Receptive Field

\[
\bar{Z}(t) = g \left[ I_0 + \int d^2 r \int_{-\infty}^{t} dt' R(\bar{r}, t-t') S(\bar{r}, t') \right]
\]

Part of synaptic

\[
g \text{ Instantaneous spike rate}
\]

Receptive Field

when the stimulus driven part is small, compared to the background \( B_0 \)

\[
\bar{Z}(t) = g \left( I_0 + \frac{d\bar{z}}{dt} \right) + g' \int d^2 r \int_{-\infty}^{t} dt' R(\bar{r}, t-t') S(\bar{r}, t')
\]

Part of \( \bar{z} \) spikes from \( \bar{z}_0 \)

\[
\bar{z}_0 \quad \text{and one spike at time } t = S_0 \delta(t - t_0) \delta(\bar{r})
\]

for a Poisson process

\[
\text{Modulation in firing rate by convolution of stimulus with receptive field}
\]

\[\text{For } R(\bar{r}, t) \equiv \sum_n \frac{F_n(\bar{r}) \delta(t)}{n} \text{ modes of receptive field}\]

\[\bar{z}(t) = \bar{z}_0 + g' \int d^2 r \int_{-\infty}^{t} dt' F(\bar{r}) \int_{-\infty}^{t} dt' G_n(\bar{r}, t-t') S(\bar{r}, t') \]

\[\text{For } S(\bar{r}, t) = X(\bar{r}) \delta(t) \text{ (In practice, rapidly changing stimulus)}\]

\[\bar{z}(t) = \bar{z}_0 + g' \int d^2 r \int_{-\infty}^{t} dt' F(\bar{r}) X(\bar{r}) \]

Rate depends on spatial pattern
Let us reverse process and ask if we can reconstruct a stimulus from the spike train.

For simplicity, let us ignore space. The previous description of receptive field gives:

\[ Z(t) = Z_0 + g \int_{-\infty}^{t} R(t-t') S(t') \mathrm{d}t' \]

Averaged and smoothed spike train

\[ \tilde{Z}(w) = Z_0 \tilde{S}(0) + g \int \tilde{R}(w) \tilde{S}(w) \mathrm{d}w \]

\[ \tilde{S}(w) = \frac{\tilde{Z}(w) - Z_0 \tilde{S}(0)}{g \tilde{R}(w)} \]

Optimal Reconstruction

How well can we reconstruct stimulus from a given spike train?

Let \( \Lambda_x(t) = \sum \delta(t-t_s) \) for \( x \)-trial

\[ S_x(t) = \int_{-T}^{t} \mathrm{d}t' \tilde{T}(t-t') \Lambda_x(t) \]

Transfer Function \((\tilde{T}(w) \equiv \tilde{R}(w)/\tau)\)

Compare \( S_{\text{actual}}(t) \) vs. \( S_{\text{predict}}(t) \)

Integrated Error

\[ = \sum_t \int_{-T}^{t} \left[ S_{\text{actual}}(t) - S_{\text{predict}}(t) \right]^2 \]

(For \( \tilde{T}(w) \), the different \( w \)'s are uncorrelated.)
$$E_{w} = \sum_{x} \left[ \tilde{S}_{o}^{\text{pred}}(w) - S_{o}(w) \right]^2$$

$$'' = \sum_{x} \left[ \hat{T}^{*}(w) \hat{A}_{o}(w) - \tilde{S}_{o}^{\text{pred}}(w) \right]^2$$

$$1 = \sum_{x} \left[ \hat{T}^{*}(w) \hat{A}_{o}(w) \hat{A}_{o}^{*}(w) + \tilde{S}_{o}^{\text{pred}}(w) \tilde{S}_{o}^{\text{pred}}(w) - \hat{T}^{*}(w) \hat{A}_{o}(w) \tilde{S}_{o}(w) \right]$$

$$\frac{2S}{\hat{T}^{*}(w)} = \sum_{x} \left[ \hat{T}^{*}(w) \hat{A}_{o}^{*}(w) - \hat{A}_{o}(w) \tilde{S}_{o}(w) \right]$$

holds for each value of $\hat{T}^{*}(w)$.

Let $\frac{2S}{\hat{T}^{*}(w)} = 0$. Therefore $\hat{T}(w) = \frac{\sum_{x} \hat{A}_{o}^{*}(w) \tilde{S}_{o}(w)}{\sum_{x} \hat{A}_{o}(w) \tilde{S}_{o}(w)}$.

So really differentiation w.r.t. a scalar.

$$\text{And } \tilde{S}_{o}^{\text{pred}}(w) = \hat{T}(w) \hat{A}_{o}(w)$$

Best Filter
Notes on Singular Value Decomposition

\[ R(t,t') = \sum_n \lambda_n F_n(t) G_n(t') \]

with \( \int d^2r F_n(r) F_m(r) = \delta_{nm} \quad \text{Orthogonal Functions} \)

\( \int dt G_n(t) G_m(t') = \delta_{nm} \)

Consider contraction to symmetric correlation matrix

\[ C(t,t') = \int d^2r R(t,r) R(r,t') \]

\[ = \sum_n \sum_m D_n^2 \delta_{nm} \int d^2r F_n(r) F_m(r) G_n(t) G_m(t') \]

\[ = \sum_n D_n^2 G_n(t) G_n(t') \]

Then \( \int dt' C(t,t') G_m(t') = \sum_n D_n^2 G_n(t) \int dt' G_n(t') G_m(t') \)

\[ = D_n^2 G_n(t) \quad \text{Eigenvalue Problem} \]

The \( F_n(t) \) are found by

\[ \int dt R(t,r) G_m(t) = \sum_n F_n(r) \int dt G_n(t) G_m(t') \]

\[ = F_m(t) \]

Bottom line: space-time modes from measured RF