Then the required value of \( R_S \) is found by substituting \( V_{GS} = -I_{DS,Q} R_S \) into the constitutive equation, i.e.,

\[ I_{DS,Q} = I_{DSS} \left( 1 + \frac{I_{DS,Q} R_S}{V_{GS\text{(off)}}} \right)^2. \]

Thus

\[ R_S = -\frac{V_{GS\text{(off)}}}{I_{DS,Q}} \left( 1 - \sqrt{\frac{I_{DS,Q}}{I_{DSS}}} \right). \]

As an example relevant to the laboratory 7 exercise with a 2N5485, for the choice \( I_{DS,Q} = 0.4 \text{ mA} \) with \( I_{DSS} = 8 \text{ mA} \) and \( V_{GS\text{(off)}} = -3 \text{ V} \), we find \( R_S = 5.8 \text{ k}\Omega \). We use the closest value 5% resistor at 5.6 k\Omega. These current sources are independent of fluctuations in the power supply voltage and largely independent of \( g_m \).

The load line for \( I_D \) versus \( V_{DS} \) is found by computing the voltage drops along the loop, i.e.,

\[ 0 = -V_{DD} + I_{DS} R_L + V_{DS} + I_{DS} R_S. \]

Thus \( I_{DS} = \frac{V_{DD} - V_{DS}}{R_S + R_L} \) and we slide along a curve of constant \( I_{DS} \) until we can no longer maintain a value of \( V_{DS} \) in the active region, i.e., \( V_{DS} > V_{GS} - V_{GS\text{(off)}} \). As before, this limits the maximum value of \( R_L \), since
$$R_s + R_L = \frac{V_{DD} - V_{DS}}{I_{DS,Q}} < \frac{V_{DD} - \left[ V_{GS} - V_{GS\text{ (off)}} \right]}{I_{DS,Q}}$$

or

$$R_s + R_L < \frac{V_{DD} - V_{GS} + V_{GS\text{ (off)}}}{I_{DS,Q}} = \frac{V_{DD} + I_{DS,Q}R_s + V_{GS\text{ (off)}}}{I_{DS,Q}} = \frac{V_{DD} + V_{GS\text{ (off)}}}{I_{DS,Q}} + R_s$$

so that

$$R_L < \frac{V_{DD} + V_{GS\text{ (off)}}}{I_{DS,Q}}$$

as for the simple current source. We slide along the flat line of $I_{DS,Q}$ so long as $V_{DS} > -V_{GS\text{ (off)}}$.

### 4. Voltage follower

![Diagram](image)

The analysis is over two loops, one to define $R_s$ and the other to define the load line.

We consider the lower loop to relate $V_{in}$ and $I_{DS}$:

$$0 = -V_{in} + V_{GS} + R_s I_{DS}$$

We get $V_{out} = I_{DS} R_s = V_{in} - V_{GS}$.

We see that the output follows the input with the addition of an offset term $V_{GS}$. Recall that $V_{GS} < 0$ so the offset is positive, but unfortunately not a constant.

We can set $I_{DS} = \frac{V_{in} - V_{GS}}{R_s}$ equal to the constitutive relation to find an expression for $V_{GS}$. The result is

$$V_{GS} = V_{GS\text{ (off)}} \left[ 1 - \frac{V_{GS\text{ (off)}}}{2RI_s^{DS,Q}} \left[ 1 + \sqrt{1 + 4 \frac{R_s I^{DS,Q}}{V_{GS\text{ (off)}}}} \left( \frac{V_{in}}{V_{GS\text{ (off)}} - 1} \right) \right] \right]$$

Simply, $V_{GS}$ changes substantially only for changes $\Delta V_{in} \ll V_{GS\text{ (off)}}$.

We consider the upper loop to relate $V_{out}$ and $I_{DS}$ and solve for the load line.

$$0 = -V_{DD} + V_{DS} + R_s I_{DS}$$

so $0 = -V_{DD} + V_{DS} + R_s I_{DS}$
and so \( I_{DS} = \frac{V_{DD} - V_{DS}}{R_S} = \frac{V_{out}}{R_S} \).

**Inclusion of a nonzero value of \( 1/g_m \):**

\[
0 = -V_{in} + V_{GS} + (1/g_m) I_{DS} + R_S I_{DS}
\]

so \( I_{DS} = \frac{V_{in} - V_{GS}}{R_S + 1/g_m} \)

and \( V_{out} = I_{DS} R_S = \frac{g_m R_S}{1 + g_m R_S} (V_{in} - V_{GS}) \).

The key is to maintain \( R_S g_m >> 1 \) or \( R_S << 1/g_m \). This is critical, as \( g_m \) is also a function of \( V_{GS} \).

4. Improved voltage follower

An improved follower may be built in which the offset voltage \( V_{GS} \) is minimized. We use a current source to define the current through \( R_1 \) and \( R_2 \), as show below.

Here we may write an expression for the equilibrium current (ignoring \( g_m \)):

\[
0 = -V_{in} + V_{GS} (Q1) + I_{DS,Q} R_1 + V_{out}
\]

But we previously solved for the self limiting current source corresponding to the lower JFET, or \( I_{DS,Q} = -\frac{V_{GS}(Q_2)}{R_2} \).

Then

\[
0 = -V_{in} + V_{GS} (Q1) - \frac{V_{GS}(Q_2)}{R_2} R_1 + V_{out}
\]

For \( R_1 = R_2 \) and matched JFETs (they are manufactured as pairs on a single substrate for this purpose), the output voltage is exactly the input voltage and we have a perfect follower with a very large input impedance.