a) $$P(x=k) = \frac{\lambda^k}{k!} e^{-\lambda}$$

x_1 and x_2 two i.i.d

$$P(x=k) = \sum_{i=0}^{k} P(X_1 = i) P(X_2 = k-i)$$

$$= \sum_{i=0}^{k} \frac{\lambda_1^i}{i!} e^{-\lambda_1} \frac{\lambda_2^{(k-i)}}{(k-i)!} e^{-\lambda_2}$$

$$= e^{-\lambda_1 - \lambda_2} \sum_{i=0}^{k} \frac{\lambda_1^i \lambda_2^{(k-i)}}{i! (k-i)!}$$

We know $$(\lambda_1 + \lambda_2)^k = \sum_{k=0}^{n} \frac{n!}{k!(n-k)!} \lambda_1^k \lambda_2^{n-k}$$

$$P(x=k) = e^{-(\lambda_1+\lambda_2)} \frac{(\lambda_1+\lambda_2)^k}{k!}$$

b) $$F_N(x) = \prod_{n=1}^{N} f_n(x)$$

$$= \prod_{n=1}^{N} \left(1 + (x-1)p_n\right)$$

$$\ln F_N(x) = \sum_{n=1}^{N} \ln\left(1 + (x-1)p_n\right)$$

$$\lim_{n\to N} \ln F_N(x) = \lim_{n\to} \sum_{n=1}^{N} \ln\left(1 + (x-1)p_n\right)$$

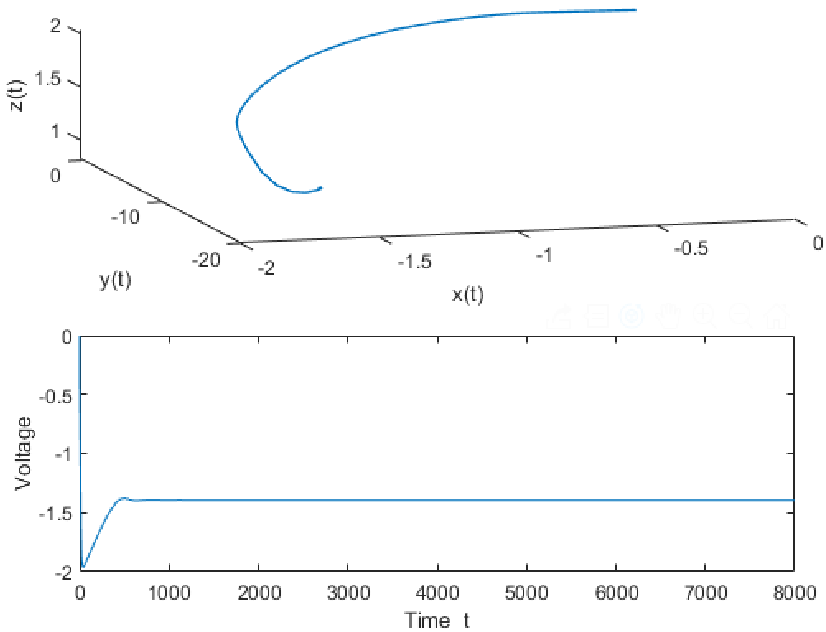
$$= \sum_{n=1}^{N} \lim \ln\left(1 + (x-1)p_n\right)$$

$$= x-1 \sum_{n=1}^{N} p_n$$

$$\lim_{N\to\infty} \sum_{n=1}^{N} p_n = \lambda$$

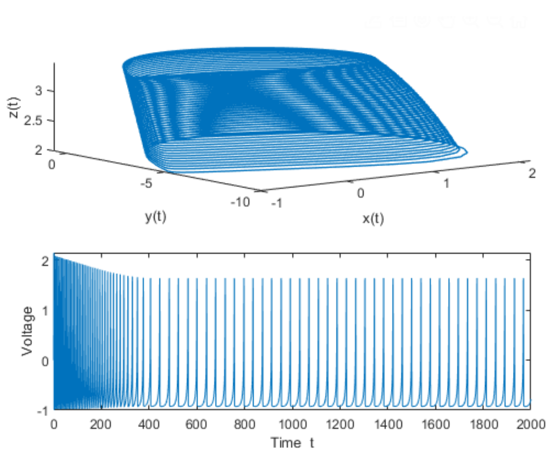
therefore $\lim_{N\to\infty} F_N(x) = \lambda(x-1)$

