The Zeeman Effect in Atomic Mercury
(Taryl Kirk - 2001)

Introduction

A state with a well defined quantum number breaks up into several sub-states when the atom is in a magnetic field. The final energies are slightly more or slightly less than the energy of the state in the absence of the magnetic field. This phenomena leads to a splitting of individual lines into separate lines when atoms radiate in a magnetic field, with the spacing of the lines dependent on the magnitude of the magnetic field. The splitting up of these spectral lines of atoms within the magnetic field is called the Zeeman Effect.

Abstract

The neutral mercury (Hg) atom in its ground state has 80 electrons in the configuration $1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 4f^{14} 5s^2 5p^6 5d^{10} 6s^2$ (where $^{2S+1}L_J$ notation is used) in which the n = 1, 2, 3, and 5 electronic energy levels are completely filled. The optical emission spectrum of Hg results from transitions of the two valence electrons between various excited two-electron configurations. The Hg spectrum therefore has many features in common with the two-electron helium system.

In a helium-like system, the total angular momentum **J** of the atom is determined solely by the total angular momentum to the two valence electrons, since the orbital and intrinsic spin angular momenta of the electrons in the closed-shell, inert core are coupled to zero. In the Russell-Sauders or **LS** coupling scheme, the **l** orbital angular momentum quantum numbers l_1 and l_2 of the two valence electrons are coupled to form a resultant angular momentum quantum number L, and similarly, the intrinsic spin angular momentum quantum numbers S_1 and S_2 are coupled to resultant intrinsic spin angular momentum quantum number S. When the conditions for the LS coupling approximation are satisfied, the operators L and S commute with Hamiltonian operator *H* for the atomic system and the allowed energy levels may be

labeled directly in terms to the angular momentum quantum numbers L and S, but not the individual quantum numbers l_1, l_2, S_1, and S_2. The total angular momentum operator **J** also commutes with *H* and therefore the total angular momentum **J** = **L** + **S** may also be sued to label atomic energy levels.

The angular momentum addition theorem restricts the possible values of an angular momentum quantum number **L** resulting from the sum of two individual angular momenta $\mathbf{l}_1$ and $\mathbf{l}_2$. A similar restriction governs the sum of $\mathbf{S}_1$ and $\mathbf{S}_2$ to form **S**, and the sum of **L** and **S** to form **J**. These angular momentum restrictions may be used to predict the quantum numbers of the low-lying excited states of the neutral Hg system. If one considers only single electron excitations of the $6s^2$ ground state, the lowest configurations should be 6s6p, 6s6d, 6s 7s, 6s 7p, and 6s7d. For a two-electron system, $S_1 = 1/2$ and $S_2 = 1/2$. So that the total intrinsic angular momentum quantum number S of the atom is limited to the values 0 and 1, corresponding to what are called singlet and triplet terms, respectively.

The electric dipole selection rules allow transitions that involve only the following changes:

(1) $\Delta S = 0$
(2) $\Delta L = \pm 1, 0$
(3) $\Delta J = 0, \pm 1,$ but not $J = 0 \not\rightarrow J = 0$
(4) $\Delta M_J = 0, \pm 1,$ but not $M_J = 0 \not\rightarrow M_J = 0$ when $\Delta J = 0$

