Spikes are the currency of neuronal computation and communication.

I-V relation for ions is nonlinear - convention is to ignore this and take \( I = G (V - V_{\text{Nernst}}) \)

\[
I(V) = z^2e \frac{D}{L} \frac{eV}{k_BT} A \frac{C(L) - C(0)e^{-\frac{z\epsilon V}{k_BT}}}{1 - e^{-\frac{z\epsilon V}{k_BT}}}
\]

\[
I(V) \rightarrow \begin{cases} 
  z^2e \frac{D}{L} \left( \frac{eV}{k_BT} \right) AC(L) & \text{if } V \gg \frac{k_BT}{e} \\
  z^2e \frac{D}{L} \left( \frac{eV}{k_BT} \right) AC(0) & \text{if } V \ll -\frac{k_BT}{e}
\end{cases}
\]
We model populations of currents - an average over all channels in one electrotonic length.
The Na\(^+\) current leads to a similar rise across different classes of neurons.

McCormick, Connors, Lighthall & Prince (J Neurophysiol 1985)
The $K^+$ current is not needed for recovery but shortens - and determines - the recovery time.

Erisir, Lau, Rudy & Leonard (J Neurophysiol 1999)
Pharmacology to dissect $\text{K}^+$ versus $\text{Na}^+$ currents

Block $\text{Na}^+$

Block $\text{K}^+$

Current [nA]

CONTROL

TTX

TEA
Fitting model parameters to the data

\[
\frac{I_0}{V(\frac{V-E_K}{eV'-1})/(eV'-1)}
\]

\[
\text{Fractional activation of } gK
\]

\[
\text{Slope } = \frac{dI}{dV} = \frac{e}{2k_BT} = \frac{nz}{2^{n+1}} \frac{c}{k_BT}
\]
Compilation of fits for dynamical model

**Na⁺ activation**

**K⁺ activation**

**Na⁺ inactivation**

\[ \begin{align*}
\dot{X} &= \begin{bmatrix}
\dot{V} \\
\dot{m} \\
\dot{h} \\
\dot{n}
\end{bmatrix} = \tilde{F}(\tilde{X}) = \\
&= \begin{bmatrix}
f_V(V,m,h,n) \\
f_m(V,m,h,n) \\
f_h(V,m,h,n) \\
f_n(V,m,h,n)
\end{bmatrix} = \\
&= \begin{bmatrix}
f_V(\tilde{X}) \\
f_m(\tilde{X}) \\
f_h(\tilde{X}) \\
f_n(\tilde{X})
\end{bmatrix}
\end{align*} \]

where

\[ f_V(V,m,h,n) = \left[ I_{\text{ext}} - G_{Na} m^3 h (V - E_{Na}) - G_K h^4 (V - E_K) - G_L (V - E_L) \right] / C \]

and

\[ f_m(V,m,h,n) = \left[ m_\infty(V) - m \right] / \tau_m(V). \]
The model accounts for the refractory period

A 2 pulse protocol was used with variable times between the 2 pulses between 5 and 28 ms. Pulse duration was 1 ms, pulse amplitude was 30 mA/cm².
The model allows one to estimate ion conductances over time.
The model allows us to estimate the "optimal" stimulus to induce a spike

(which is your HW exercise!)

Aguera y Arcas, Fairhall & Bialek (Neural Comp 2003)
The calculation for a Na\textsuperscript{+}-K\textsuperscript{+} cell is not so different from what is seen for a "complex" neuron.
Lastly, the model predicts a discontinuous input (current) - output (spike rate) relation.

BTW, this is modified by the addition of inactivating $K^+$ channels.
Modeling allows one to graph the I-V relation versus time.

Outward current:
\[ |i_K| > |i_{Na}| \]

Inward current:
\[ |i_{Na}| > |i_K| \]
The initial, fast Na+ dynamics can be modeled by a cubic
The equilibrium curves, or nullclines

- **W nullcline**
- **V nullcline**
- **Stable Equilibrium**
Examples (referenced to the notes)
Comparison of trajectories at 100:1 versus 11:1 temporal differences in time-scale of the "Fast" to "Slow" variables