2.1a) The Hindmarsh-Rose model with ε=0.002 has one stable fixed point for I_{app} = 1 (for example):

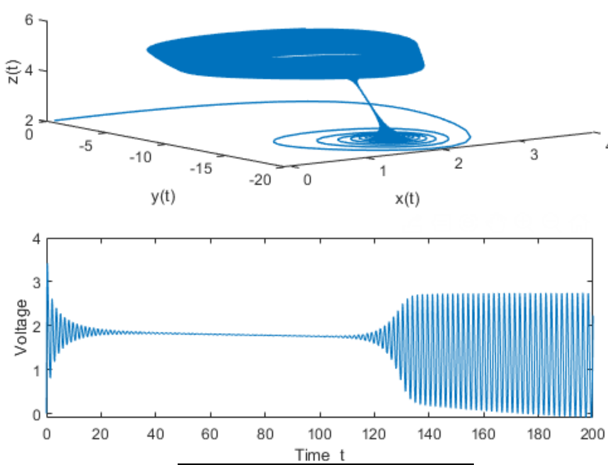


I_{app} = 1, T = 8000

2.1b) The region such that the model has sustained spike trains is approximately 3.38 to 14.72:

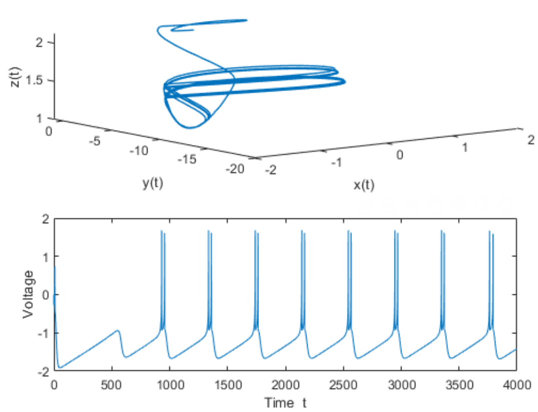


I_{app} = 3.38



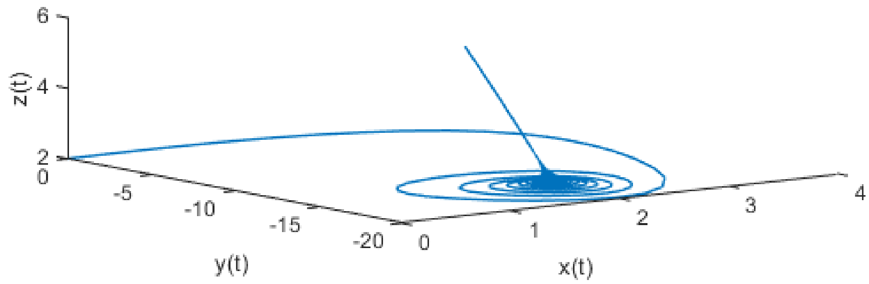
I_{app} = 14.72

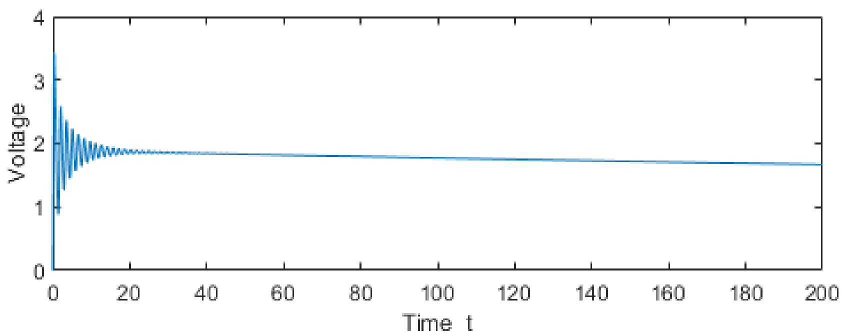
The minimum current such that the model has bursting oscillations is approximately 1.27:



