Problem 1 (KG).

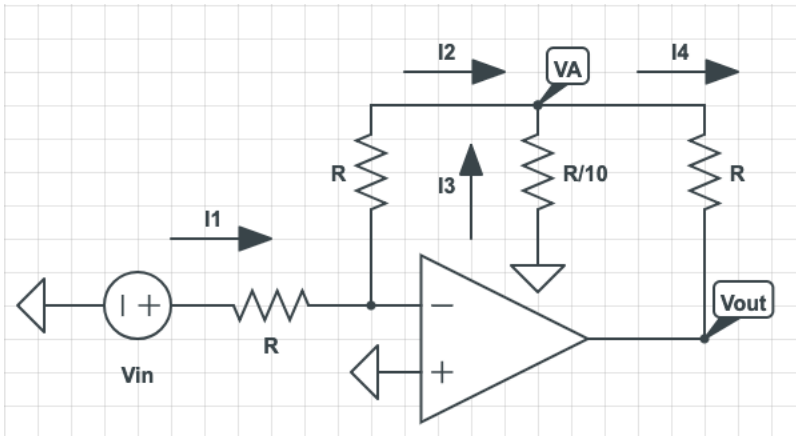

Figure 1: Current flow through the inverting node (V_-) to V_{out}. You can define the current directions differently and get the same result below, but you must keep track of your signs!

Ohm's Law:

$$V_- - V_{in} = I_1 R \tag{1}$$
$$V_A - V_- = I_2 R \tag{2}$$
$$V_A - 0V = I_3(R/10) \tag{3}$$
$$V_{out} - V_A = I_4 R \tag{4}$$

KCL:

$$I_4 = I_2 + I_3 \tag{5}$$
$$I_1 = I_2 \tag{6}$$

Op-Amp:

$$A(V_+ - V_-) = V_{out} \tag{7}$$

This is great because we have 7 equations with 7 unknowns, so we can solve V_{out} in terms of V_{in}. We simplify the equations below starting with (4):

$$\begin{aligned} V_{out} - V_A &= (I_2 + I_3)R && \text{(substitute (5) into (4))} \\ &= \left(\frac{V_A - V_-}{R} + \frac{10V_A}{R}\right) R && \text{(substitute (2) and (3))} \\ V_{out} &= 12V_A - V_- && \text{(simplify)} \end{aligned}$$

Observe that $I_1 = I_2 \implies V_A = 2V_- - V_{in}$. We substitute this in our equation below:

$$\begin{aligned} V_{out} &= 12(2V_- - V_{in}) - V_- \\ &= 12\left(-2\frac{V_{out}}{A} - V_{in}\right) + \frac{V_{out}}{A} && \text{(substitute (7) where } V_+ = 0V\text{)} \\ &= -23\frac{V_{out}}{A} - 12V_{in} && \text{(simplify)} \\ &= -12V_{in}\left(\frac{A}{A+23}\right) && \text{(full expression)} \end{aligned}$$

Hence, our answer is $\boxed{\lim_{A\to\infty} V_{out} = -12V_{in}}$

Problem 2 (BW).

This circuit is very similar to the load line example that is in your Load Line analysis handout on the PHYS 120 website. However, instead of an ideal voltage source, we have a photodiode. A photodiode acts like a current source in our problem, like what you observed in one of your previous labs. Also, instead of resistors to adjust voltage gain, we have an Op Amp.

The photodiode operates in Quadrant III of the Diode I-V curve, which means it generates a negative current in the direction of the diode (opposite of conventional current) while maintaining a negative voltage potential. If the diode is pointing up, that really means conventional current (flow of positive charge) is flowing down into ground.

Recall from the KCL analysis of current flowing into V_- from Section 4.6 of the previous lab. Since the Op Amp terminals have infinite input impedance ($A = \infty$), absolutely no current can flow into the Op Amp. To guarantee this, current is actually flowing right past the V_- node towards V_{out}. The photocurrent is the same current that flows through the upper-most diode. This physically occurs because V_{out} maintains a negative potential.

$$I_{Light} + (-I_D) = 0$$
$$I_{Light} = I_D$$

Recall from Lab 4 that the voltage drop across a diode is not linearly proportional to the current running through it, nor is it generally $0.65V$. The relation between diode voltage and current is given in the problem as follows:

$$I_D = I_0 \left(e^{\frac{qV_D}{k_B T}} - 1 \right)$$
$$\approx I_0 \left(e^{\frac{qV_D}{k_B T}} \right) \text{ As you will probably put more than } 0.025V \text{ over this diode during operation}$$

Where I_0 is the reverse saturation current, a very small constant of magnitude $\approx 10^{-10}$ to 10^{-12} in silicon diodes. This approximation also implies that there is no minimum reverse saturation current

Rearranging to solve for V_D at large values of V_D, we arrive at the following relation for voltage over the diode:

$$V_D = \ln \left[\frac{I_D}{I_0} \right] \frac{k_B T}{q}$$

Now, what we arrived at is actually the final solution, in the opposite direction. How do we know that? Let's look at how the Op Amp factors into this circuit

The Differential Op Amp relation: $V_{out} = A(V_+ - V_-)$ changes to $V_{out} = A(-V_-)$ since V_+ is grounded at $0V$.

