University of California, San Diego

"Experiment of Modern Physics"
Zeeman Effect

Phys 173
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Zeeman Effect

Theory:

The Fine structure of the hydrogen atom in the absence of an external magnetic field has energies:

$$E = -\frac{Z^2 \alpha^2 m_e c^2}{n^2} - \frac{(Z^2 \alpha^2)^2}{2} \left[3 - \frac{4n}{j+½} \right]$$

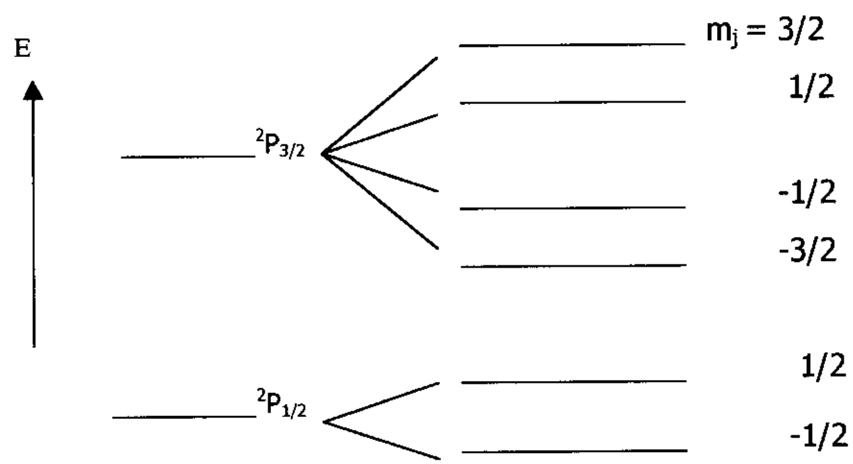
where Z is the atomic number, α is a constant (1/137), and n and j are usual quantum numbers.

Recall total angular momentum, J, is the sum of the total momentum number L plus the total spin number.

$J = L + S, \quad |L-S| \le j \le |L+S|$

Each of the j^{th} state has 2j+1 degenerate states m_j, $-j \le mj \le j$.

When a weak external magnetic field is acting upon the hydrogen atom, the magnetic moment of the circulating electron and the intrinsic dipole moment of the spin interact with the magnetic field resulting in a split of the energy into non-degenerate levels (see figure 1).



The interaction with the external magnetic field has the following Hamiltonian:

$$H_{zeeman} = \frac{e}{2mc} (L + 2S)\, \mathbf{B}$$

Using First order perturbation theory, the energy shifts have magnitude:

$\Delta E = g \mu_o B m_j$, where $\mu_o = \frac{e h}{4\pi mc}$ and $g = 1 + \frac{j(j+1)+s(s+1)-L(L+1)}{2j(j+1)}$

The spectrum of mercury under a weak magnetic field can be used to show the linear relationship between the energy splitting and the magnitude of the magnetic field and therefore to determine the experimental values of μ_o.

The mercury atom has two electrons outside a closed shell with following configuration: $1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 4f^{14} 5s^2 5p^6 5d^{10} 6s^2$. Having two valence electrons results in configuration similar to that of the helium atom with spin number s=1 and total angular momentum j, with values: j= L +1, L, L- 1.

The different energy transitions between energy states satisfy the following rules:

$\Delta L = \pm 1$
$\Delta j = 0, \pm 1$, except transitions from j=0 to j=0.
Transitions between triplet and singlet states are not allowed