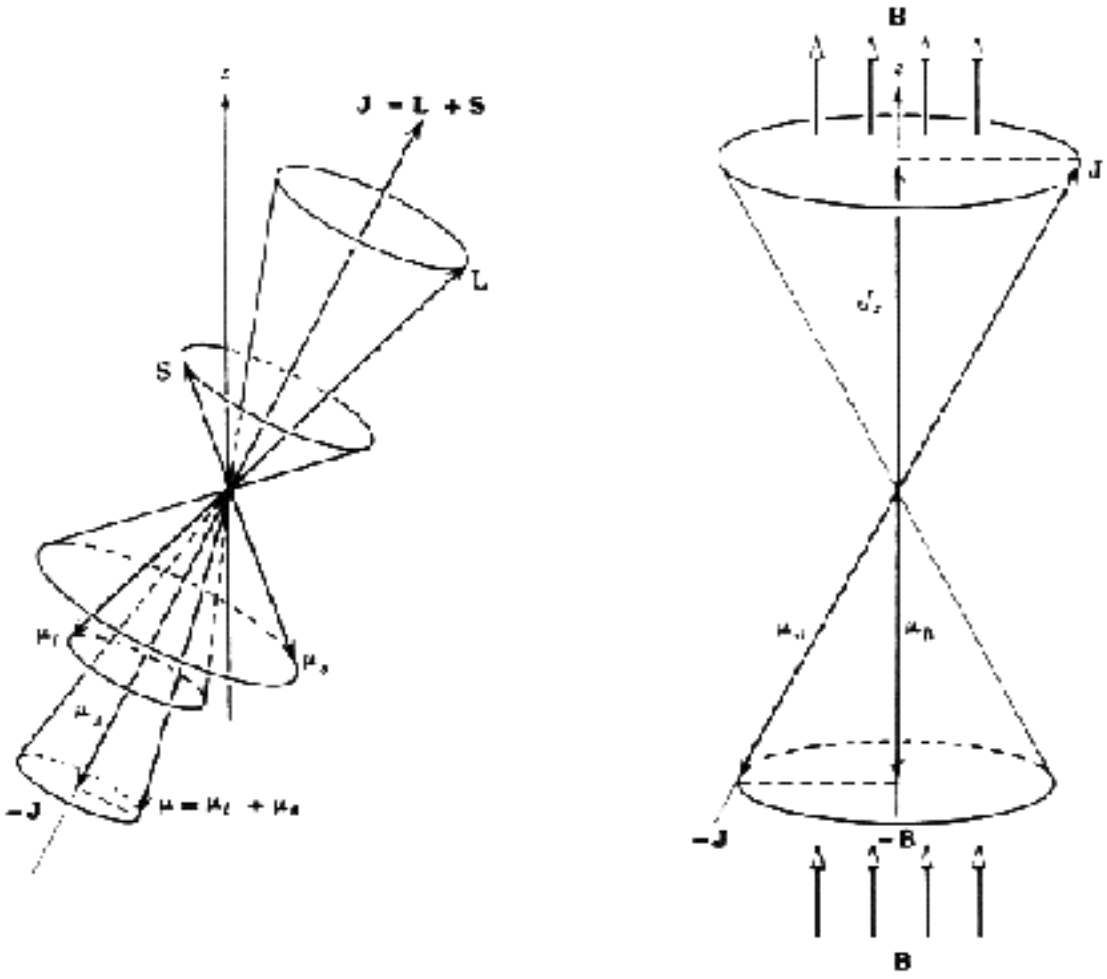


Figure 1. Geometry of the Zeeman effect. On the left, the total dipole moment m precesses around the total angular momentum **J**. On the right, **J** precesses much more slowly about the magnetic field.

The total magnetic dipole moment of the electron is

$$\mu = \mu_1 + \mu_2 = -(\mu_b / h)(\boldsymbol{L} + 2\boldsymbol{S}), \qquad (1)$$

where μ_b is the Bohr Magneton

Because of the difference in the orbital and spin gyromagnetic ratios of the electron, the total magnetic dipole moment is not in general parallel to

$$J = L + S \qquad (2)$$

So, as L and S precess about J, the total dipole moment μ also precesses about J. Assuming the external field to be in the z direction, this field causes J to precess about the z-axis. If the external field is much weaker that 1 Tesla (10,000 Gauss), then the precession of J around the z - axis will take place much more slowly that the precession of μ around J. The Hamiltonian of the Zeeman effect is

$$\Delta H_z = - \mu \cdot B = - \mu_B B, \tag{3}$$

where μ_B is the projection of the dipole moment onto the direction of the field, the z-axis. Because of the difference in the precession rates, it is reasonable to evaluate μ_b by first evaluating the projection of μ onto J, called μ_J, and then evaluating the projection of this onto B, thus giving some average projection of μ onto B. First, the projection of μ onto J is

$$\mu_J = (\mu \cdot J) / J = - (\mu_b / h) [(L + 2S) \cdot (L + S)] / J. \tag{4}$$

Then

$$\mu_B = \mu_J [(J \cdot B) / J B] = \mu_J (J_z / J) = - (\mu_b / h) [(L + 2S) \cdot (L + S) J_z] / J^2 \tag{5}$$

