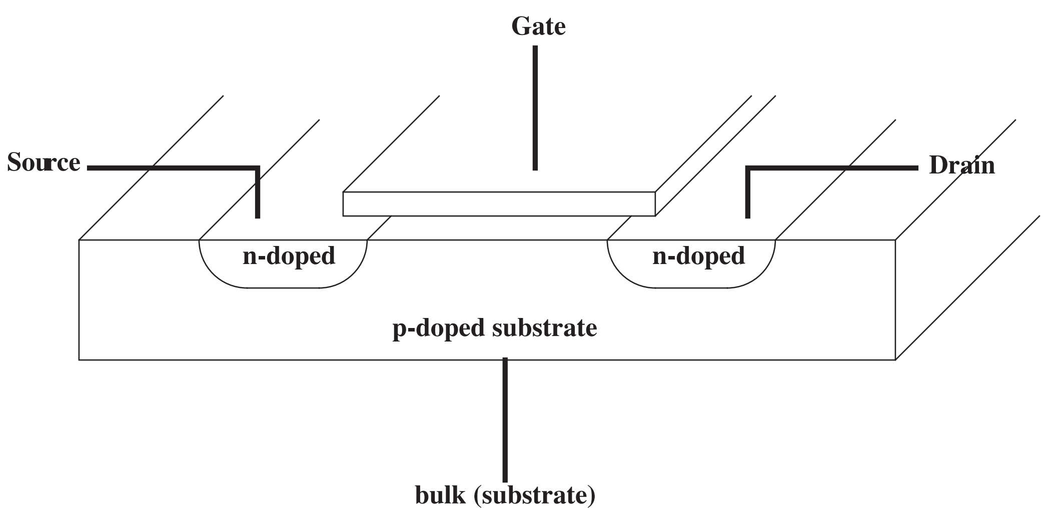
MOSFET Physics: The Long Channel Approximation

A basic n-channel MOSFET (Figure 1) consists of two heavily-doped n-type regions, the Source and Drain, that comprise the main terminals of the device. The gate is made of heavily doped polysilicon, while the bulk of the device is p-type and is typically rather lightly doped. In much of what follows, we will assume that the substrate (bulk) terminal is at the same potential as the Source. However, it is extremely important to keep in mind that the substrate constitutes a fourth terminal, whose influence cannot always be ignored.

FIGURE 1. n-channel MOSFET

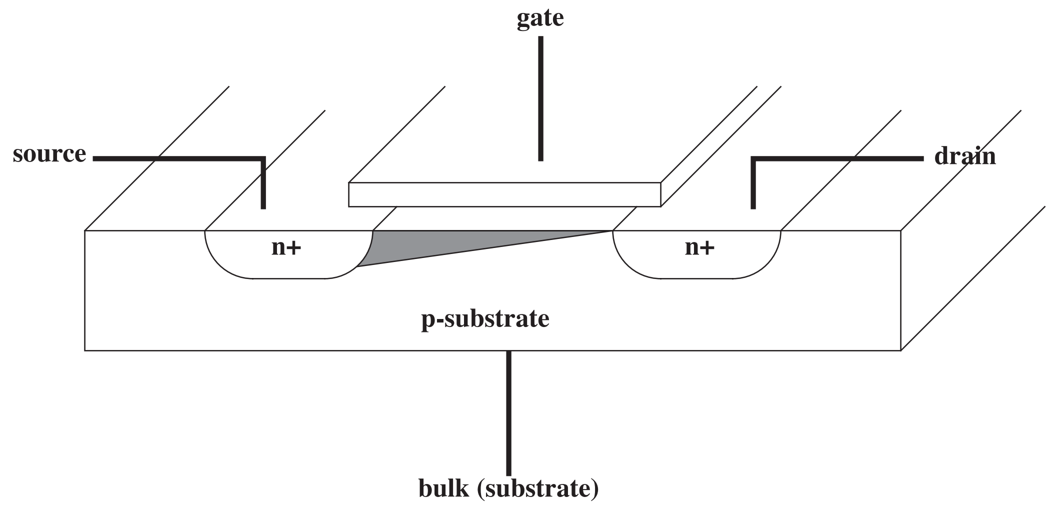


As an increasing positive voltage is applied to the Gate, holes are progressively repelled

away from the surface of the substrate. At some particular value of Gate voltage, denoted the threshold voltage $V_{GS}(off)$, the surface becomes completely depleted of charge. Further increases in Gate voltage induce an *inversion layer*, composed of electrons supplied by the Source (or Drain), that constitutes a conductive path ("channel") between Source and Drain.

