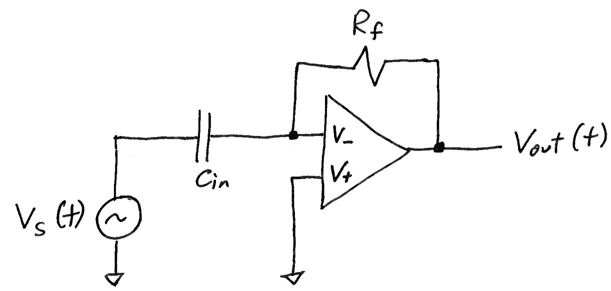
Notes on an Op Amp Differentiator Circuit; Physics 120: David Kleinfeld, Spring 2017

An operation amplifier may be configured as a differentiator, AKA a high pass filter, using a capacitor as the source impedance and a resistor as the feedback impedance in an inverting amplifier configuration.



The above circuit can be analyzed in the time domain as:

$$0 = C_{In}\frac{\mathrm{d}\left[V_-(\mathrm{t}) - V_S(\mathrm{t})\right]}{\mathrm{dt}} + \frac{V_-(\mathrm{t}) - V_{Out}(\mathrm{t})}{\mathrm{R}_F}$$

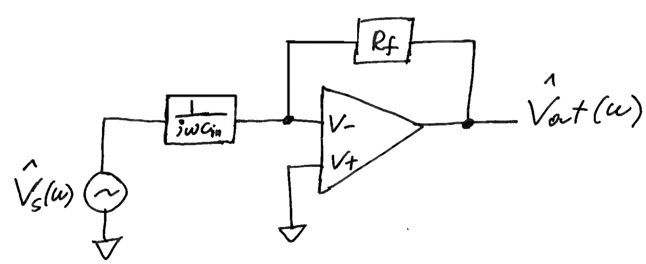
$$V_{Out}(\mathrm{t}) = A\left[V_+(\mathrm{t}) - V_-(\mathrm{t})\right]$$

$$V_+(\mathrm{t}) = 0$$

and for A $\rightarrow \infty$ yields:

$$V_{Out}(\mathrm{t}) = -\mathrm{R}_F\mathrm{C}_{In}\frac{\mathrm{d}\left[V_S(\mathrm{t})\right]}{\mathrm{dt}}$$

In the frequency domain we have

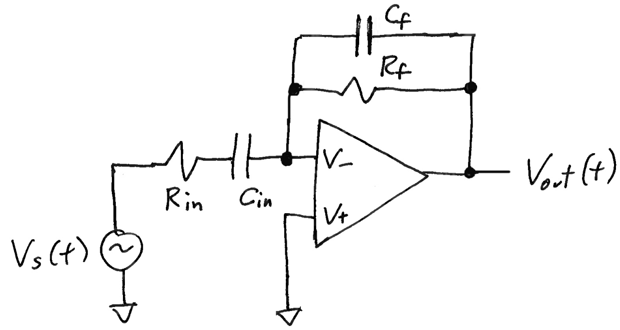


and

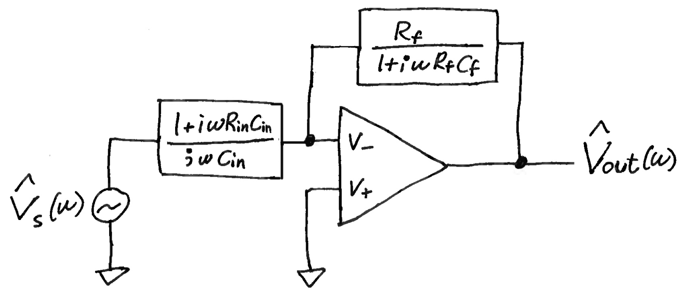
$$\hat{V}_{Out}(\omega) = -\frac{\mathrm{R}_\mathrm{F}}{1/i\omega\mathrm{C}_\mathrm{In}}\hat{V}_S(\omega) = -i\omega\mathrm{R}_\mathrm{F}\mathrm{C}_\mathrm{In}\ \hat{V}_S(\omega).$$

This is an ideal differentiator. One problem is that the closed loop gain $|G(\omega)| = \omega R_F C_S$ grows, even as the frequency approaches the value where the open loop gain $A(\omega)$ experiences large phase shifts that lead to positive feedback with associated distortion and saturation of the output. We thus need to "roll-off" the closed loop gain at high frequencies so that it varies as $|G(\omega)| \propto 1/\omega$ rather than as $|G(\omega)| \propto \omega$.

We add a resistor in series with the input so that the gain will saturate at high frequencies. We add a capacitor in parallel with the feedback so that the output will further turn integrative at high frequencies. The new circuit looks like:



In the frequency domain, the combined components yield a circuit with both complex source and feedback impedances:



Thus

$$\hat{V}_{Out}(\omega) = -\frac{R_F}{1 + i\omega R_F C_F} \times \frac{i\omega C_{In}}{1 + i\omega R_{In} C_{In}} \hat{V}_S(\omega) = \cdots = -i\omega R_F C_{In} \frac{1 - \omega^2 R_F C_F R_{In} C_{In} - i\omega\left(R_F C_F + R_{In} C_{In}\right)}{\left[1 + \left(\omega R_F C_F\right)^2\right]\left[1 + \left(\omega R_{In} C_{In}\right)^2\right]} \hat{V}_S(\omega)$$

The magnitude of the closed loop gain is just:

$$\left|G(\omega)\right| = \left|\frac{\hat{V}_{Out}(\omega)}{\hat{V}_S(\omega)}\right| = \frac{\omega R_F C_{In}}{\sqrt{\left[1 + \left(\omega R_F C_F\right)^2\right]\left[1 + \left(\omega R_{In} C_{In}\right)^2\right]}}$$

and clearly has the desired fall-off properties with increasing frequency.

FOR HOMEWORK:

(1) Draw a Bode magnitude plot, *log* $|G(\omega)|$ versus *log* ω, with $1 \gg R_{In}C_{In} \gg R_F C_F$. Identity the break points, the slope in each portion of the plot, *e.g.*, $d|G(\omega)|/d\omega = \omega R_F C_{In}$ for limiting case of $\omega \to 0$, and the maximum gain.

(2) Write an expression for the phase of $G(\omega)$ as a function of ω.

(3) Draw a Bode phase plot, phase{$G(\omega)$} versus *log* ω, with $1 \gg R_{In}C_{In} \gg R_F C_F$. Identity the breaks and the asymptotic value of the phase. Compare with the value for the "ideal" differentiator.