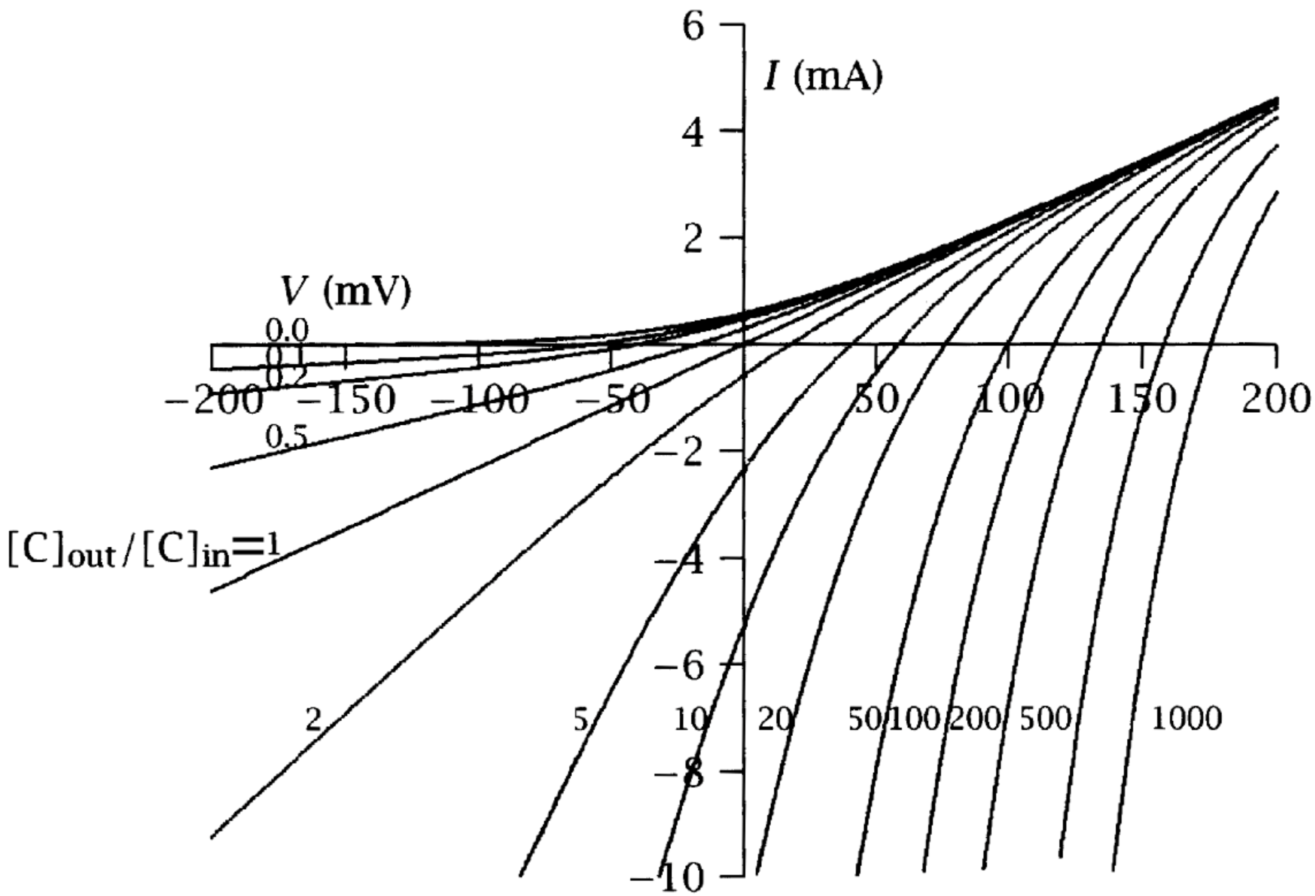
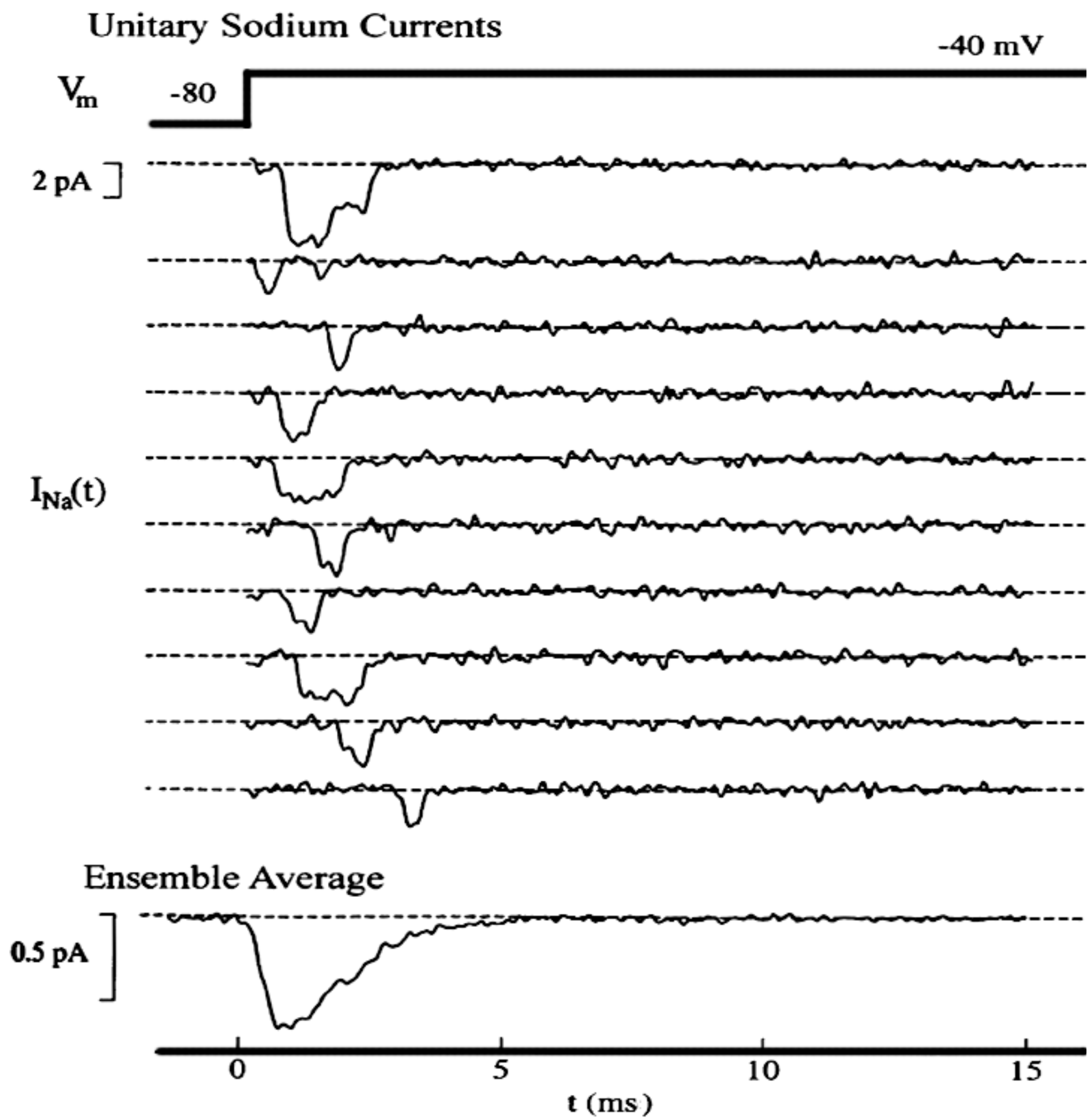
## I-V relation for ions is nonlinear - convention is to ignore this and take I = G (V - $V_{Nernst}$)!