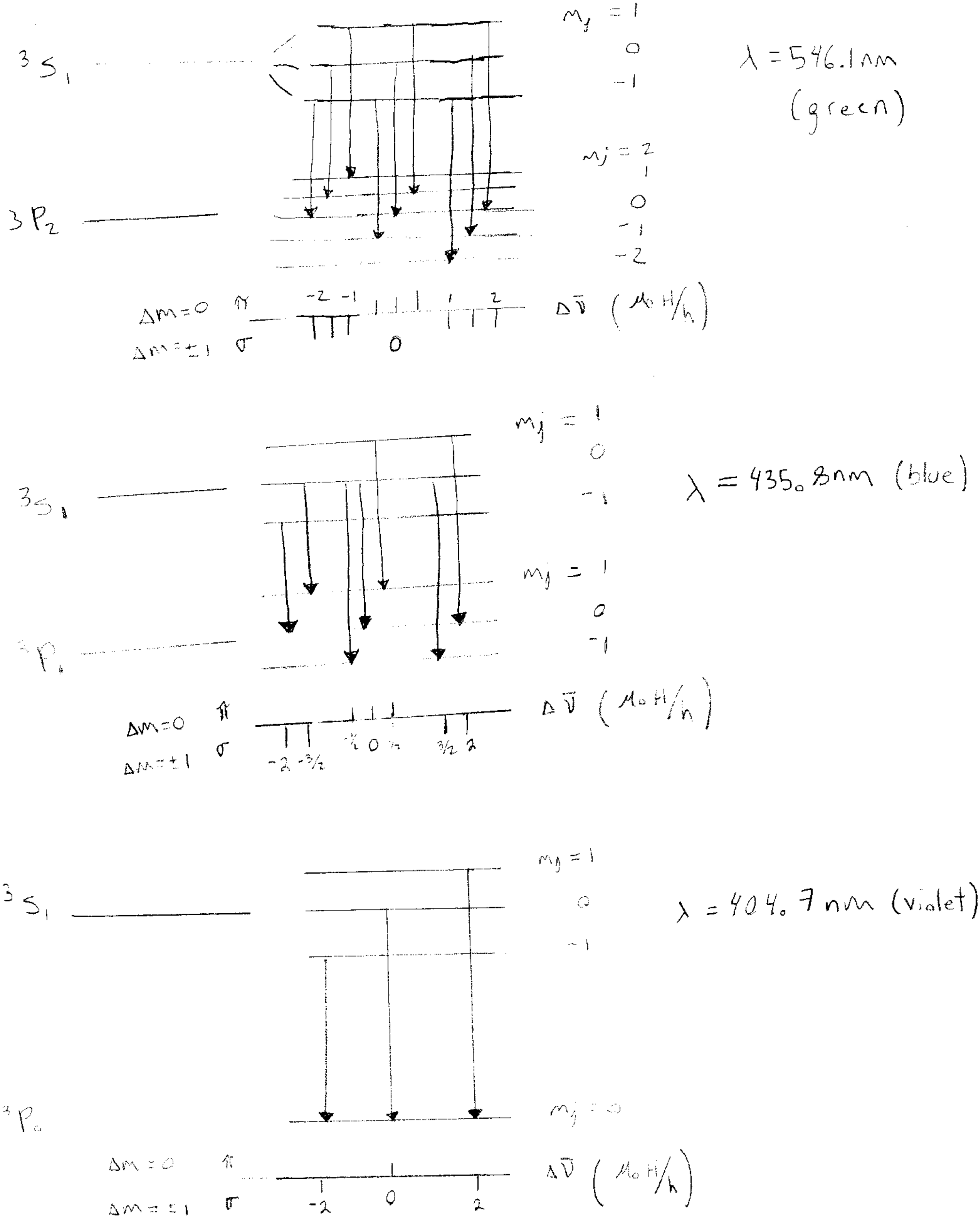
This study will focus in transitions from 6s7s to 6s6p. Specifically transitions from singlet state $^3S_1$ to states $^3P_2$, $^3P_1$, $^3P_0$, which correspond to wavelengths of 546.1 nm (green), 435.8 nm (blue) and 404.7 nm (violet) respectively this transitions are shown in figure 2.

The procedures and explanations for this experiment are found in Experiments in Modern Physics (Melissinos, 1966), chapter 7 sections 2,5 and 6 (see attachment)

Data:

In this experiment the filters 546nm and 436nm were used with a Pi polarization. Since Pi polarization isolates 3 components at these two wavelengths (see figure #2) and otherwise a sigma polarization would result in 6 and 4 components respectively and would be harder to resolve.
The 405nm filter was not used due to the given resolving limitation of the digital camera.