Since $A = \infty$ in this problem, $V_- = -\frac{V_{out}}{A} \approx 0$.

Since the diode is the only thing between V_{out} and V_-, this implies $V_{out} = -V_D$.

$$\boxed{V_{out} = -\ln \left[\frac{I_D}{I_0} \right] \frac{k_B T}{q} = -\ln \left[\frac{I_{Light}}{I_0} \right] \frac{k_B T}{q}}$$

Problem 3 (SH).

In general, Op-Amp problems are easily solved by considering the Op-Amp voltage output relation:

$$V_{\text{out}} = A(V^+ - V^-)$$

This motivates us to find V^+ and V^-, both of which are simple voltage dividers in frequency domain. Using complex impedances:

$$\hat{V}^+ = \frac{Z_R}{Z_R + Z_C}\hat{V}_{\text{in}} = \frac{RCj\omega}{1 + RCj\omega}\hat{V}_{\text{in}} \qquad \hat{V}^- = \frac{Z_C}{Z_R + Z_C}\hat{V}_{\text{out}} = \frac{1}{1 + RCj\omega}\hat{V}_{\text{out}}$$

Thus, our expression for V_{out}:

$$\hat{V}_{\text{out}} = A\left(\frac{RCj\omega}{1 + RCj\omega}\hat{V}_{\text{in}} - \frac{1}{1 + RCj\omega}\hat{V}_{\text{out}}\right)$$

Rearranging, we quickly arrive at:

$$\boxed{\hat{V}_{\text{out}} = \frac{ARCj\omega}{A + 1 + RCj\omega}\hat{V}_{\text{in}}}$$

In the limit of $A \to \infty$:

$$\boxed{\hat{V}_{\text{out}} \approx \frac{ARCj\omega}{A}\hat{V}_{\text{in}} = RCj\omega\hat{V}_{\text{in}}}$$

We can check that, because we have negative feedback here, we can also solve this problem using the approximation $V^- \approx V^+$, which gives the same final answer.

The magnitude of the transfer function is the absolute value while the phase is the arctangent of the imaginary component over the real component. As there is no real component in this instance, the argument of arctan approaches ∞, which yields $\pi/2$. Thus, we may also represent our answer as:

$$\boxed{\hat{V}_{\text{out}} = RC\omega e^{j\pi/2}\hat{V}_{\text{in}}}$$

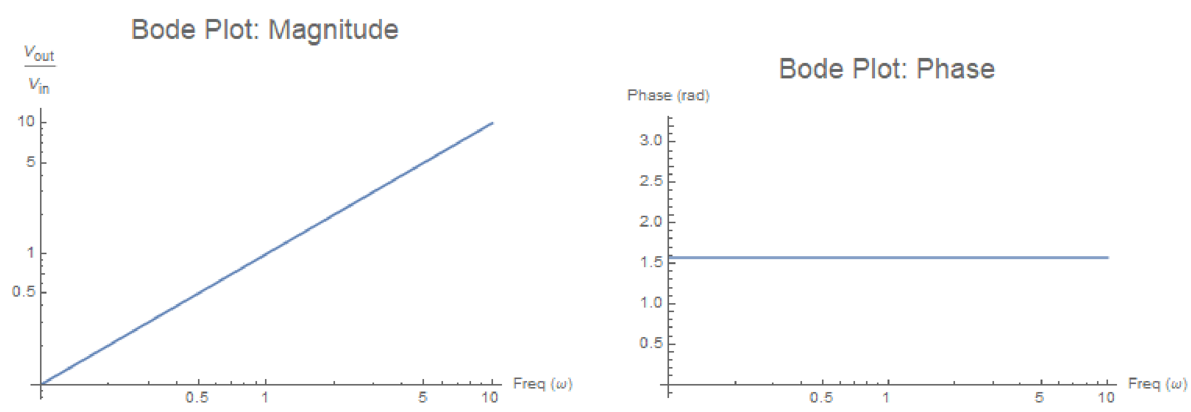

Figure 2: Bode plot. Values chosen: $R = 100$ kΩ and $C = 10$ μF.