$$I(V) = z^2 e \frac{D}{L}\frac{eV}{k_BT} A \frac{C(L) - C(0)e^{-\frac{zeV}{k_BT}}}{1 - e^{-\frac{zeV}{k_BT}}}$$

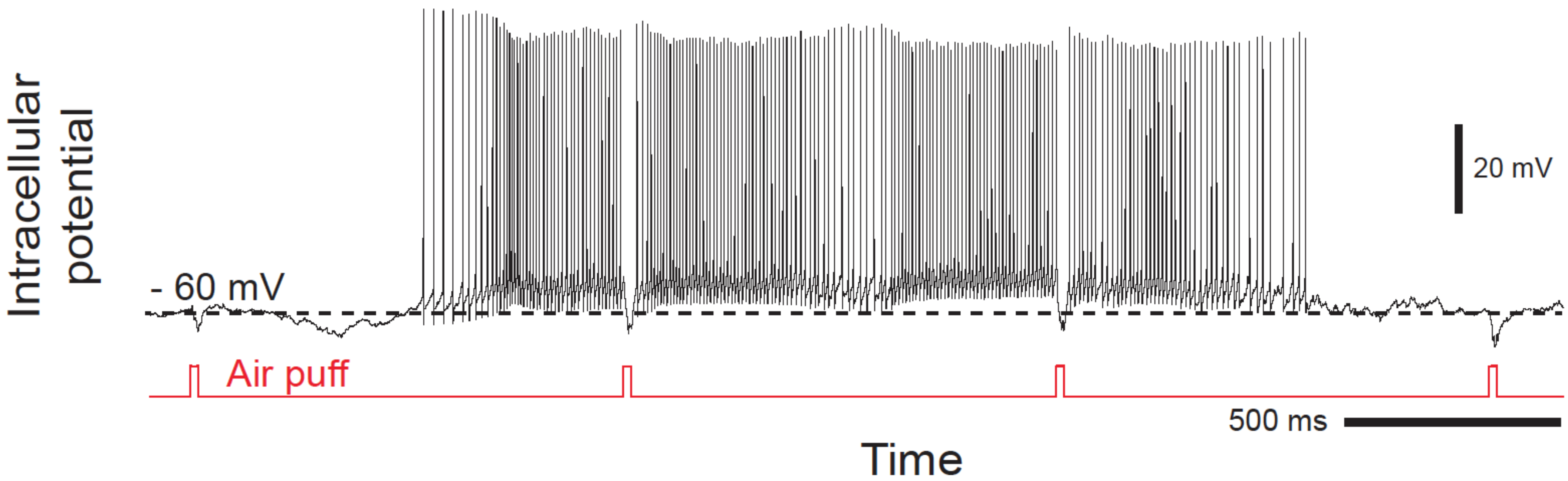
$$I(V) \rightarrow \begin{cases} z^2 e \frac{D}{L}\left(\frac{eV}{k_BT}\right) AC(L) & \text{if } V >> \frac{k_BT}{e} \\ z^2 e \frac{D}{L}\left(\frac{eV}{k_BT}\right) AC(0) & \text{if } V << -\frac{k_BT}{e} \end{cases}$$