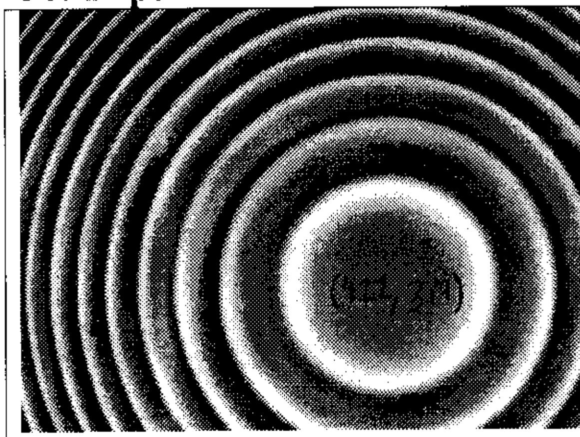
The data acquired was the following:



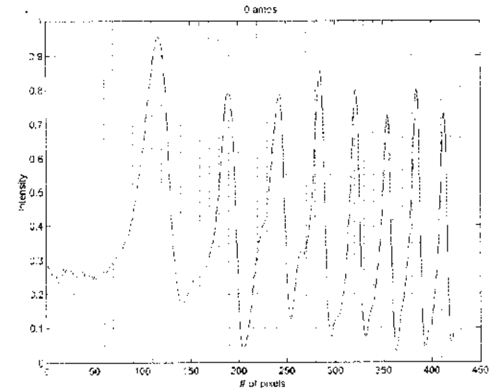
$^3S_1$
$m_j = 1$
0
−1
λ = 546.1 nm
(green)
$^3P_2$
$m_j = 2$
1
0
−1
−2
Δm = 0 π
Δm = ±1 σ
−2 −1 0 1 2
$\Delta\bar{\nu}$ $(\mu_0 H / h)$
$^3S_1$
$m_j = 1$
0
−1
λ = 435.8 nm (blue)
$^3P_1$
$m_j = 1$
0
−1
Δm = 0 π
Δm = ±1 σ
−2 −3/2 −1/2 0 1/2 3/2 2
$\Delta\bar{\nu}$ $(\mu_0 H / h)$
$^3S_1$
$m_j = 1$
0
−1
λ = 404.7 nm (violet)
$^3P_0$
$m_j = 0$
Δm = 0 π
Δm = ±1 σ
−2 0 2
$\Delta\bar{\nu}$ $(\mu_0 H / h)$