The foregoing discussion implicitly assumes that the potential across the semiconductor surface is a constant, that is, there is zero Drain-to-Source voltage. With this assumption, the induced inversion charge is proportional to the Gate voltage above the threshold, and the induced charge density is constant along the channel. However, if we do apply a positive Drain voltage, V, the channel potential must increase in some manner from zero at the Source end to V at the Drain end. The net voltage available to induce an inversion layer therefore decreases as one approaches the Drain end of the channel. Hence, we expect the induced channel charge density to vary from a maximum at the Source (where V_{GS} minus the channel potential is largest) to a minimum at the drain end of the channel (where V_{GS} minus the channel potential is smallest), as shown by the shaded region representing charge density in figure 2:

FIGURE 2. n-channel MOSFE T (shown at boundary between Ohmic region and Active region)



Specifically, the channel charge density has the following form:

$$\rho_n(x) = -C_{ox}\{[V_{GS} - V(x)] - V_{GS}(off)\} \tag{1}$$

where $\rho_n(x)$ is the charge density at position x, C_{ox} is ε_{ox}/t_{ox} and $V(x)$ is the channel potential at position x. We follow the convention of defining the x-direction as along the channel. Note also that C_{ox} is a capacitance *per unit area*. The minus sign simply reflects that the charge is made up of electrons in this nMOS example.

This last equation is all we really need to derive the most important equations governing the terminal characteristics.

Drain Current in the Ohmic Region

The Ohmic region of operation is defined as one in which V_{GS} is large enough (or V_{DS} small enough) to guarantee the formation of an inversion layer the whole distance from source to drain. From our expression for the channel charge density, we see that it has a zero value when

$$[V_{GS} - V(x)] - V_{GS}(off) = 0 \tag{2}$$

The charge density thus first becomes zero at the drain end at some particular voltage. Therefore the boundary for the Ohmic region is defined by

$$[V_{GS} - V_{DS}] - V_{GS}(off) = 0 \rightarrow V_{DS} = V_{GS} - V_{GS}(off) \equiv V_{DSAT} \tag{3}$$

As long as V_{DS} is smaller than V_{DSAT}, the device will be in the Ohmic region of operation.

Having derived an expression for the channel charge and defined the linear region of operation, we are now in a position to derive an expression for the device current in terms of the terminal variables. Current is proportional to charge times velocity, so we've just about got it:

$$I_D = -W \rho_n(x) v(x) \tag{4}$$

The velocity at low fields (remember, this is the "long channel" approximation) is simply the product of mobility and electric field. Hence,

$$I_D = -W\rho_n(x) \mu_n E \tag{5}$$

where W is the width of the device.

Substituting now for the channel charge density, we get:

$$I_D = -WC_{ox}[V_{GS} - V(x) - V_{GS}(off)] \mu_n E \tag{6}$$

Next, we note that the (x-directed) electric field E is simply (minus) the gradient of the voltage along the channel. Therefore,

$$I_D = \mu_n C_{ox} W [V_{GS} - V(x) - V_{GS}(off)] \frac{dV}{dx} \tag{7}$$

so that

$$I_D dx = \mu_n C_{ox} W [V_{GS} - V(x) - V_{GS}(off)] dV \tag{8}$$

Next, integrate along the channel and solve for I_D:

$$\int_0^L I_D dx = I_D L = \int_0^{V_{DS}} \mu_n C_{ox} W [V_{GS} - V(x) - V_{GS}(off)] dV \tag{9}$$

At last, we have the following expression for the Drain current in the Ohmic region:

$$I_D = \mu_n C_{ox} \frac{W}{L} \left\{ [V_{GS} - V_{GS}(off)] V_{DS} - \frac{V_{DS}^2}{2} \right\} \tag{10}$$

Note that the relationship between Drain current and Drain-to-Source voltage is nearly linear for small V_{DS}. Thus, a MOSFET in the Ohmic region behaves as a voltage-controlled resistor.

Drain Current in Saturation

When V_{DS} is high enough so that the inversion layer does not extend all the way from Source to Drain, the channel is said to be "pinched off." In this case, the channel charge ceases to increase, causing the total current to remain constant despite increases in V_{DS}.

Calculating the value of this current is easy; all we have to do is substitute V_{DSAT} for V_{DS} in our expression for current:

$$I_D = \mu_n C_{ox} \frac{W}{L} \left\{ [V_{GS} - V_{GS}(off)]\, V_{DSAT} - \frac{V_{DSAT}^2}{2} \right\} \qquad (11)$$

which simplifies to:

$$I_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L} [V_{GS} - V_{GS}(off)]^2 \qquad (12)$$

Hence, in saturation, the Drain current has a square-law dependence on the Gate-Source voltage, and is (ideally) independent of Drain voltage.