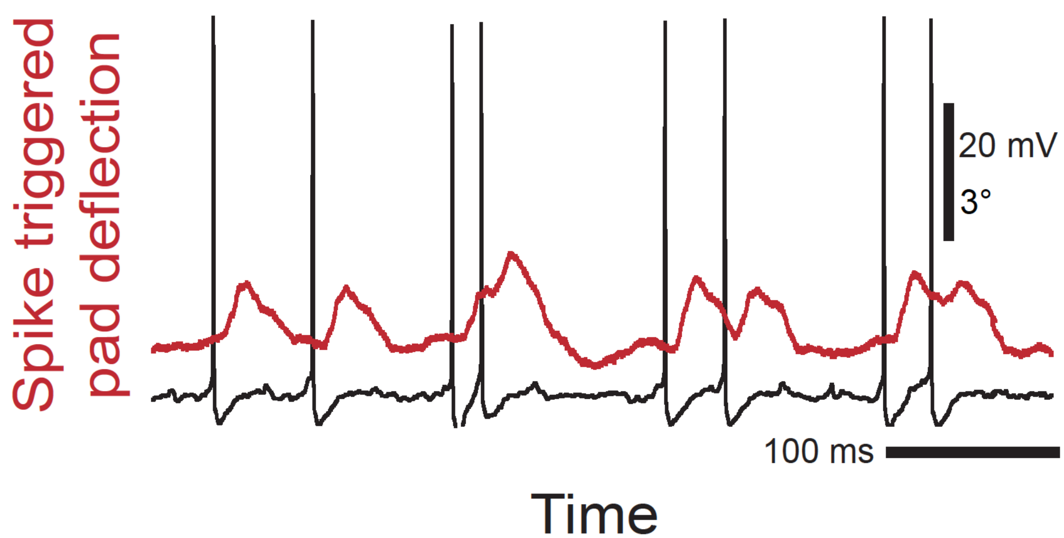


**We model populations of currents - an average over all channels in one electrotonic length**



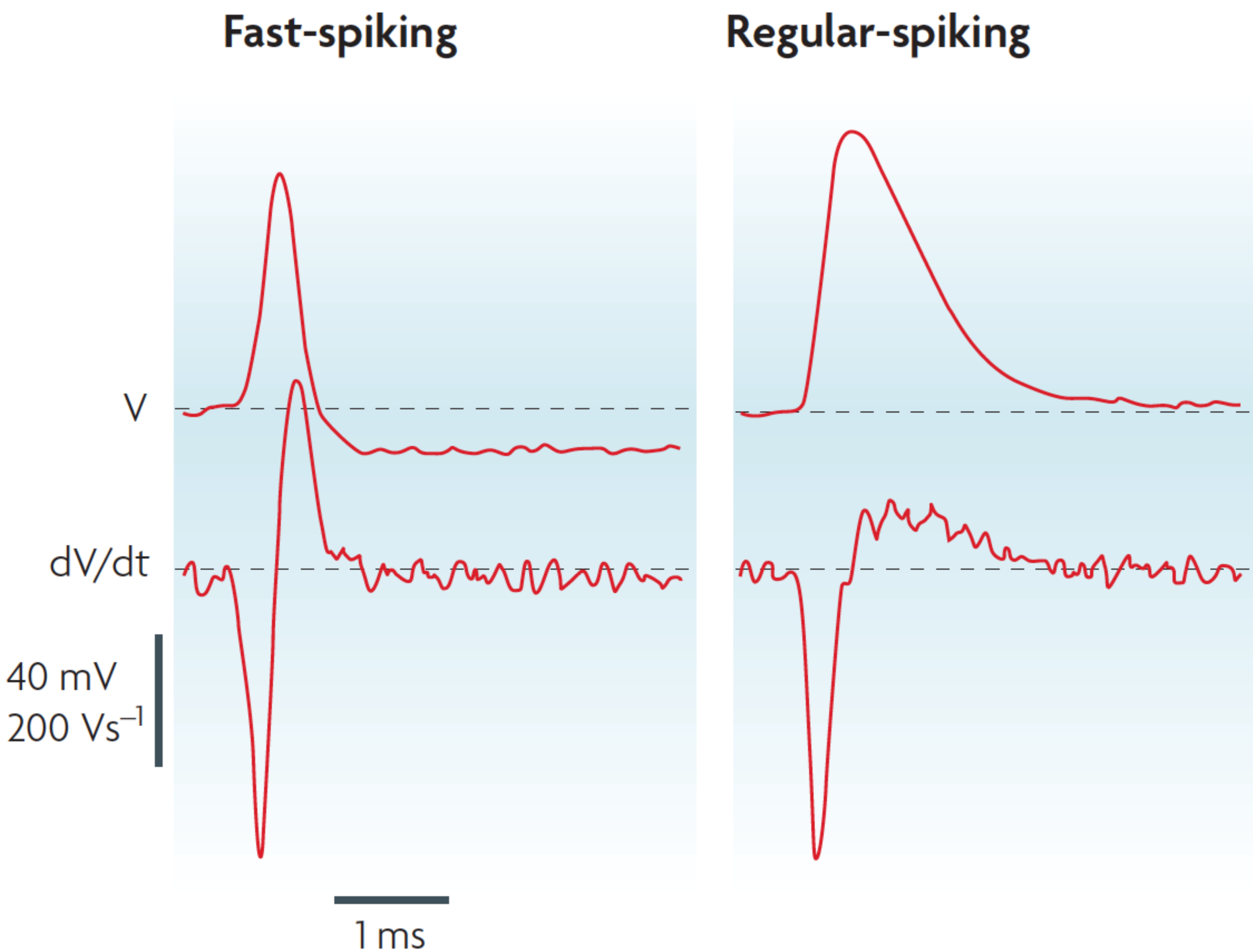
## Spikes are the currency of neuronal computation and communication



