University of California, San Diego

“Experiment of Modern Physics”
Zeeman Effect

Phys 173
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Leonel Romero
Zeeman Effect

**Theory:**

The fine structure of the hydrogen atom in the absence of an external magnetic field has energies:

\[
E = -\frac{Z^2 \alpha^2 m_e e^2}{n^2} - \left(\frac{Z^2 \alpha^2}{2}\right)^2 \left[\frac{3 - \frac{4n}{j + \frac{1}{2}}}{j + \frac{1}{2}}\right],
\]

where \(Z\) is the atomic number, \(\alpha\) is a constant \((1/137)\), and \(n\) and \(j\) are usual quantum numbers.

Recall total angular momentum, \(J\), is the sum of the total momentum number \(L\) plus the total spin number.

\[J = L + S, \quad |L - S| \leq j \leq |L + S|\]

Each of the \(j\)th state has \(2j + 1\) degenerate states \(m_j, -j \leq m_j \leq j\).

When a weak external magnetic field is acting upon the hydrogen atom, the magnetic moment of the circulating electron and the intrinsic dipole moment of the spin interact with the magnetic field resulting in a split of the energy into non-degenerate levels (see figure 1).

```
           m_j = 3/2
           ____________
          /             |
  2^p_{3/2}  /                | 1/2
          /         \              |
         /           |          -1/2
  2^p_{1/2} /                | 1/2
          /         \              |
         /           |          -1/2
```

The interaction with the external magnetic field has the following Hamiltonian:

\[
H_{\text{Zeeman}} = \frac{e}{2mc} (L + 2S) \mathbf{B}
\]

Using First order perturbation theory, the energy shifts have magnitude:
\[ \Delta E = g \mu_0 B m_j, \text{ where } \mu_0 = \frac{e \hbar}{4\pi mc} \text{ and } g = 1 + \frac{j(j+1)+s(s+1)-l(l+1)}{2j(j+1)} \]

The spectrum of mercury under a weak magnetic field can be used to show the linear relationship between the energy splitting and the magnitude of the magnetic field and therefore to determine the experimental values of \(\mu_0\).

The mercury atom has two electrons outside a closed shell with following configuration: 1s\(^2\)2s\(^2\)2p\(^6\)3s\(^2\)3p\(^6\)3d\(^10\)4s\(^2\)4p\(^6\)4d\(^10\)4f\(^{14}\)5s\(^2\)5p\(^6\)6s\(^2\). Having two valence electrons results in configuration similar to that of the helium atom with spin number \(s=1\) and total angular momentum \(j\), with values: \(j=L+1, L, L-1\).

The different energy transitions between energy states satisfy the following rules:

\[ \Delta l = \pm 1 \]
\[ \Delta j = 0, \pm 1, \text{ except transitions from } j=0 \text{ to } j=0. \]
Transitions between triplet and singlet states are not allowed.

This study will focus in transitions from 6s7s to 6s6p. Specifically transitions from singlet state \(^3\)S\(_1\) to states \(^3\)P\(_2\), \(^3\)P\(_1\), \(^3\)P\(_0\), which correspond to wavelengths of 546.1 nm (green), 435.8 nm (blue) and 404.7 nm (violet) respectively this transitions are shown in figure 2.

The procedures and explanations for this experiment are found in Experiments in Modern Physics (Melissinos, 1966), chapter 7 sections 2, 5 and 6 (see attachment)

**Data:**
In this experiment the filters 546nm and 436nm were used with a Pi polarization. Since Pi polarization isolates 3 components at these two wavelengths (see figure #2) and otherwise a sigma polarization would result in 6 and 4 components respectively and would be harder to resolve.
The 405nm filter was not used due to the given resolving limitation of the digital camera.

**The data acquired was the following:**
$^3S_1$   

$^3P_2$   

$^3S_1$   

$^3P_0$   

$^3S_1$   

$^1P_1$   

$\lambda = 546.1 \text{ nm (green)}$

$\lambda = 435.8 \text{ nm (blue)}$

$\lambda = 404.7 \text{ nm (violet)}$
546 Pi polarization

0 amps

12 amps

12.5 amps

13 amps
436 Pi Polarization

0 amps

9 amps

12 amps

12.5 amps
From the data above, the following table was obtained:

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<td>5</td>
<td>6</td>
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<td>249</td>
<td>288</td>
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<td>358</td>
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<tr>
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<tr>
<td>Component c</td>
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<td>272</td>
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<td>Current = 12.5 amps</td>
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<table>
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In order to know the magnitude of the magnetic field from the current the magnet was calibrated in the linear range to the following relation:

\[ B = 0.46 \pm 0.04 + (0.48 \pm 0.005) I \]

where \( B \) is the magnetic field (kiloGauss) and \( I \) is the current in amps.

Notice that this relationship only holds in the range between 0 and 15 amps.
The conversion factor from pixel to microns is the following:

1 pixel = 7.4 Microns

Knowing the radii of the different rings at different orders as well as the magnitude of the magnetic field and the spacing (t) of the interferometer which in our case is: \( t = 10.03 \text{ mm} \), the difference in frequency between components can be calculated from:

\[
\Delta a = R_{p+1,a}^2 - R_{p,a}^2 \\
\delta_{ab} = R_{p,b}^2 - R_{p,a}^2, \text{ where } a \text{ and } b \text{ are the component index, } p \text{ is the ring order} \text{ and } R \text{ the radius.}
\]

Then the change in frequency \( \Delta \nu \) is:

\[
\Delta \nu = \frac{-\delta_{ab}}{(2\Delta t)} \quad , \text{ } t \text{ is the spacing of the Fabry-Perot etalon (interferometer)}
\]

and finally,

\[
\mu_r = \frac{\Delta \nu}{( \Delta g B )}, \text{ is } g_f - g_i \text{ for the given wavelength.}
\]

Using the formulas above, the results obtained are shown in the following table:
CONCLUSION:

The overall mean value of the bohr magneton obtained using the 546nm filter, $4.4 \times 10^5 \pm 5 \times 10^6 \text{cm}^{-1}/\text{Gauss}$, agrees within error bars with the standard value of $4.669 \times 10^6 \text{cm}^{-1}/\text{Gauss}$. Whereas the value obtained with the 436nm filter ($4.1 \times 10^5 \pm 5 \times 10^6 \text{cm}^{-1}/\text{Gauss}$) differs from the standard value by 12%.
This inaccuracy with a shorter wavelength is due to the resolving limitation of the setup used. In order to improve the results at high frequencies a magnifying lens between the camera and the image from the converging lens after the interferometer could be used but then not many ring orders would appear on the screen of the camera.

REFERENCES: