Receptive Field

\[ Z(t) = g \left[ I_0 + \int d^2r \int_{-\infty}^{t} dt' R(\mathbf{r}, t-t') S(\mathbf{r}, t) \right] \]

Probability of no spikes in the interval \([0, t]\) and one spike in \((t, t+dt)\) is

\[ e^{-\int_0^t dt' Z(t')} Z(t) \]

for a Poisson process

For \( R(\mathbf{r}, t) = \sum_n \lambda_n F_n(\mathbf{r}) G_n(t) \)

Modulation of firing rate by convolution of stimulus with receptive field

For \( S(\mathbf{r}, t) = \chi(\mathbf{r}) S(t) \) (in practice rapidly varying stimulus)

Rate depends on spatial pattern
Let us reverse process and ask if we can reconstruct a stimulus from the spike train.

For simplicity, let us ignore space. The previous description of receptive field gives

\[ Z(t) = Z_0 + \int_{-\infty}^{t} \delta(t-t') R(t-t') S(t') \delta t' \]

\[ \tilde{Z}(\omega) = Z_0 \delta(\omega) + G^* R(\omega) \tilde{S}(\omega) \]

\[ \tilde{S}(\omega) = \tilde{Z}(\omega) - Z_0 \delta(\omega) \]

\[ G^* R(\omega) \]

**Optimal Reconstruction**

How well can we reconstruct stimulus from a given spike train?

Let \( \Lambda_2(t) = \sum_{\text{spikes}} \delta(t-t_s) \) for \( t \rightarrow +\infty \).

\[ S_2(t) = \int dt' T(t-t') \Lambda_2(t') \]

\[ \text{Transfer Function} \quad (\tilde{T}(\omega) \cdot \tilde{R}(\omega))^2 \]

Compare \( S_{\text{actual}}(t) \) vs. \( S_{\text{predicted}}(t) \)

\[ \text{Integrated Error} = \sum_{t} \int dt \left[ S_{\text{actual}}(t) - S_{\text{predicted}}(t) \right]^2 \]

\[ \text{Fourier Transform} \sqrt{\text{different \( \omega \)'s are uncorrelated.}} \]
We shall see next how the filter has a simple form and reduces to the expression for the reverse correlation or spike-triggered average, when the correlation between spikes is zero. This holds for low spike rates; at high rates the refractory period leads to correlations in the rate.

The transfer function is just the cross-correlation, divided by the autocorrelation to correct for trends of correlation among the neuronal response $\Lambda(\omega)$.

We shall see next how the filter has a simple form - and reduces to the expression for the reverse correlation or spike-triggered average, when the correlation between spikes is zero. This holds for low spike rates; at high rates the refractory period leads to correlations in the rate.
How do we interpret the result for the optimal filter? Look at the case of \( \Lambda(w) \) for a spike train.

First - the numerator is just the cross power, or the Fourier transform of the cross-correlation, i.e.

\[
\sum_{w} \Lambda^*(w) \tilde{S}(w) = \sum_{w} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Lambda^*(t) \tilde{S}(t-c) e^{iwt} dt \, dw
\]

But \( \sum_{w} \Lambda(t) \rightarrow \sum_{\text{spike times}} \delta(t-t_s) \)

\[
\therefore \sum_{w} \Lambda^*(w) \tilde{S}(w) = \sum_{\text{spike times}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(t-t_s) \delta(t-c) e^{iwt} dt \, dw
\]

\[
\therefore = \sum_{\text{spike times}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(t-t_s) \delta(t-c) \sum_{\text{spike times}} \delta(t-t_s) \delta(t-c) e^{iwt} dt \, dw
\]

Spike Trigger Stimulus Aug,

Second - the denominator is just the power, or the Fourier transform of the auto-correlation

\[
\sum_{w} |\Lambda(w)|^2 = \sum_{t} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(t-t_s) \delta(t-c) e^{iwt} dt \, dw
\]

\[
\therefore = \sum_{t} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(t-t_s) \delta(t-c) \sum_{\text{spike times}} \delta(t-t_s) \delta(t-c) e^{iwt} dt \, dw
\]

\[
\therefore = \sum_{\text{spike times}} N \delta(t-t_s) \delta(t-c)
\]

\[
\therefore = N
\]
We can write that for the case of uncorrelated spikes

\[
T(t) = \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} e^{i\omega t} T_n(\omega) = \frac{1}{N} \int_{-\infty}^{\infty} \sum_{n=-}\infty^{\infty} e^{i\omega t} T_n(\omega) S_n(\omega)
\]

\[
\hat{S}(\omega) = \frac{1}{N} \sum_{\text{spike}} S(\omega, t)
\]

Reverse Correlation Formula.

This formula is very easy to apply. Want to sum up stimulus at time \( t \) (s) the past for all spikes.

\[
\alpha = 1
\]

\[
\alpha = 2
\]

\[
\vdots
\]

\[
\hat{S} = \sum_{\text{spike}} S(\omega, t)
\]

\[
T(t)
\]

Increasing - \( \alpha \)

Sum of stimuli for all spikes for each value of \( \omega \).
Notes on Singular Value Decomposition

\[ R(t, t) = \sum_n \lambda_n F_n(t) G_n(t) \]

with \( \int \delta r F_n(r) F_m(r) = \delta_{nm} \) \( n \neq m \) \( \text{orthonormal} \)
and \( \int \delta t G_n(t) G_m(t) = \delta_{nm} \) \( n \neq m \) \( \text{functions} \)

Consider contraction to symmetric convolution matrix

\[ C(t, t') = \int \delta r R(r, t) R(r, t') \]

\[ = \sum_n \sum_m D_{nm} \int \delta r F_n(r) F_m(r) G_n(t) G_m(t') \]

\[ = \sum_n D_{nn} G_n(t) G_n(t') \]

Then \( \int \delta t' C(t, t') G_m(t') = \sum_n D_{nm}^2 G_n(t) \int \delta t' G_n(t') G_m(t') \)

\[ = D_{nm}^2 G_m(t) \]

Eigenvalue problem

and the \( F_n(t) \) are found by

\[ \int \delta t R(r, t) G_m(t) = \sum_n F_n(r) \int \delta t G_n(t) G_m(t) \]

\[ = F_n(t) \]

\[ \int \delta t G_n(t) G_m(t) = \delta_{nm} \] \( n \neq m \) 

Bottom line: space-time modes from pseudo RF