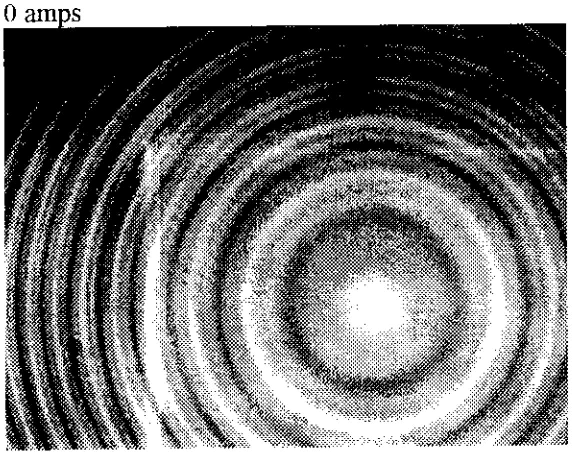
Figure 2

546 Pi polarization

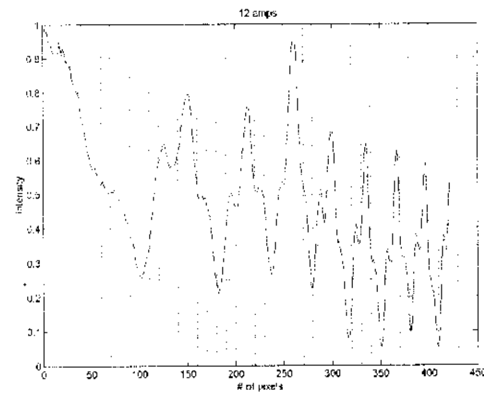


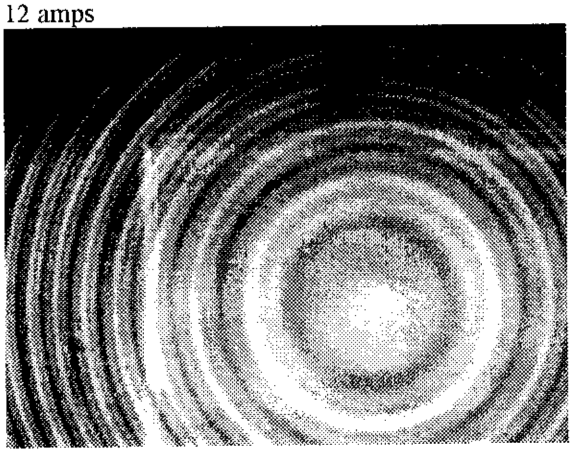
0 amps



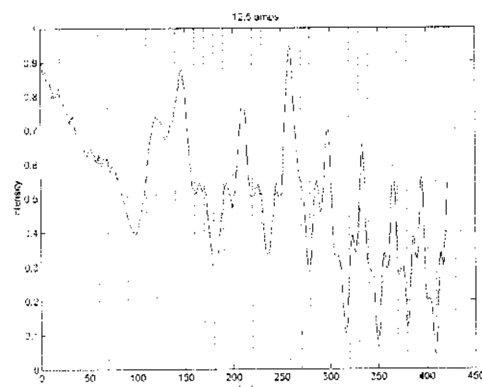


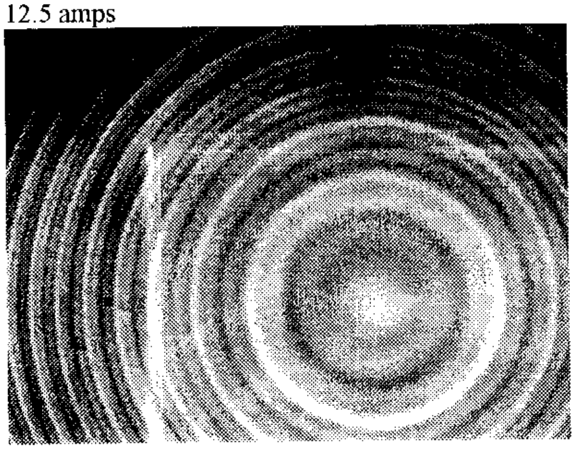
12 amps



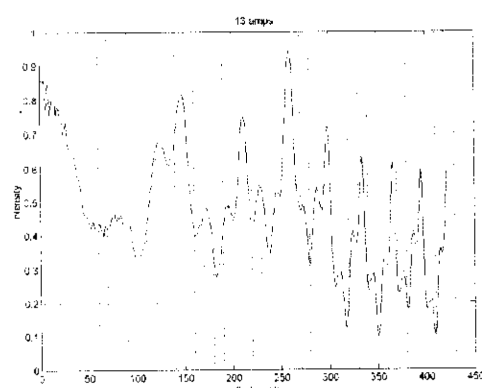


12.5 amps

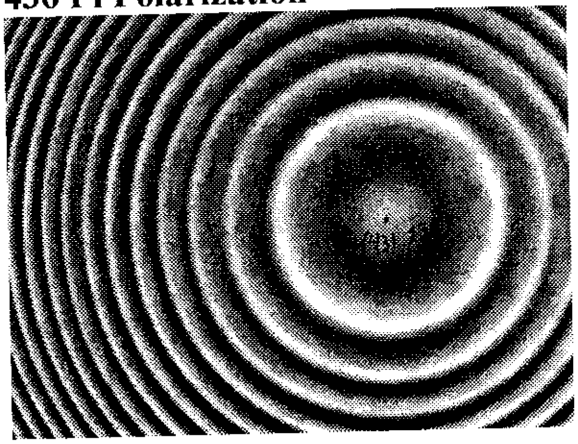




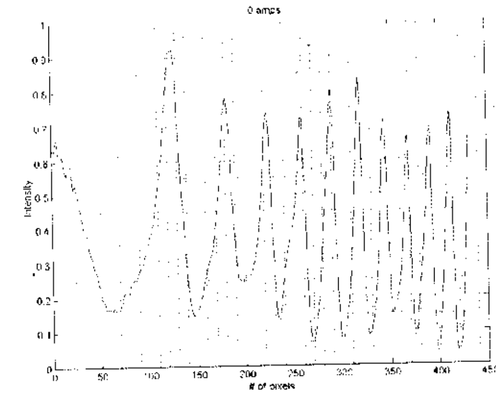