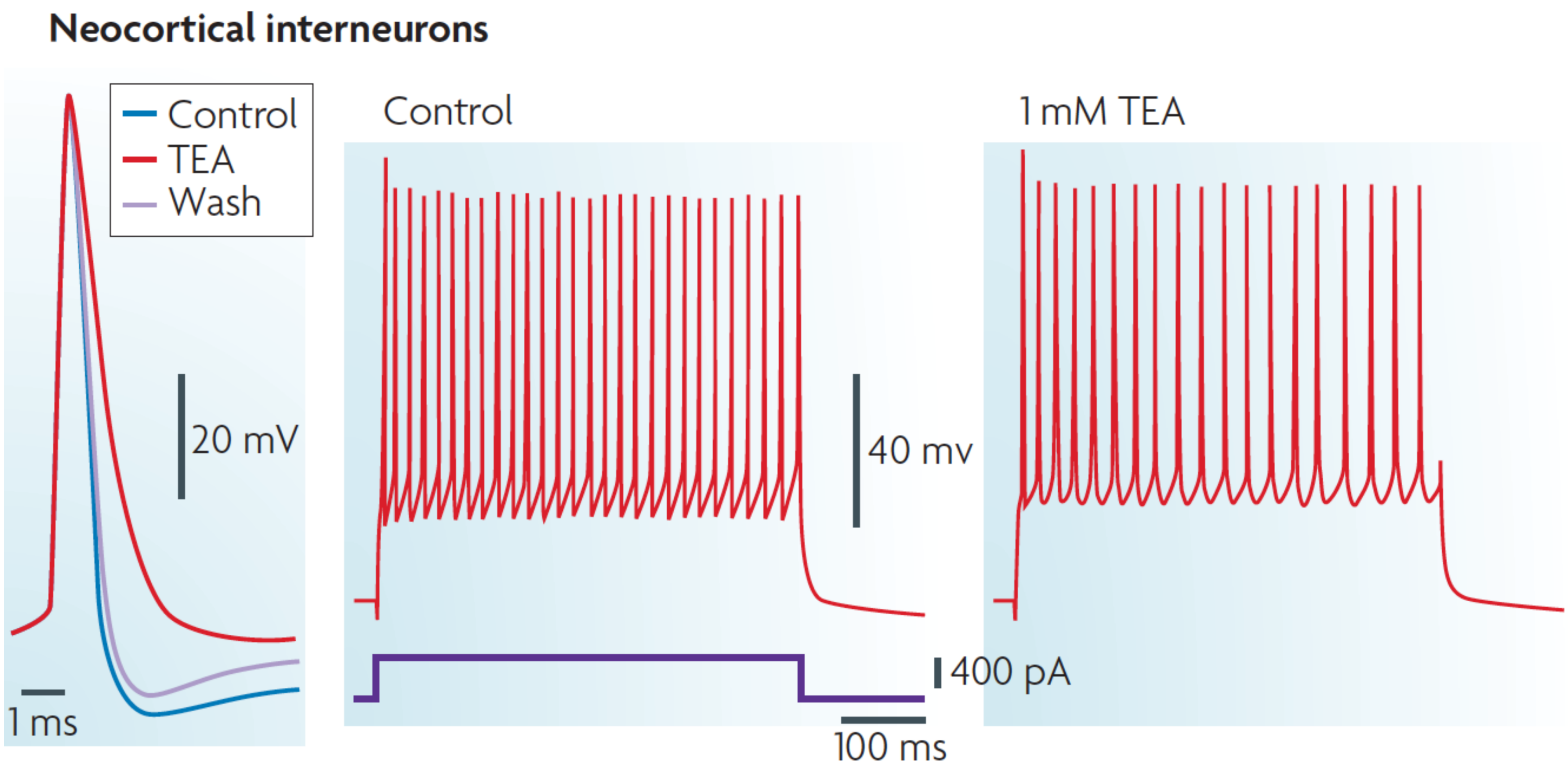

Bellavance, Takatoh, Lu, Demers, Kleinfeld, Wang & Deschênes (Neuron 2017)

## The $Na^+$ current leads to a similar rise across different classes of neurons



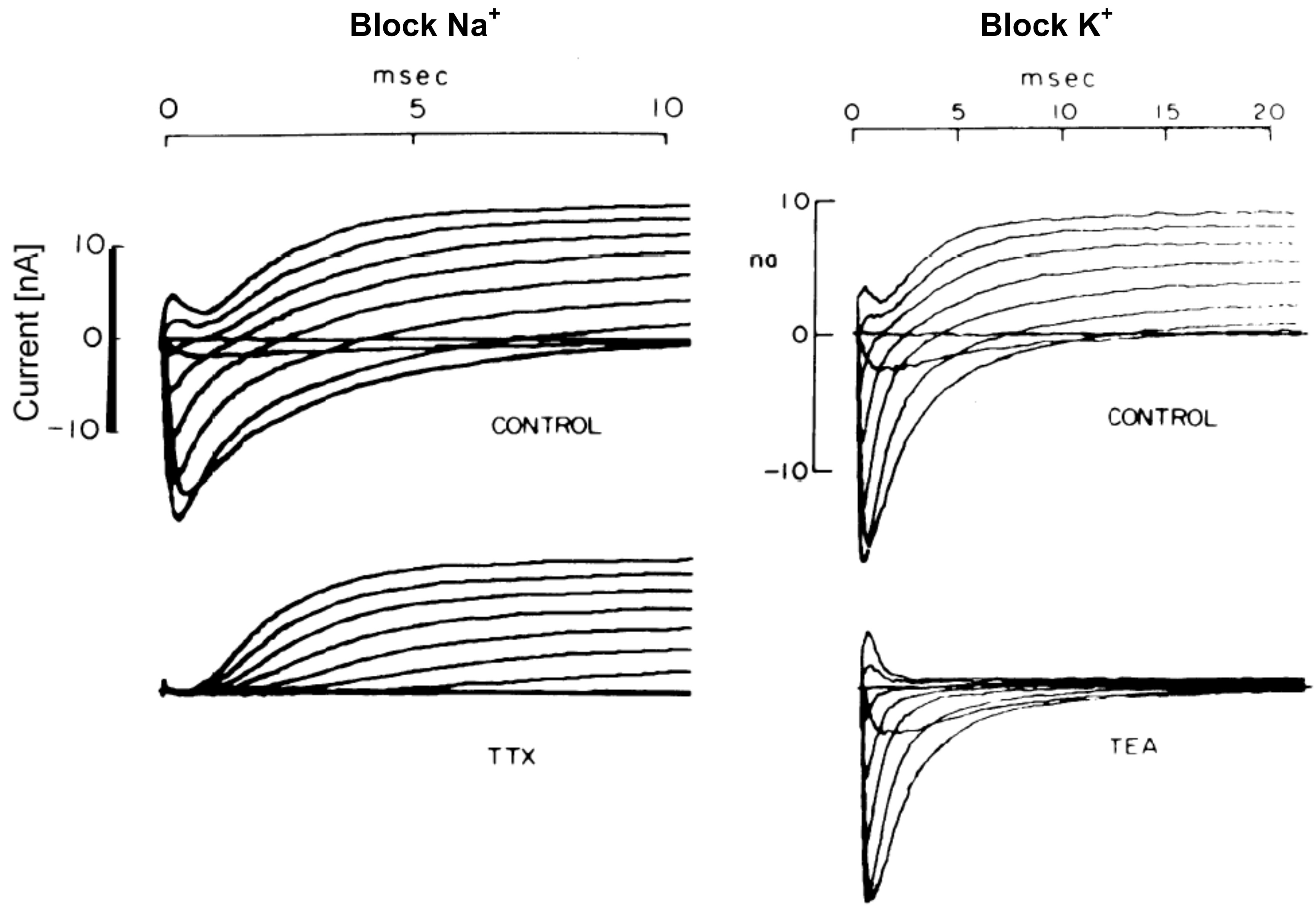

McCormick, Connors, Lighthall & Prince (J Neurophysiol 1985)