Evaluating the dot product using again that $J^2 = L^2 + S^2 + 2L \cdot S$, this becomes

$$\mu_B = - (\mu_b / h) [(3J^2 + S^2 - L^2) J_z] / 2J^2. \tag{6}$$

So when first order perturbation theory is applied, the energy shift is

$$\Delta E_z = \mu_b B g m_J, \tag{7}$$

where

$$g = 1 + \{[j(j +1) + s(s+1) - l(l+1)] / 2j(j +1)\} \tag{8}$$

is called the Landé *g* factor for the particular state being considered. Note that if S = 0, then j = 1 so *g* = 1, and if l = 0, j = s so *g* = 2. The Landé *g* factor thus gives some effective gyromagnetic ratio for the electron when the total dipole moment is partially from the orbital angular momentum and partially from the spin. From equation (8), it can be seen that the energy shift caused by the Zeeman effect is linear in B and m_j, so for a set of states with particular values of n, l, and j, the individual states with different m_j will be equally spaced in energy, separate by $\mu_b B g$. However, the spacing will in general be different for a set of states with different n, l, and j due to the difference in the Landé *g* factor.

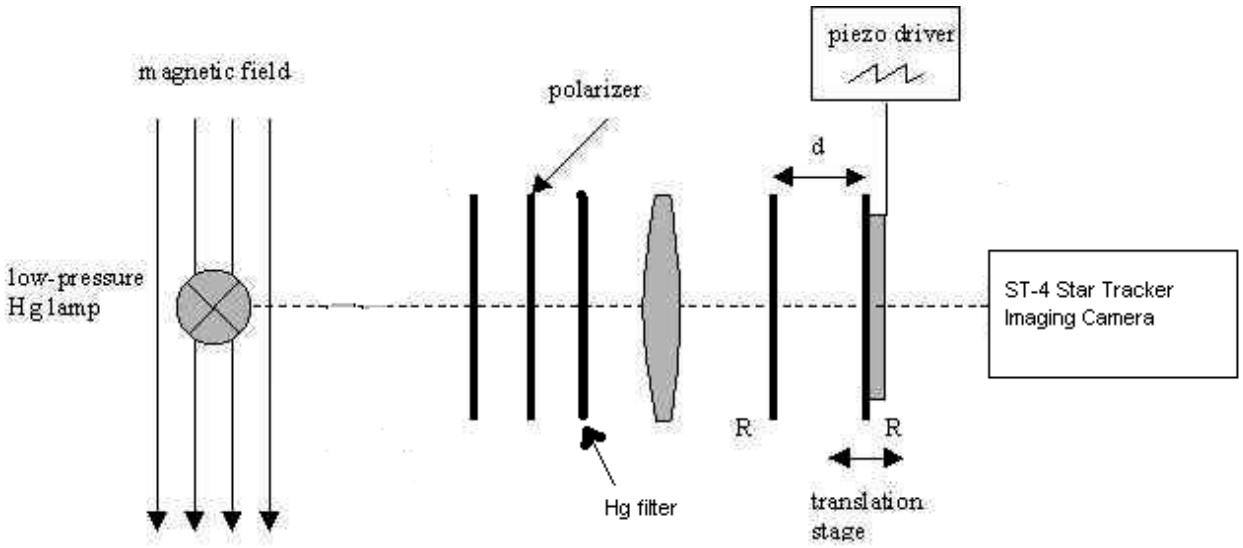


Figure 2. Diagram of the set up for experiment.

Procedure:

(1) Be sure to remove the Hg Pen-Ray lamp from between the electromagnet pole pieces before turning on the magnet power supply. {Note: This is to assure that the Hg lamp is not crushed by the pole pieces as they are drawn together when the magnet current is turned on}.

(2) Turn the electromagnet power supply and advance the magnet current to 20 amps.. With the current at 20 amps., determine if the Hg lamp can be placed between the pole tips. If the lamp cannot be placed between the pole pieces you will have to remove the Hg lamp, turn the magnet current down to zero and rotate the pole piece adjustment so that the pole pieces are farther apart. With the Hg lamp removed, turn the magnet back up to 20 amps. See if the Hg lamp can now be put between the pole pieces.

