Homework 8 Solutions

Note Title 1/4/2006

14 The centripetal acceleration is

$a_c = r\omega^2$, where $r = \frac{d}{2} = \frac{1}{2}(5\ mi) = \frac{5}{2}(1609\ m)$

$\because$ we want $a_c = g$

$\therefore\ \omega = \sqrt{\frac{a_c}{r}} = \sqrt{\frac{2g}{d}} = \sqrt{\frac{9.8\ m/s^2 \times 2}{5(1609\ m)}} = \boxed{0.05\ rad/s}$

(4)

17 The angular velocity of the disk 3.0 s after starting from rest is:

$\omega = 78\ \frac{rev}{min}\left(\frac{2\pi\ rad}{1\ rev}\right)\left(\frac{1\ min}{60\ s}\right) = 8.2\ rad/s$

and the bug follows a circular path of radius

$r = 5.0\ in\left(\frac{1\ m}{39.37\ in}\right) = 0.13\ m$

(a) The constant angular acceleration of the disk is

$\alpha = \frac{\omega - \omega_0}{t} = \frac{8.2\ rad/s - 0}{3.0\ s} = 2.7\ rad/s^2$

The tangential acceleration of the bug is $a_t = r\alpha$,

thus $a_t = (0.13\ m)(2.7\ rad/s^2) = \boxed{0.35\ m/s^2}$

(b) When the disk is rotating at its final angular velocity, $\omega = 8.2\ rad/s$, and

$v_t = r\omega = (0.13\ m)(8.2\ rad/s) = \boxed{1.0\ m/s}$

(c) Since both r and α are constant, the tangential acceleration, $a_t = r\alpha$, is also constant. Thus at $t = 1.0\ s$, $a_t = \boxed{0.35\ m/s^2}$

The tangential velocity of the bug is

$v_t = (v_t)_{t=0} + a t = 0 + (0.35\ m/s^2)(1.0\ s) = 0.35\ m/s$

and the centripetal acceleration is

7.20 Since $F_c = m\frac{v_t^2}{r} = mr\omega^2$, the needed angular velocity is

$$\omega = \sqrt{\frac{F_c}{mr}} = \sqrt{\frac{4.0\times10^{-11}\text{ N}}{(3.0\times10^{-16}\text{ kg})(0.150\text{ m})}}$$

$$= (9.4\times10^{2}\text{ rad/s})\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right) = \boxed{1.5\times10^{2}\text{ rev/s}}$$

7.22 $$a_c = \frac{v_t^2}{r} = \frac{\left[\left(86.5\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\right]^2}{61.0\text{ m}}\left(\frac{1\,g}{9.80\text{ m/s}^2}\right) = \boxed{0.966\,g}$$

7.35 (a) The gravitational force must supply the required centripetal acceleration, so

$\frac{Gm_E m}{r^2} = m\left(\frac{v_t^2}{r}\right)$. This reduces to $r = \frac{Gm_E}{v_t^2}$, which gives

$$r = \left(6.67\times10^{-11}\,\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{(5.98\times10^{24}\text{ kg})}{(5000\text{ m/s})^2} = 1.596\times10^{7}\text{ m}.$$

The altitude above the surface of the Earth is then

$$h = r - R_E = 1.596\times10^{7}\text{ m} - 6.38\times10^{6}\text{ m} = \boxed{9.58\times10^{6}\text{ m}}.$$

(b) The time required to complete one orbit is

$$T = \frac{\text{circumference of orbit}}{\text{orbital speed}} = \frac{2\pi(1.596\times10^{7}\text{ m})}{5000\text{ m/s}} = 2.00\times10^{4}\text{ s} = \boxed{5.57\text{ h}}.$$