## The $K^+$ current is not needed for recovery but shortens - and determines - the recovery time

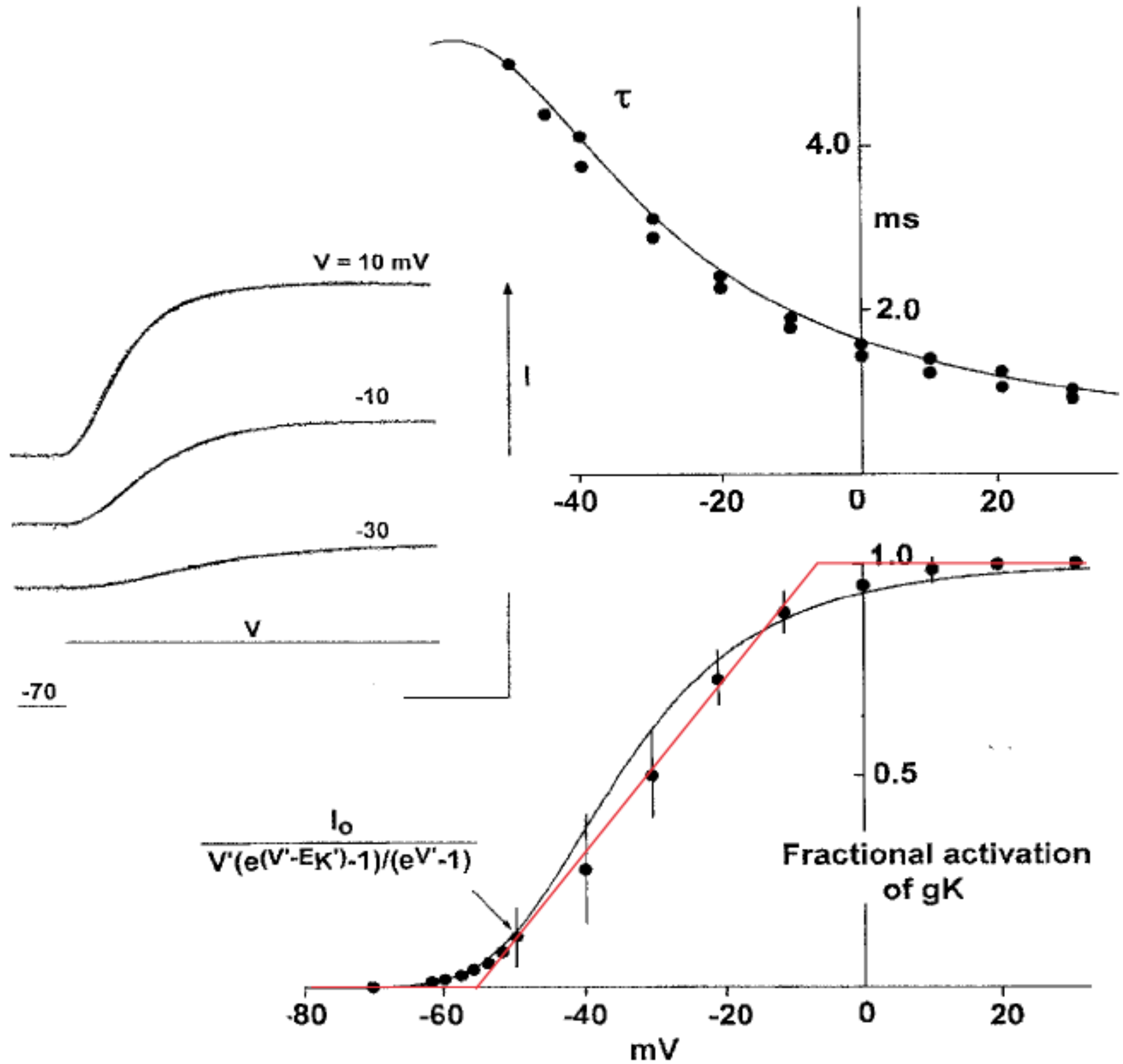



Erisir, Lau, Rudy & Leonard (J Neurophysiol 1999)