(3) Measure the magnetic field at the center of the region between the pole tips using the Hall effect guassmeter. {Note: The Hall effect guassmeter should be oriented perpendicular to the magnetic field centered as much as possible.}

(4) Make sure that all the lenses are cleaned. {Note: Use kiwi-wipes and removing the dust of the lenses using a dust-off for lenses is also very helpful.}

(5) Turn on the Mercury lamp. {Note: The best resolution occurs when the magnitude is set at least 80.}

(6) Turn on Model ST-4 Star Tracker Imaging Camera, and make sure there is connection between the camera and program. Adjust the lenses used to transport light from the Hg lamp to the ST-4 Star Tracker. Focus the camera using the CCD OPS 4.21 program. {Note: Use the diagram to set up the lenses.}

(7) Obtain spectra for the following Hg line: 4358.4 Å $^3S_1$ Æ $^3P_1$ (blue light) electron triplet coupling

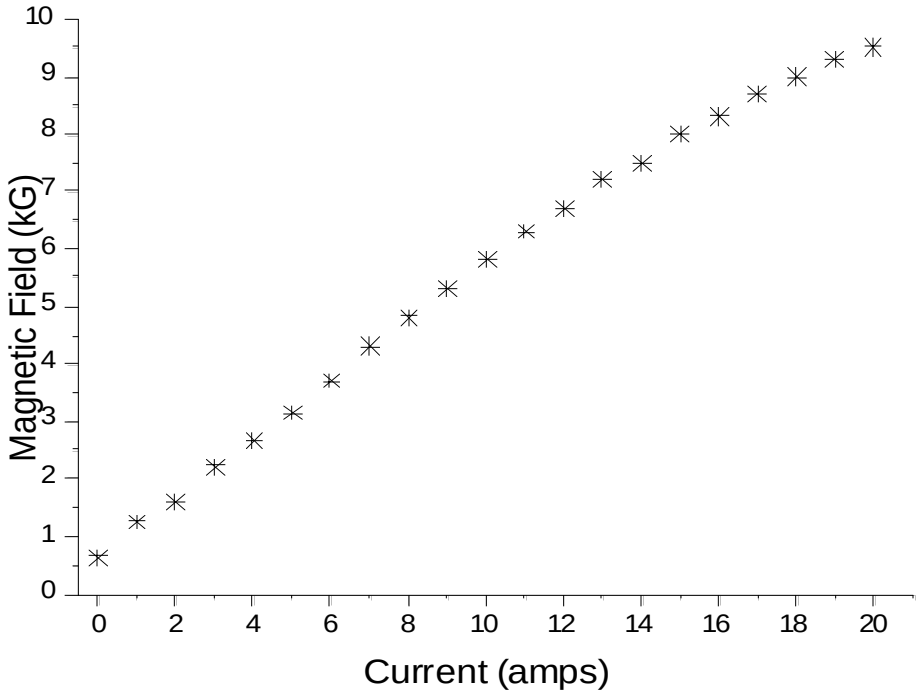


Figure 3. calibration for magnetic

This transition is to be observed under the following conditions:

(a) magnet off / no polarizer,
(b) magnet current at 18 amps. / no polarizer,
(c) magnet current at 16.5 amps. / polarizer aligned for transmission of light that has linear polarization parallel to B,
(d) magnet current at 17 amps. / polarizer aligned for transmission of light that has linear polarization perpendicular to B.
(e) observe rings for no polarizer and linear polarization oriented both parallel and perpendicular to B, with the magnetic current ranging from 0-18 amps.

(8) When step (7) is completed, immediately remove the Hg lamp and measure the magnetic field between the pole tips with the magnet current set at 20 amps.

Analysis.