37 Kepler's third law gives $T^2 = \left(\frac{4\pi^2}{GM_J}\right)r^3 \Rightarrow$

$$M_J = \frac{4\pi^2}{G}\frac{r^3}{T^2} = \frac{4(3.14)^2}{6.673\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2}\frac{(4.22\times10^{5}\times10^{3}\text{ m})^3}{(1.77\times24\times3600\text{ s})^2}$$

$$= \boxed{1.90\times10^{27}\text{ kg}}$$

38 (a) $PE = -G \frac{M_E m}{r}$ where $r = R_E + h \Rightarrow$

$$PE = -G \frac{M_E m}{R_E + h} = -6.673 \times 10^{-11} N \cdot m^2/kg^2 \frac{5.98 \times 10^{24} kg (100 kg)}{6.38 \times 10^6 m + 2 \times 10^6 m}$$

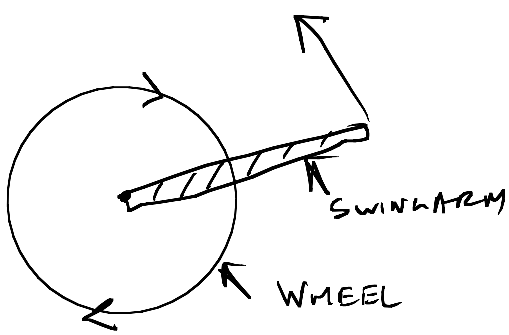
$$= \boxed{-4.76 \times 10^9 J}$$

(b)
$$F = G \frac{M_E m}{r^2} = G \frac{M_E m}{(R_E + h)^2}$$

$$= 6.673 \times 10^{-11} N \cdot m^2/kg \frac{5.98 \times 10^{24} kg (100 kg)}{(6.38 \times 10^6 m + 2 \times 10^6 m)^2}$$

$$= \boxed{568 N}$$ toward the Earth.

C7)



IN ORDER TO CONSERVE ANGULAR MOMENTUM, AS THE SPEED OF THE WHEEL INCREASES, THE NOSE OF THE BIKE MUST LIFT.

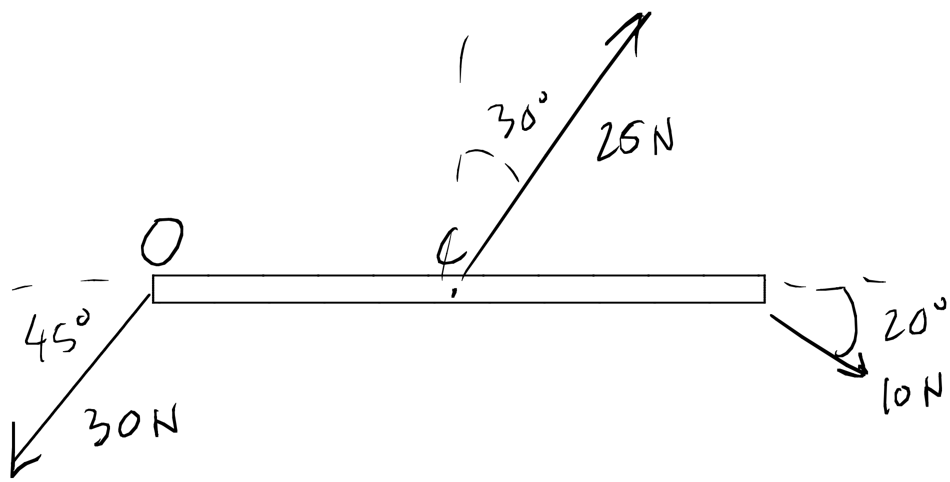
C16) THIS QUESTION HAS 2 ANSWERS. THE CAT ROTATES TO CONSERVE ANGULAR MOMENTUM. THE ACTUAL MOTION IS SLIGHTLY COMPLICATED AND INVOLVES MULTIPLE STAGES. SEE ME (BEN) IF YOU WANT A FULL EXPLANATION.

1 The magnitude of the torque $\vec{\tau}$ of the force $\vec{F}$ is given by $\tau = rF\sin\theta$, where r is the length of the position vector, θ the angle between $\vec{F}$ and $\vec{r}$.