## Pharmacology to dissect $K^+$ versus $Na^+$ currents



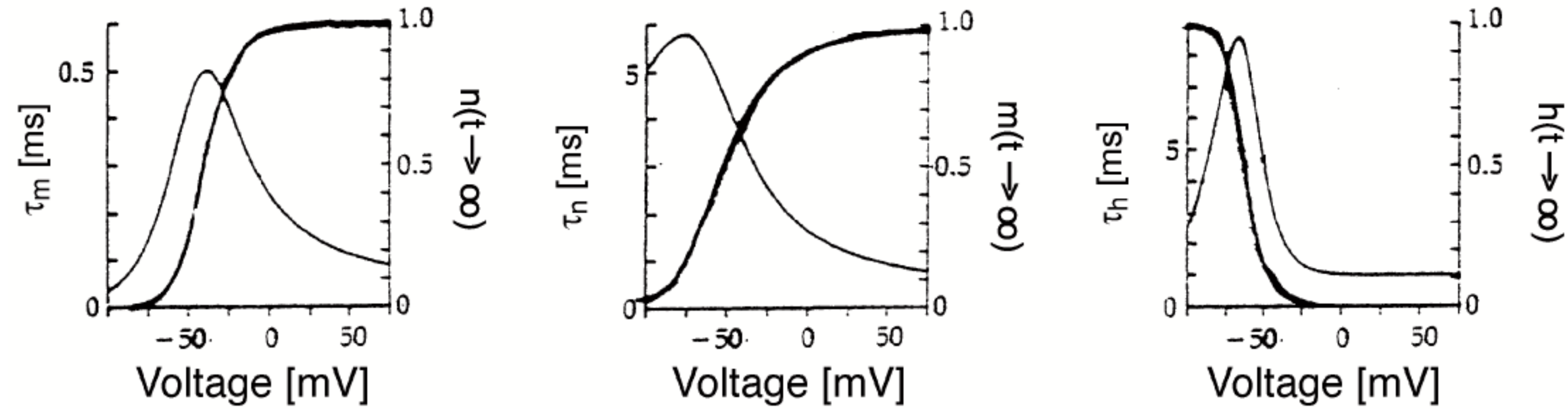
# Fitting model parameters to the data




$$\text{Slope} = dP/dV = \frac{e}{2k_BT} = \frac{nz}{2^{n+1}}\,\frac{e}{k_BT}$$

# Compilation of fits for dynamical model

**$Na^+$ activation** **$K^+$ activation** **$Na^+$ inactivation**

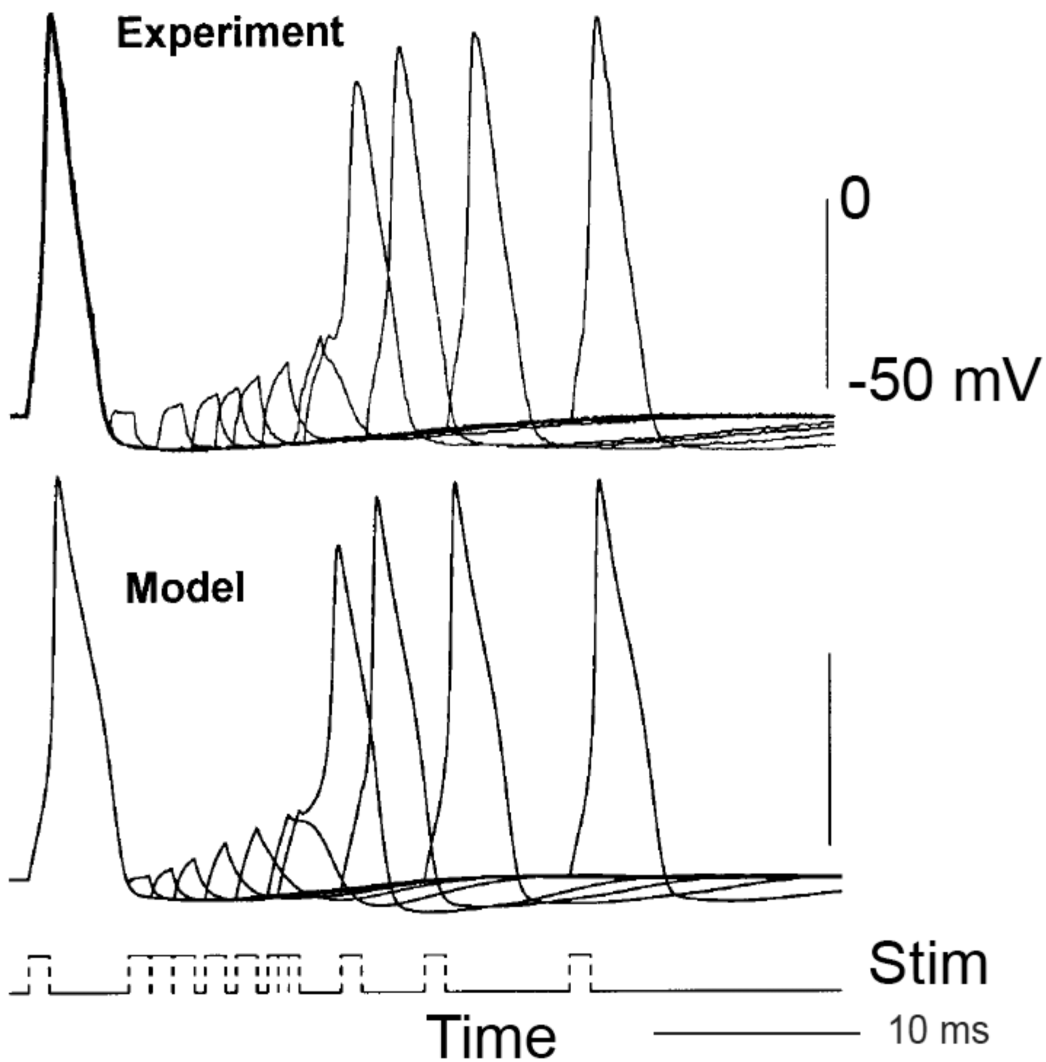



$$\dot{\vec{X}} = \begin{bmatrix} \dot{V} \\ \dot{m} \\ \dot{h} \\ \dot{n} \end{bmatrix} = \vec{F}(\vec{X}) = \begin{bmatrix} f_V(V,m,h,n) \\ f_m(V,m,h,n) \\ f_h(V,m,h,n) \\ f_n(V,m,h,n) \end{bmatrix} = \begin{bmatrix} f_V(\vec{X}) \\ f_m(\vec{X}) \\ f_h(\vec{X}) \\ f_n(\vec{X}) \end{bmatrix}$$

where $f_V(V,m,h,n) = \left[I_{ext} - \overline{G}_{Na} m^3 h (V - E_{Na}) - \overline{G}_K n^4 (V - E_K) - G_L (V - E_L)\right] / C$

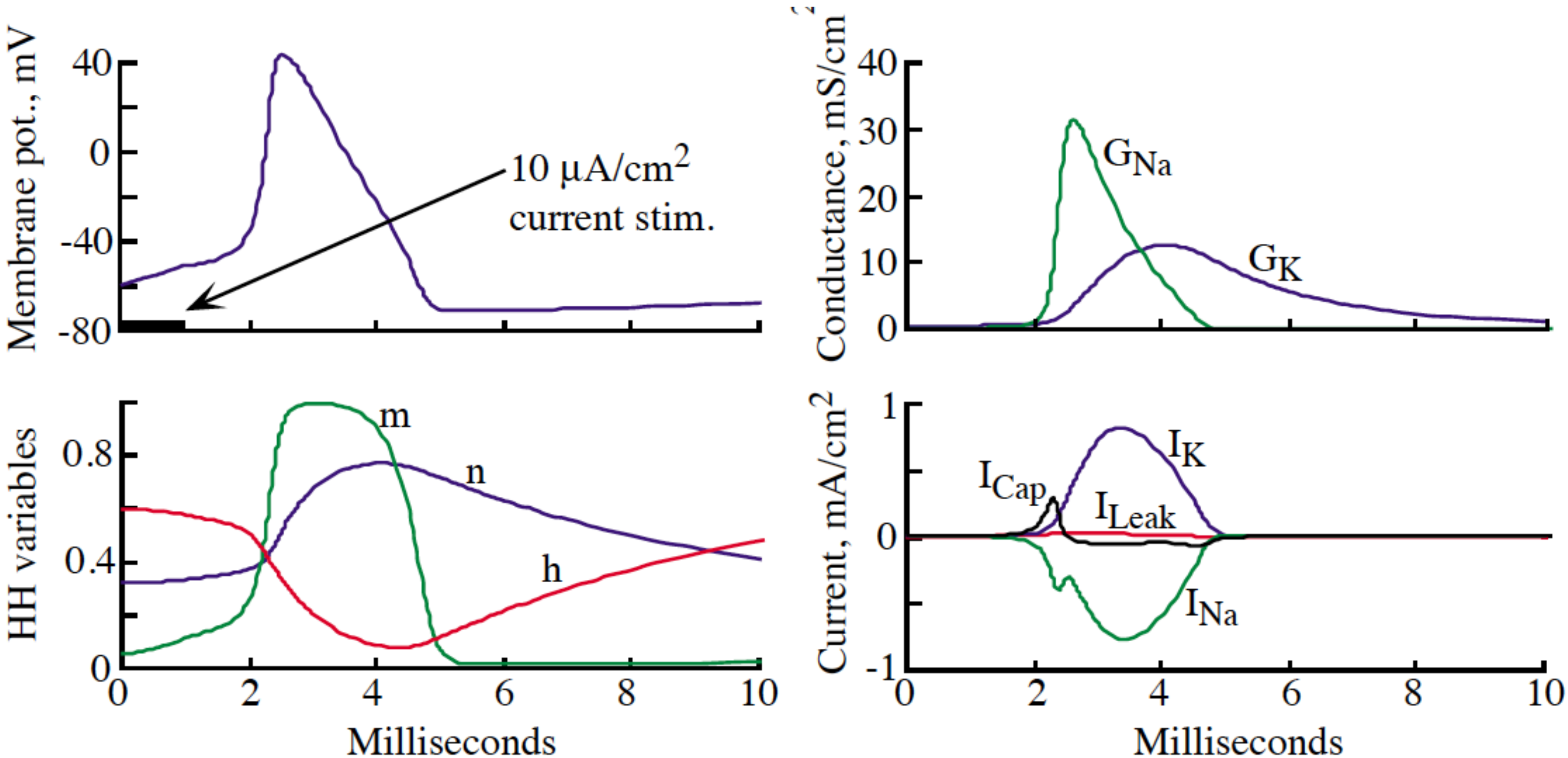
and $f_m(V,m,h,n) = \left[m_\infty(V) - m\right] / \tau_m(V)$.