13 amps

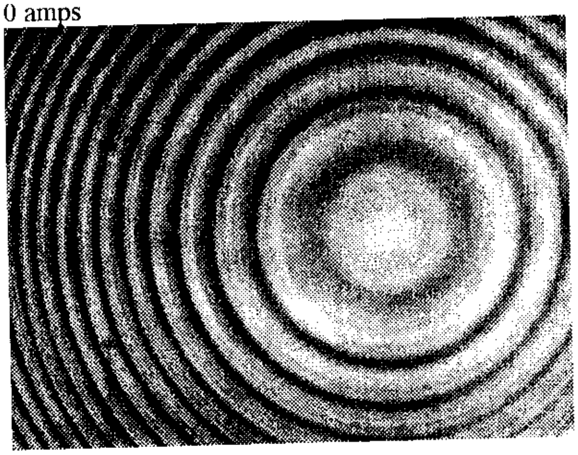


436 Pi Polarization

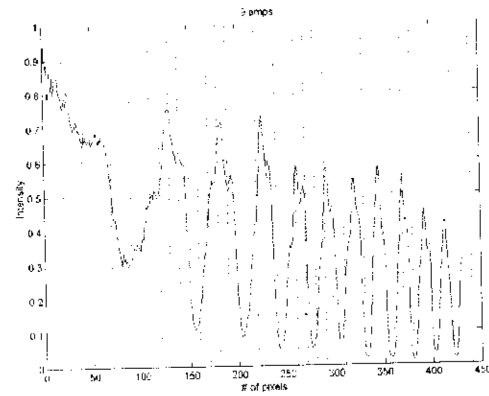


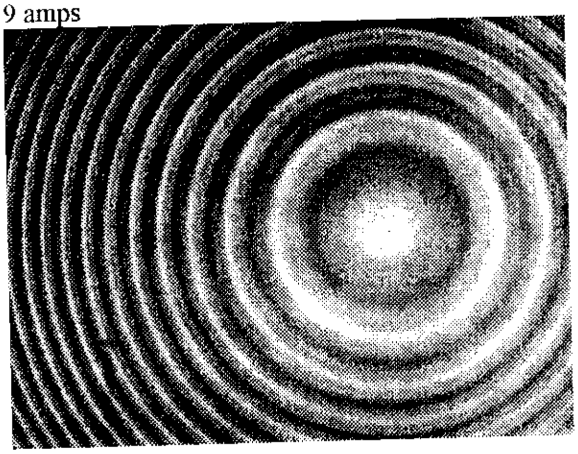
0 amps



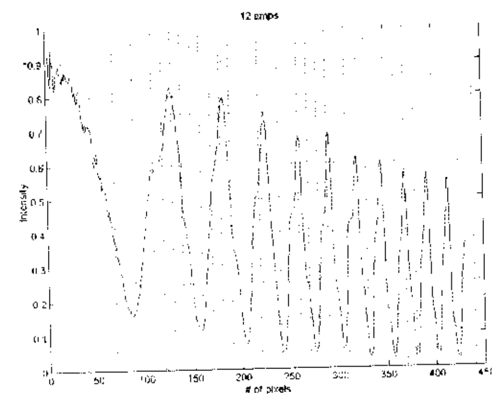


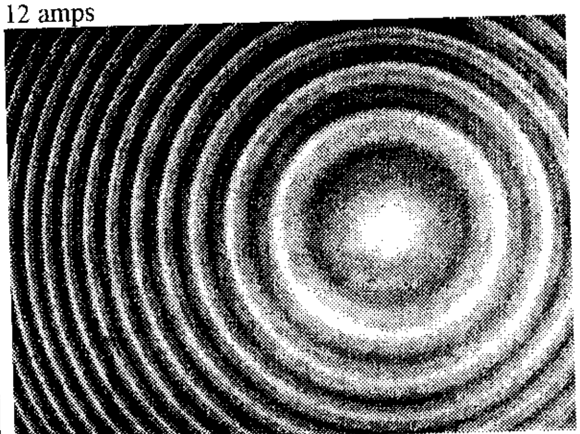
9 amps



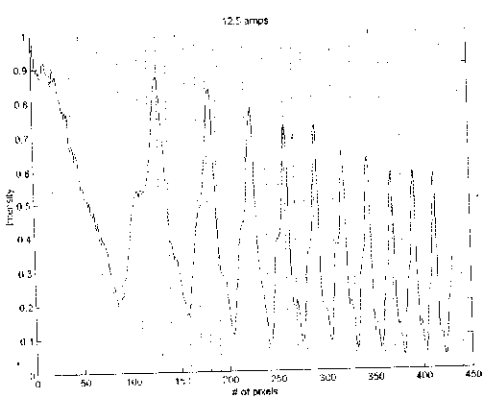


12 amps





12.5 amps



From the data above, the following table was obtained:

Wavelength 546 nm									
Current= 12 amps									
Ring Order	1	2	3	4	5	6	7	8	9
Radius (pixels)									
Component a	125	196	249	288	325	358	388		
Component b	150	212	260	300	339	365	397		
Component c	178	225	272	308	343	375	405		
Current =12.5 amps									
a	120	193	245	286	325	358	386		
b	145	210	258	298	334	366	395		
c	168	226	273	307	343	375	403		
Current = 13 amps									
a	124	195	245	287	324	358	388		
b	146	212	258	298	334	365	394		
c	172	228	273	312	345	377	405		
Wavelength 436 nm									
Current = 9 amps									
a	114	173	218	254	284	314	340	367	389
b	130	184	226	260	292	320	346	370	392
c	142	193	232	268	298	325	350	374	397
Current = 12 amps									
a	110	172	216	253	285	312	340	365	386
b	130	184	225	261	292	320	345	370	391
c	142	194	232	269	300	325	352	378	396
Current = 12.5 amps									
a	110	170	217	251	284	314	340	362	384
b	130	182	226	260	292	320	345	370	391
c	145	196	235	270	300	327	353	375	398

In order to know the magnitude of the magnetic field from the current the magnet was calibrated in the linear range to the following relation:
$B = 0.46 \pm 0.04 + (0.48 \pm 0.005)\, I$, where B is the magnetic field (kiloGauss) and I is the current in amps.
Notice that this relationship only holds in the range between 0 and 15 amps.

The conversion factor from pixel to microns is the following:
1 pixel = 7.4 Microns

Knowing the radii of the different rings at different orders as well as the magnitude of the magnetic field and the spacing (t) of the interferometer which in our case is: t= 10.03 mm, the difference in frequency between components can be calculated from:

$\Delta_a = R^2_{(p+1),a} - R^2_{p,a}$
$\delta_{ab} = R^2_{p,b} - R^2_{p,a}$, where a and b are the component index, p is the ring order and R the radius.
Then the change in frequency $\Delta\nu$ is:

$$\Delta\nu = \frac{\delta_{ab}}{(2\Delta t)}$$, t is the spacing of the Fabry_Perot etalon (interferometer)

and finally,

$\mu_o = \frac{\Delta\nu}{(\Delta g\ B)}$, is $g_f - g_i$ for the given wavelength.

Using the formulas above, the results obtained are shown in the following table:

RESULTS:

λ=546 nm

Current: 12.0 amps
Magnetic Field: 6220 +/-0.1 gauss

Component	R_1^2	Δ_{12}	R_2^2	Δ_{23}	R_3^2	Δ_{23}	R_4^2	Δ_{45}	R_5^2	Δ_{56}	R_6^2	Δ_{67}	R_7^2	Δ_{78}	R_8^2	Δ_{89}	R_9^2 (mm²)
A	0.856	1.248	2.104	1.292	3.395	1.147	4.542	1.242	5.784	1.234	7.018	1.226	8.244				
δ_{ab}	0.376		0.357		0.307		0.386		0.509		0.277		0.387				
B	1.232	1.229	2.461	1.241	3.702	1.227	4.928	1.365	6.293	1.002	7.295	1.335	8.631				
δ_{bc}	0.503		0.311		0.350		0.266		0.149		0.405		0.351				
C	1.735	1.037	2.772	1.279	4.051	1.143	5.195	1.248	6.442	1.258	7.701	1.281	8.982				
$<\Delta>$		1.171		1.270		1.172		1.285		1.165		1.281					

$<\delta_{ab}>$	σ_δ	$\Delta\nu(cm^{-1})$	σ_ν	μ_o (cm^{-1}/Gauss)	σ_μ
0.371	0.0279	0.151	0.01	$4.8x10^{-5}$	$7x10^{-6}$

$<\delta_{bc}>$	σ_δ	$\Delta\nu(cm^{-1})$	σ_ν		
0.334	0.0418	0.135	0.01	$4.3x10^{-5}$	$1x10^{-5}$