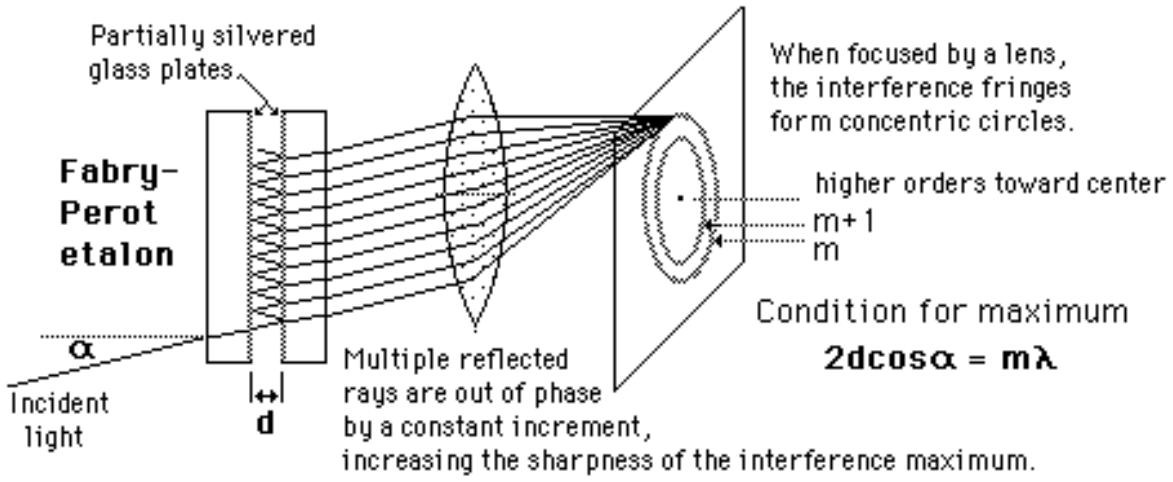
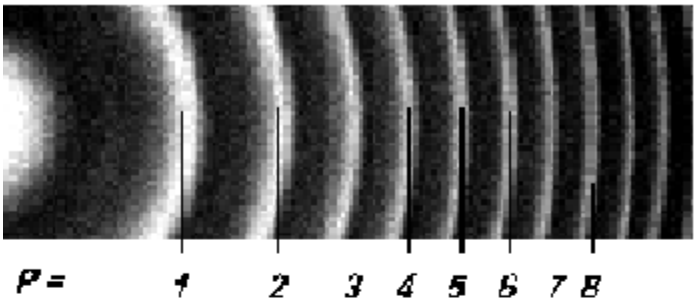


Figure 4. This diagram illustrates how the rings are produced by the Fabry-Perot etalon.



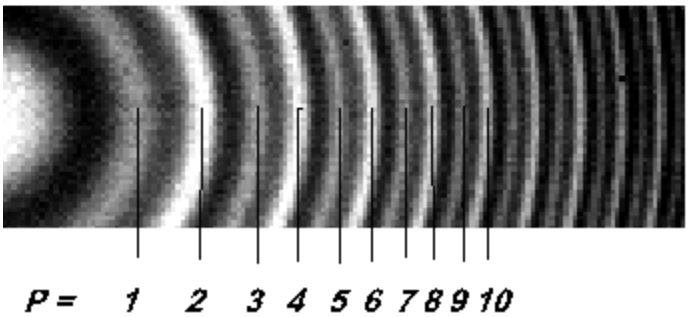


Figure 5. These are interference ring patterns formed at 0 Gauss (on top) and 9 kiloGauss (on bottom). The radii of the associated orders are to be measured from the center of the bands as shown above.

(1) The initial step in the reduction of the data is the measurement of the diameters (or radii) of the rings. A chart should be made of the radii of the Fabry-Perot Patterns:

Ring P	Radius R_P (cm)	R^2_P (mm^2)	$(R^2_{P+1} - R^2_P)$ (mm^2)

(2) Then the data has to be broken down to its components:

Component	Ring numbers				
	1		2	3	4 . . . etc.
a	R^2_{1a}	D^1_{12}	R^2_{2a} . . . *etc.*		
	$\_^1_{ab}$		$\_^2_{ab}$		
b	R^2_{1b}	. . . *etc.*			
.					
.					
.					
etc.					

Footnote: $D_a = R^2_{(P+1),a} - R^2_{P,a}$ & $\_^1_{ab} = R^2_{1a} - R^2_{1b}$

(3) Calculate: Dn bar (where Dn bar = $(\_^P_{ab}/D) \bullet (1/2t)$, where t is the spacing in between the plates of the Fabry-Perot interferometer (normally $t \approx 0.5002$ cm) and $D = \langle D \rangle = (1/k)(D_{12} + D_{23} + ... + D_{k,k+1})$.

(4) Using the value of B from the magnet's calibration and $g(^3S_1) = 2$, determine the value for $g(^3P_1)$ and its uncertainty. Compare your value for $g(^3P_1)$ to the Landé g-factor prediction of equation (8).

(5) For the transition $^3S_1$ ---> $^3P_1$, discuss briefly the spectra taken with the linear polarizer at the various fields.