## The model accounts for the refractory period



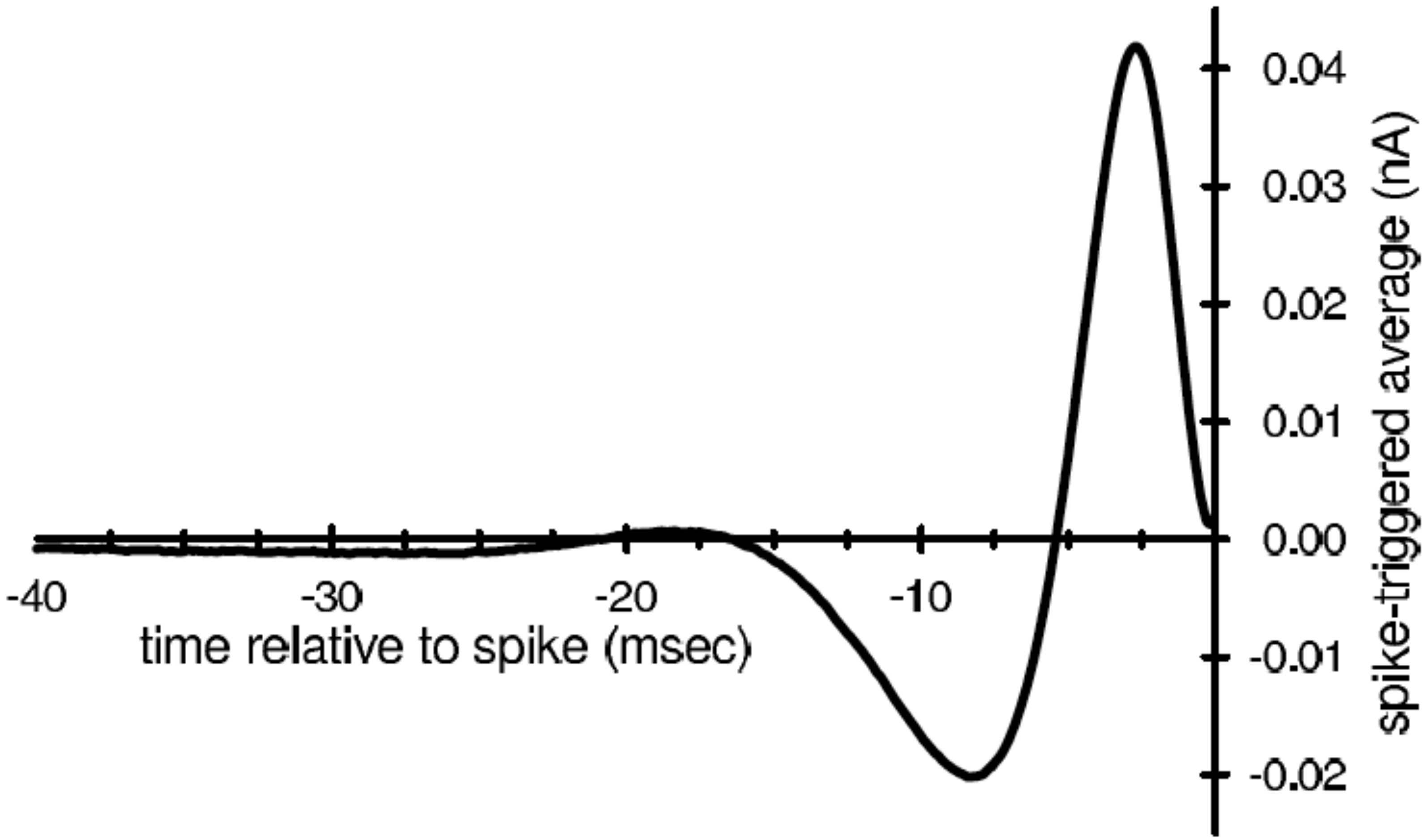

*A* 2 pulse protocol was used with variable times between the 2 pulses between 5 and 28 ms. Pulse duration was 1 ms, pulse amplitude was 30 mA/cm$^2$ .