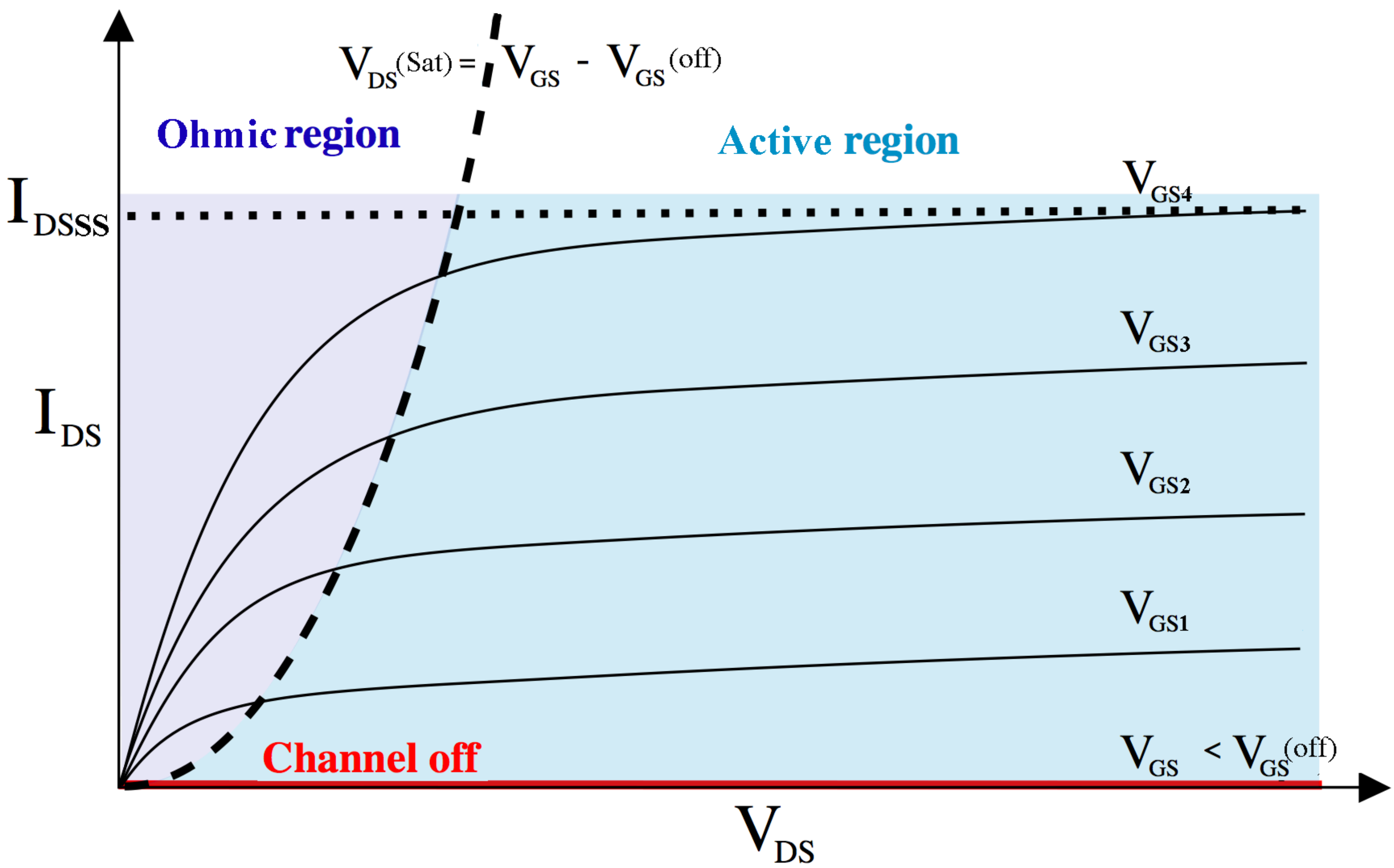
The transconductance of such a device in saturation is easily found from differentiating our expression for drain current:

$$g_m = \mu_n C_{ox} \frac{W}{L} [V_{GS} - V_{GS}(off)] \qquad (13)$$

which may also be expressed as:

$$g_m = \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D} \qquad (14)$$

Thus, a long-channel MOSFET's transconductance depends only on the square-root of the bias current.


$V_{DS}(Sat) = V_{GS} - V_{GS}(off)$
Ohmic region
Active region
I_{DSSS}
I_{DS}
V_{GS4}
V_{GS3}
V_{GS2}
V_{GS1}
Channel off
$V_{GS} < V_{GS}(off)$
V_{DS}