$<\Delta>_{net}$	σ_Δ
1.224	0.02

Current: 12.5 amps
Magnetic Field: 6460 +/-0.1 gauss

Component	R_1^2	Δ_{12}	R_2^2	Δ_{23}	R_3^2	Δ_{23}	R_4^2	Δ_{45}	R_5^2	Δ_{56}	R_6^2	Δ_{67}	R_7^2
A	0.789	1.251	2.040	1.247	3.287	1.192	4.479	1.305	5.784	1.234	7.018	1.141	8.159
δ_{ab}	0.363		0.375		0.358		0.384		0.325		0.317		0.385
B	1.151	1.264	2.415	1.230	3.645	1.218	4.863	1.246	6.109	1.227	7.335	1.208	8.544
δ_{bc}	0.394		0.382		0.436		0.298		0.334		0.365		0.350
C	1.546	1.251	2.797	1.284	4.081	1.080	5.161	1.281	6.442	1.258	7.701	1.193	8.894
$<\Delta>$		1.255		1.254		1.163		1.277		1.240		1.181	

$<\delta_{ab}>$	σ_δ	$\Delta\nu(cm^{-1})$	σ_ν	μ_o (cm^{-1}/Gauss)	σ_μ
0.358	0.0103	0.145	0.005	$4.5x10^{-5}$	$3x10^{-6}$

$<\delta_{bc}>$	σ_δ	$\Delta\nu(cm^{-1})$	σ_ν		
0.366	0.0168	0.148	0.0072	$4.6x10^{-5}$	$4x10^{-6}$

$<\Delta>_{net}$	σ_Δ
1.228	0.02

Current: 13 amps
Magnetic Field: 6700 +/-0.1 gauss

Component	R_1^2	Δ_{12}	R_2^2	Δ_{23}	R_3^2	Δ_{23}	R_4^2	Δ_{45}	R_5^2	Δ_{56}	R_6^2	Δ_{67}	R_7^2
A	0.842	1.240	2.082	1.205	3.287	1.224	4.511	1.238	5.748	1.270	7.018	1.226	8.244
δ_{ab}	0.325		0.379		0.358		0.352		0.360		0.277		0.257
B	1.167	1.294	2.461	1.184	3.645	1.218	4.863	1.246	6.109	1.187	7.295	1.205	8.501
δ_{bc}	0.453		0.386		0.436		0.468		0.409		0.488		0.481
C	1.620	1.227	2.847	1.235	4.081	1.249	5.331	1.187	6.518	1.265	7.783	1.199	8.982
$<\Delta>$		1.254		1.208		1.230		1.224		1.241		1.210	

$<\delta_{ab}>$	σ_δ	$\Delta\nu(cm^{-1})$	σ_ν	μ_o (cm^{-1}/Gauss)	σ_μ
0.330	0.0174	0.134	0.007	$4.0x10^{-5}$	$4x10^{-6}$

$<\delta_{bc}>$	σ_δ	$\Delta\nu(cm^{-1})$	σ_ν		
0.446	0.0143	0.181	0.005	$5.4x10^{-5}$	$3.x10^{-6}$

$<\Delta>_{net}$	σ_Δ
1.228	0.01

$<\mu_o>$ $4.4x10^{-5}$ cm^{-1}/Gauss

σ_μ $5x10^{-6}$

λ=436 nm

Current: 9 amps
Magnetic Field: 4780 +/-0.1 gauss

Component	R_1^2	Δ_{12}	R_2^2	Δ_{23}	R_3^2	Δ_{23}	R_4^2	Δ_{45}	R_5^2	Δ_{56}	R_6^2	Δ_{67}	R_7^2	Δ_{78}	R_8^2	Δ_{89}	R_9^2 (mm^2)						
a	0.712	0.927	1.639	0.964	2.602	0.930	3.533	0.884	4.417	0.982	5.399	0.931	6.330	1.045	7.376	0.911	8.286	$\langle\delta_{ab}\rangle$	σ_δ	$\Delta\nu$(cm^{-1})	σ_ν	μ_o (cm^{-1}/Gauss)	σ_μ
δ_{ab}	0.214		0.215		0.195		0.169		0.252		0.208		0.225		0.121		0.128	0.192	0.0147	0.101	0.008	4.3x10^{-5}	7E^{-6}
b	0.925	0.929	1.854	0.943	2.797	0.905	3.702	0.967	4.669	0.938	5.607	0.948	6.556	0.941	7.497	0.918	8.415	$\langle\delta_{bc}\rangle$	σ_δ	$\Delta\nu$(cm^{-1})	σ_ν		
δ_{bc}	0.179		0.186		0.150		0.231		0.194		0.177		0.152		0.163		0.216	0.183	0.0091	0.096	0.005	4.06E-5	4E^{-6}
c	1.104	0.936	2.040	0.908	2.947	0.986	3.933	0.930	4.863	0.921	5.784	0.924	6.708	0.952	7.660	0.971	8.631	$\langle\Delta\rangle_{net}$	σ_Δ				
$\langle\Delta\rangle$		0.930		0.938		0.940		0.927		0.947		0.934		0.979		0.933		0.941	0.0059				