**The model allows one to estimate ion conductances over time**



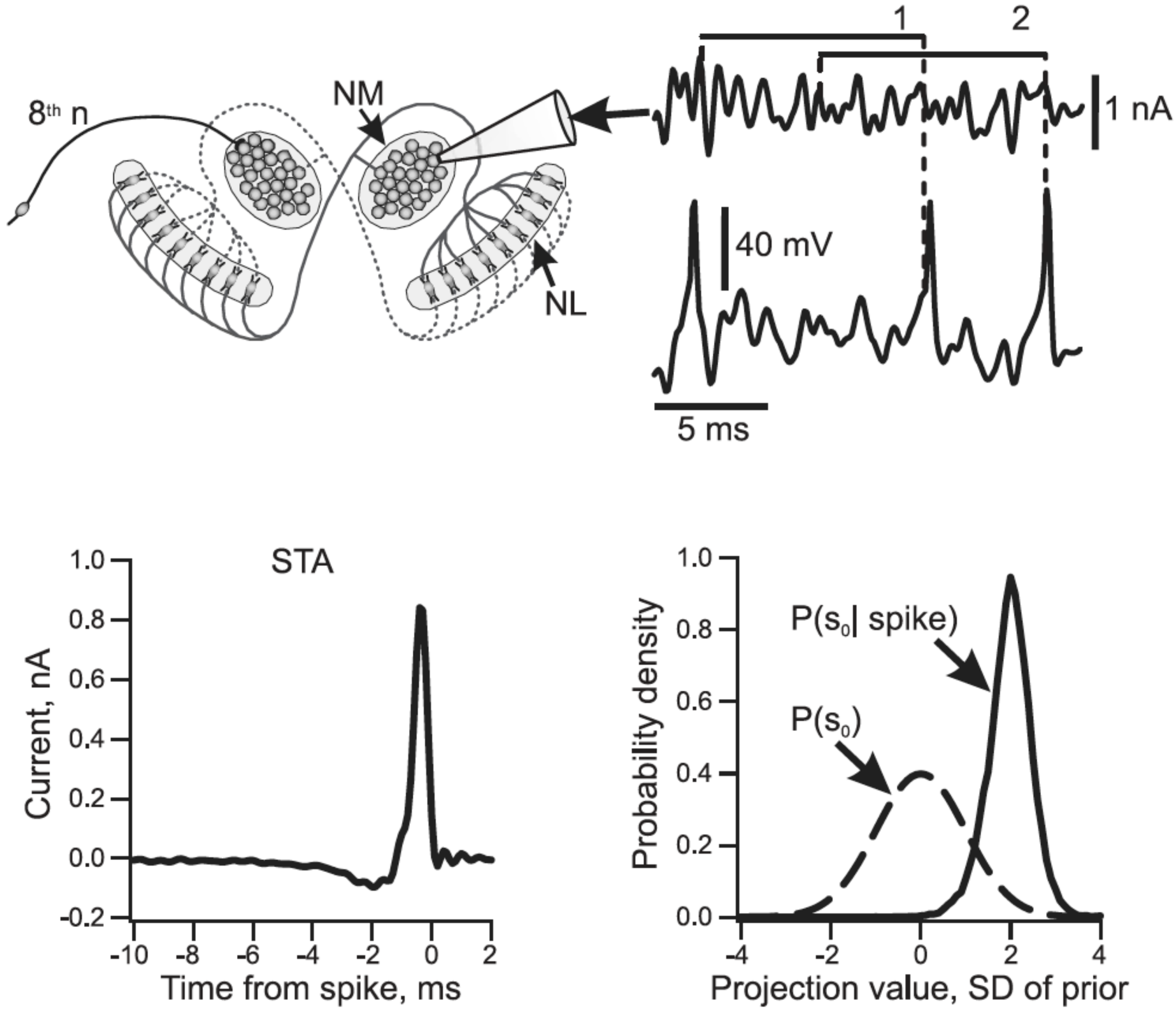
**The model allows us to estimate the "optimal" stimulus to induce a spike**

**(which is your HW exercise!)**



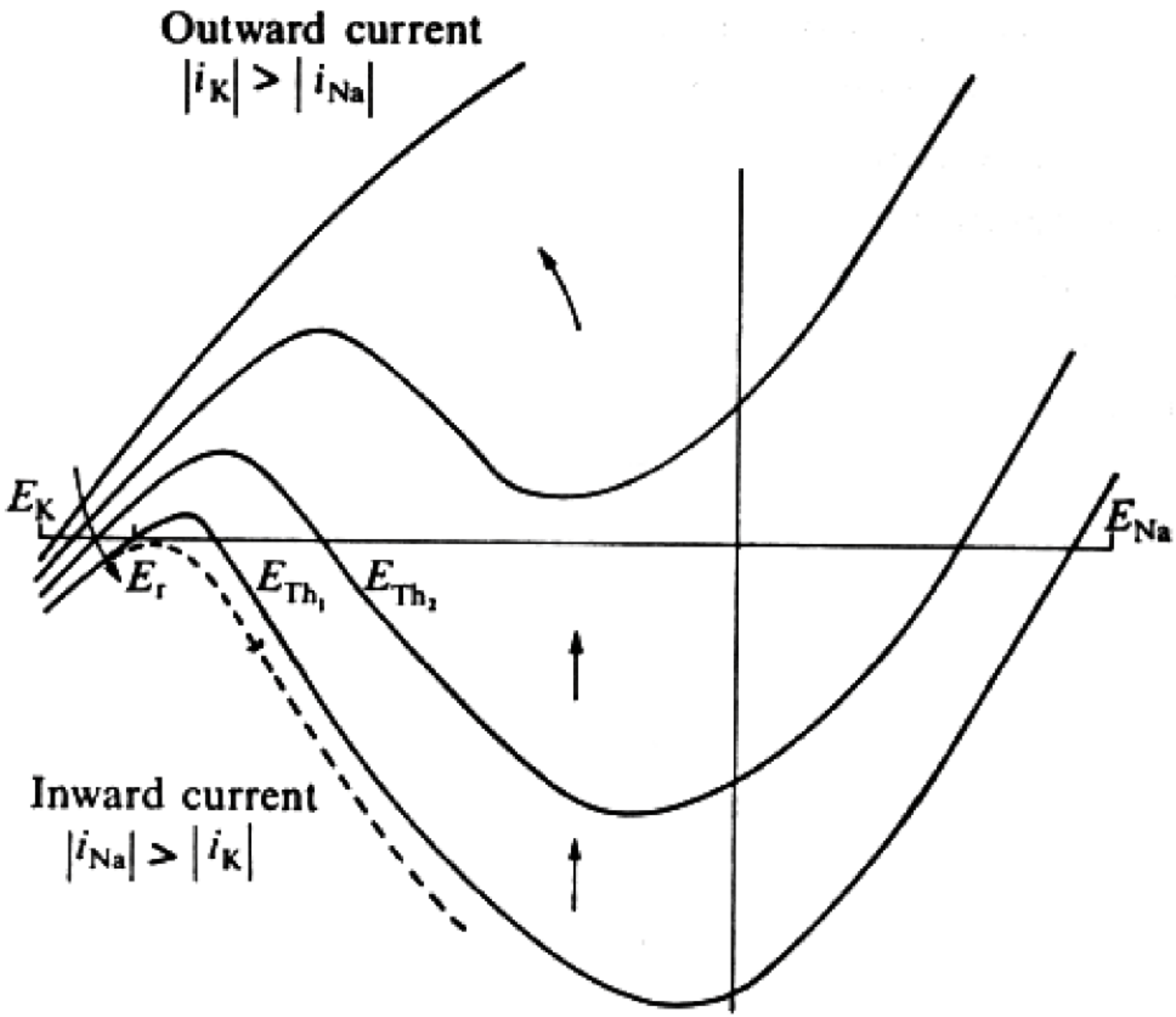

Aguera y Arcas, Fairhall & Bialek (Neural Comp 2003)

## The calculation for a $Na^{+}$-$K^{+}$ cell is not so different from what is seen for a "complex" neuron




Slee, Higgs, Fairhall & Spain (J Neurosci 2005)

Modeling allows one to graph the I-V relation versus time



**Lastly, the model predicts a discontinuous input (current) - output (spike rate) relation**

**BTW, this is modified by the addition of inactivating $K^+$ channels**