An operation amplifier may be configured as a differentiator, AKA a high pass filter, using a capacitor as the source impedance and a resistor as the feedback impedance in an inverting amplifier configuration.

The above circuit can be analyzed in the time domain as:

\[ 0 = C_{\text{in}} \frac{d[V_-(t) - V_S(t)]}{dt} + \frac{V_-(t) - V_{\text{out}}(t)}{R_F} \]

\[ V_{\text{out}}(t) = A [V_+(t) - V_-(t)] \]

\[ V_+(t) = 0 \]

and for \( A \to \infty \) yields:

\[ V_{\text{out}}(t) = -R_F C_{\text{in}} \frac{d[V_S(t)]}{dt} \]

In the frequency domain we have

\[ \hat{V}_{\text{out}}(\omega) = -R_F \frac{1}{i\omega C_{\text{in}}} \hat{V}_S(\omega) = -i\omega R_F C_{\text{in}} \hat{V}_S(\omega) \]

This is an ideal differentiator. One problem is that the closed loop gain \( |G(\omega)| = \omega R_F C_s \) grows, even as the frequency approaches the value where the open loop gain \( A(\omega) \) experiences large phase shifts that lead to positive feedback with associated distortion and saturation of the output. We thus need to "roll-off" the closed loop gain at high frequencies so that it varies as \( |G(\omega)| \propto 1/\omega \) rather than as \( |G(\omega)| \propto \omega \).
We add a resistor in series with the input so that the gain will saturate at high frequencies. We add a capacitor in parallel with the feedback so that the output will further turn integrative at high frequencies. The new circuit looks like:

In the frequency domain, the combined components yield a circuit with both complex source and feedback impedances:

Thus

\[
\hat{V}_{\text{out}}(\omega) = - \frac{R_F}{1 + i\omega R_F C_f} \times \frac{i\omega C_{\text{in}}}{1 + i\omega R_{\text{in}} C_{\text{in}}} \hat{V}_s(\omega) = \cdots = -i\omega R_f C_{\text{in}} \frac{1}{1 + (\omega R_F C_f)^2} \frac{1}{1 + (\omega R_{\text{in}} C_{\text{in}})^2} \hat{V}_s(\omega)
\]

The magnitude of the closed loop gain is just:

\[
|G(\omega)| = \left| \frac{\hat{V}_{\text{out}}(\omega)}{\hat{V}_s(\omega)} \right| = \frac{\omega R_f C_{\text{in}}}{\sqrt{1 + (\omega R_F C_f)^2} \sqrt{1 + (\omega R_{\text{in}} C_{\text{in}})^2}}
\]

and clearly has the desired fall-off properties with increasing frequency.

FOR HOMEWORK:
(1) Draw a Bode magnitude plot, \(\log |G(\omega)|\) versus \(\log \omega\), with \(1 \gg R_{\text{in}} C_{\text{in}} \gg R_f C_f\). Identity the break points, the slope in each portion of the plot, e.g., \(d|G(\omega)|/d\omega = \omega R_f C_{\text{in}}\) for limiting case of \(\omega \to 0\), and the maximum gain.
(2) Write an expression for the phase of \(G(\omega)\) as a function of \(\omega\).
(3) Draw a Bode phase plot, \(\text{phase}\{G(\omega)\}\) versus \(\log \omega\), with \(1 \gg R_{\text{in}} C_{\text{in}} \gg R_f C_f\). Identity the breaks and the asymptotic value of the phase. Compare with the value for the "ideal" differentiator.