Thus, when $\theta = \frac{\pi}{2}$, F reaches its minimum value:

$$F_{min} = \left.\frac{\tau}{r \cdot \sin\theta}\right|_{\theta=\frac{\pi}{2}} = \frac{40.0\,N\cdot m}{(30.0\text{ cm}) \times 1} = \frac{40.0\,N\cdot m}{0.30\,m} = \boxed{133N}$$

8.3)



a) TORQUE AT C:

2.0 METER · 25N · Cos(30°)

TORQUE AT END:

− 10N · 4 METERS · Sin(20°)

TOTAL: 30 N·M ↻

b) TORQUE AT O:

− 30N · −2 m · Sin 45

TORQUE AT END:

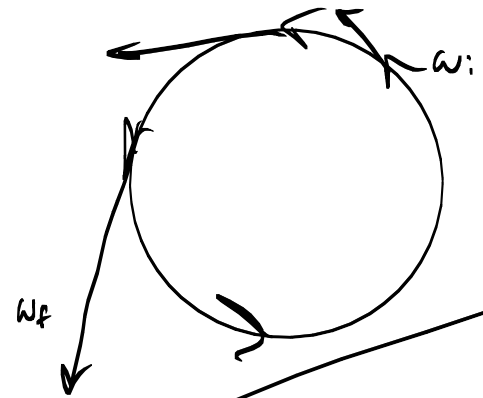
2.0 · −10N · Sin 20°

TOTAL: 36N ↻

Homework Addendum - Week 8

Note Title 3/6/2006

13)



$37 \text{ rev.} \cdot 2\pi = \Delta\theta$

$t = 3.0\ s$

$\omega_f = 98.0\ r/s$

$\omega_f^2 = \omega_i^2 + 2\alpha(\Delta\theta$

$\omega_f = \omega_i + \alpha t$

$\omega_i = \omega_f - \alpha t$

$\omega_f^2 = \omega_f^2 - 2\omega_f \alpha t + \alpha^2 t^2 + 2\alpha(\Delta\theta)$

$\alpha^2 t^2 + \alpha(\Delta\theta \cdot 2 - 2\omega_f t) = 0$

$\alpha[\alpha t^2 + 2\Delta\theta - 2\omega_f t] = 0$

Trivial Solution $\alpha = 0$

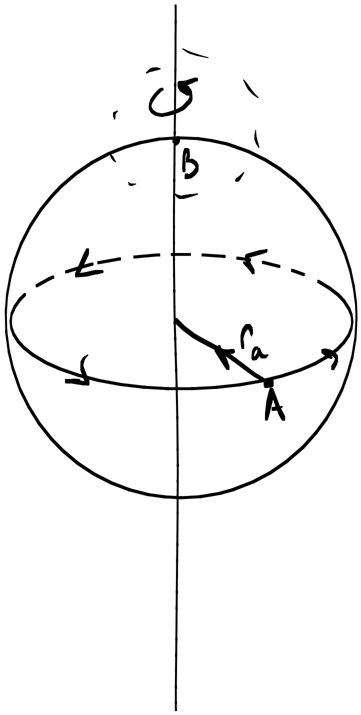
Units make sense.

Nontrivial:

$$\alpha = \frac{2[\omega_f t - \Delta\theta]}{t^2} = \frac{2[98\ r/s \cdot 3s - 37 \cdot 2\pi r]}{3.0\ s^2}$$

$$\boxed{= 13.7\ r/s^2}$$

15.)



A: $a_c = \frac{v^3}{r_a} =$

$r_{earth} = 6378$ Km

1 DAY $= 2\pi = 24$ HOURS $= 86400$

$V_{EQUATOR} = \frac{2\pi r_e}{T} = 463$ m/s

$a_c = \frac{v^2}{r_a} = 3.3 \times 10^{-2}$ m/s² INWARD.

b:) AT THE NORTH POLE, THIS IS EASY.

$V = \frac{2\pi r_b}{T} = 0,$ SO $a_c = 0.$