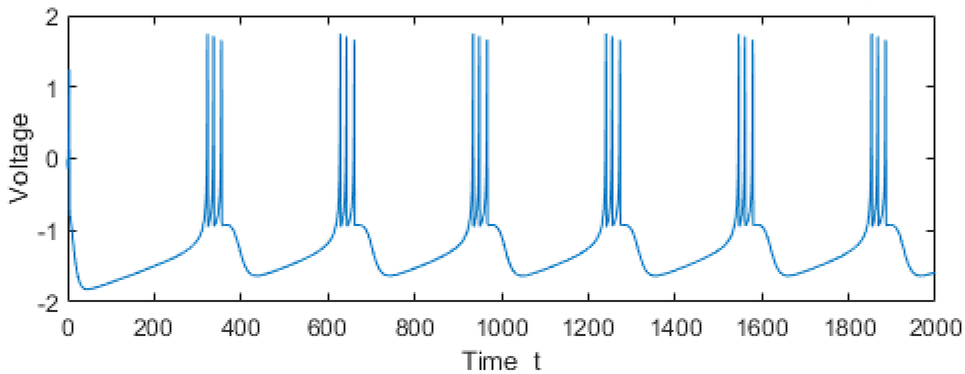
I_{app} = 1.27

2.1c) If I_{app} is too large, the system will have a stable fixed point and will not produce oscillations. Below is the result for I_{app} = 15:

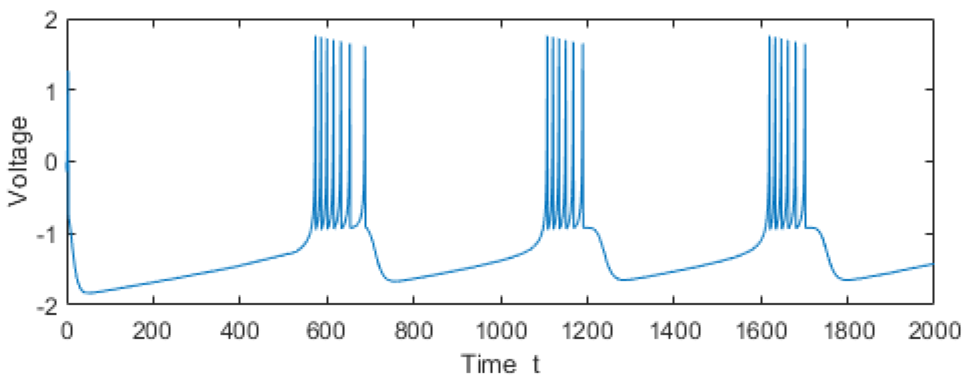




2.2a) When epsilon is decreased from 0.002, there are more spikes per burst and the burst frequency decreases:



ε = 0.002



ε = 0.001

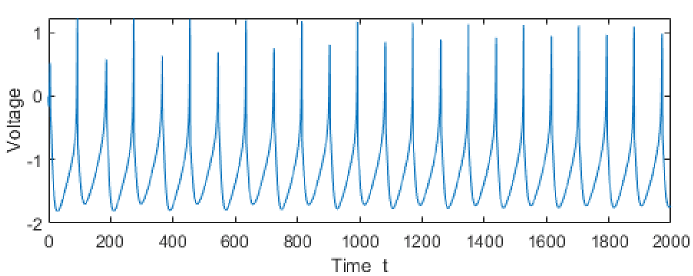
2.2b) ε = 0.005, there are fewer spikes per burst (only 2 as opposed to 3 for ε = 0.002) and burst frequency increased.

ε = 0.01, now there is only one spike per burst, and burst frequency increased again.

ε = 0.02, now there is again only one spike per burst, and now there are two spike amplitudes (every other spike is larger than the first).

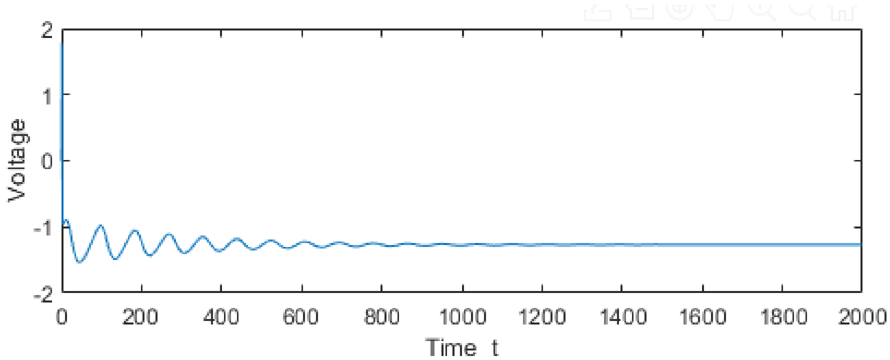
ε = 0.05, now the system is attracted to a stable fixed point and does not oscillate.

2.2c) for ε = 0.02, the model bursts (one spike per burst) with initial conditions [0,0,2]:

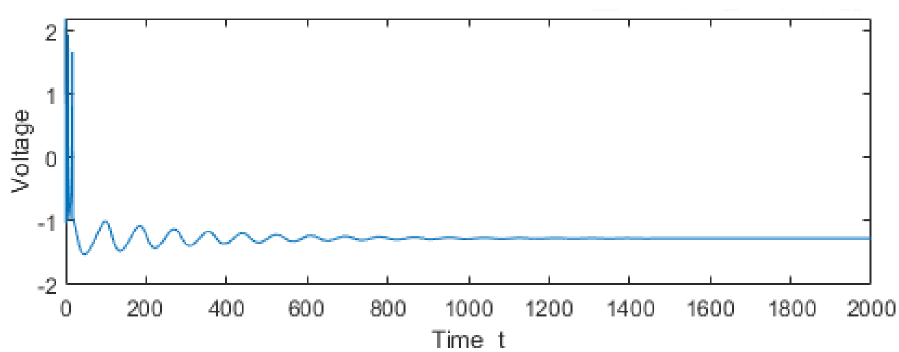


IC = [0,0,2]

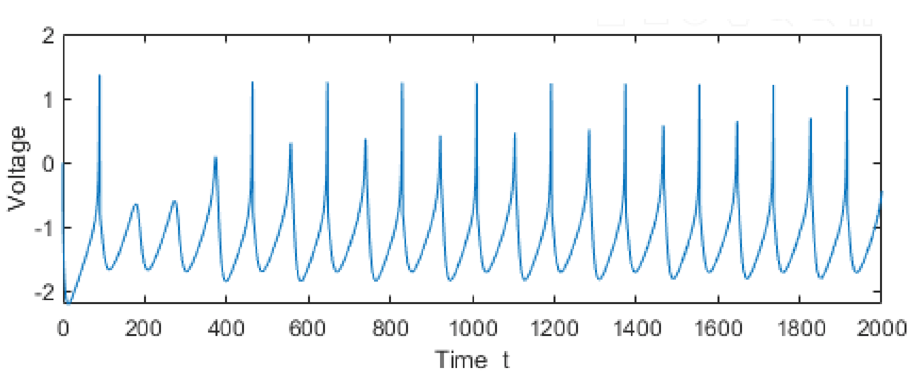
However with other initial conditions, the model does not burst or spike. I could also spike differently than for IC = [0,0,2] (see IC = [0,0,5]). **Yes the behavior does depend on initial conditions.**



IC = [0,0,1]



IC = [1,0,0]



IC = [0,0,5]