Current: 12 amps
Magnetic Field: 6220 +/-0.1 gauss

a	0.663	0.957	1.620	0.935	2.555	0.950	3.505	0.943	4.448	0.883	5.331	1.000	6.330	0.965	7.295	0.864	8.159	$\langle\delta_{ab}\rangle$	σ_δ	$\Delta\nu$(cm^{-1})	σ_ν	μ_o (cm^{-1}/Gauss)	σ_μ
δ_{ab}	0.263		0.234		0.217		0.225		0.221		0.277		0.188		0.201		0.213	0.227	0.009	0.12086	0.006	3.89E-5	5E^{-6}
b	0.925	0.929	1.854	0.918	2.772	0.958	3.730	0.939	4.669	0.938	5.607	0.910	6.518	0.979	7.497	0.875	8.372	$\langle\delta_{bc}\rangle$	σ_δ	$\Delta\nu$(cm^{-1})	σ_ν		
δ_{bc}	0.179		0.207		0.175		0.232		0.259		0.177		0.267		0.328		0.215	0.227	0.01	0.12089	0.009	3.89E-5	8E^{-6}
c	1.104	0.957	2.061	0.886	2.947	1.015	3.962	0.966	4.928	0.856	5.784	1.001	6.785	1.039	7.824	0.763	8.587	$\langle\Delta\rangle_{net}$	σ_Δ				
$\langle\Delta\rangle$		0.948		0.913		0.974		0.949		0.892		0.970		0.994		0.834		0.93	0.01				

Current: 12.5 amps
Magnetic Field: 6460 +/-0.1 gauss

a	0.663	0.920	1.583	0.996	2.579	0.871	3.450	0.967	4.417	0.982	5.399	0.931	6.330	0.846	7.176	0.899	8.075	$\langle\delta_{ab}\rangle$	σ_δ	$\Delta\nu$(cm^{-1})	σ_ν	μ_o (cm^{-1}/Gauss)	σ_μ
δ_{ab}	0.263		0.231		0.218		0.252		0.252		0.208		0.188		0.321		0.297	0.248	0.01	0.13247	0.008	4.1E-5	5E^{-6}
b	0.925	0.888	1.814	0.983	2.797	0.905	3.702	0.967	4.669	0.938	5.607	0.910	6.518	0.979	7.497	0.875	8.372	$\langle\delta_{bc}\rangle$	σ_δ	$\Delta\nu$(cm^{-1})	σ_ν		
δ_{bc}	0.226		0.290		0.227		0.290		0.259		0.248		0.306		0.204		0.302	0.261	0.01	0.13974	0.006	4.3E-5	4E^{-6}
c	1.151	0.952	2.104	0.920	3.024	0.968	3.992	0.936	4.928	0.927	5.855	0.968	6.824	0.877	7.701	0.974	8.674	$\langle\Delta\rangle_{net}$	σ_Δ				
$\langle\Delta\rangle$		0.920		0.967		0.915		0.957		0.949		0.937		0.901		0.916		0.933	0.008				

$\langle\mu_o\rangle$ 4.1E^{-5} cm^{-1}/Gauss
σ_μ 5E^{-6}

CONCLUSION:

The overall mean value of the bohr magneton obtained using the 546nm filter, $4.4x10^{-5} \pm 5x10^{-6} cm^{-1}$/Gauss, agrees within error bars with the standard value of $4.669x10^{-5} cm^{-1}$/Gauss. Whereas the value obtained with the 436nm filter ($4.1x10^{-5} \pm 5x10^{-6} cm^{-1}$/Gauss) differs from the standard value by 12%.

This inaccuracy with a shorter wavelength is due to the resolving limitation of the setup used. In order to improve the results at high frequencies a magnifying lens between the camera and the image from the converging lens after the interferometer could be used but then not many ring orders would appear on the screen of the camera.

REFERENCES:

Melissinos, Adrian C. "Experiments in Modern Physics". Academic Press, New York 1966. ch 5.2,5.5,5.6.

Whity, Harvey E. "Introduction to Atomic Spectr". McGraw Hill Book Company, NY and London 1934. ch 10.