

## Physics 1B - Quiz 3 (12 Feb 2007)

### Formulas for Electrostatics

The force on charge  $q_1$  from charge  $q_2$  is  $\vec{F}_{12} = k_e \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12}$ , where the direction vector  $\hat{r}_{12}$  points from  $q_2$  to  $q_1$  and the proportionality constant is  $k_e = 8.99 \times 10^9 \text{ Nm}^2/\text{C}^2$ .

Note that the permittivity of free space is  $\epsilon_0 \equiv \frac{1}{4\pi k_e} = 8.85 \times 10^{-12} \text{ C}^2/(\text{Nm}^2) = 8.85 \times 10^{-12} \text{ A}^2 \text{ s}^4/(\text{kg m}^3)$ .

Note that the unit of elemental electronic charge is  $e^- = -1.62 \times 10^{-19} \text{ C}$ .

We note the Taylor's expansion  $(1+x)^n = 1 + nx + \dots$ , which is useful when  $nx \ll 1$ . For example,

$$\frac{1}{(r+d)^2} = \frac{1}{r^2} \left(1 + \frac{d}{r}\right)^{-2} = \frac{1}{r^2} \left(1 - 2\frac{d}{r} + \dots\right) \approx \frac{1}{r^2} - 2\frac{d}{r^3} \text{ for } d \ll r.$$

The force on a test charge  $q_0$  induced by an electric field, denoted  $\vec{E}$ , is  $\vec{F} = q_0 \vec{E}$ .

### Formulas for Fields and Potentials

The electric flux through a surface is  $\Phi_e \equiv \sum_{\text{All Surfaces}} EA_{\perp} = \sum_{\text{All Surfaces}} EA \cos \theta$ , where  $A_{\perp} = A \cos \theta$  is the component of the area whose normal lies parallel to the electric field..

Gauss' Law relates the total flux through a closed surface to the total net charge enclosed by the surface, *i.e.*,  $\Phi_e = 4\pi k_e Q_{\text{Total}}$ .

The electric field produced by a point charge  $q$  at the origin, *i.e.*,  $\vec{r} = 0$ , is  $\vec{E} = k_e \frac{q}{r^2} \hat{r}$  where  $\hat{r}$  is the radius vector in spherical coordinates.

The electric field produced by a line charge, with charge per unit length  $\lambda$ , is  $\vec{E} = 2k_e \frac{\lambda}{r} \hat{r}$ , where the line is defined to lie along the  $\hat{z}$  axis and  $\hat{r}$  is the radius vector in cylindrical coordinates.

The electric field produced by a surface charge, with charge per unit area  $\sigma$ , is  $\vec{E} = 2\pi k_e \sigma \hat{n}$ , where the surface lies in the  $\hat{x}$ - $\hat{y}$  plane and  $\hat{z}$  corresponds to the normal to the  $\hat{x}$ - $\hat{y}$  plane in Cartesian coordinates.

Work-Energy Theorem:  $W = \Delta KE + \Delta PE$

$$\text{Electric potential: } \Delta V = -E \Delta x \cos \theta, \text{ where } \Delta V = \frac{\Delta PE}{Q}$$

$$V = k_e \frac{q}{r} \text{ a distance } r \text{ away from a point charge } q.$$

### Formulas on Current, Resistance and Capacitance

$$\text{Current: } I = \frac{\Delta Q}{\Delta t}$$

Capacitance:  $Q = C \Delta V$  where  $C = \frac{\kappa}{4\pi k_c} \frac{A}{d}$  for parallel plates and  $\kappa$  is the dielectric constant

$$I = C \frac{\Delta V}{\Delta t}$$

$$\text{Energy Stored} = \frac{1}{2} Q \Delta V = \frac{1}{2} C (\Delta V)^2 = \frac{1}{2C} Q^2$$

Resistance:  $V = I R$  where  $R = \rho \frac{l}{A}$  and  $\rho$  is the resistivity in Ohm-m.

	Series	Parallel
Capacitors	$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots$	$C_{eq} = C_1 + C_2 + C_3 + \dots$
Resistors	$R_{eq} = R_1 + R_2 + R_3 + \dots$	$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$

$$\text{Power Dissipated} = IV = I^2R = V^2/R$$

- Kirchoff's Laws:
- 1) Sum of voltage drops around any loop is zero, *i.e.*, gains = losses
  - 2) Sum of current flow at a node is zero, *i.e.*, total current in = total current out

## Finally! The Quiz

1. For current flow in a resistor, which statement is true?

- A. The acceleration of the charge carrier is proportional to the voltage drop across the resistor
- B. The velocity of the charge carriers is proportional to the voltage drop
- C. The velocity of the charge carriers is a constant that depends solely on the type of material
- D. The velocity of the charge carriers is a constant that depends solely on the temperature
- E. The velocity of the charge carriers is a constant that depends solely on the geometry

\* Students should be given  $I = nqvdA$  \*

2. A 1.50 V battery is connected across a 4 F capacitor, as shown below in figure 1. What is the energy stored in the capacitor subsequent to removal of the battery?

- A. XXX J 6.0 J (E=CV)
- B. YYY J 4.5 J correct
- C. zero
- D. XXX W 6.0 W
- E. YYY W 4.5 W

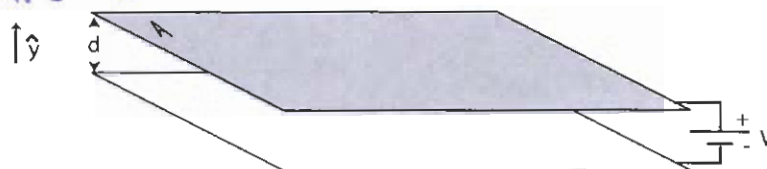


Figure 1

3. The resistivity of cytoplasm, the solution inside of a cell, is  $100 \Omega \text{ cm}$ . What is the resistance of a cylinder of cytoplasm that is  $1.0 \text{ mm}$  long and  $2.0 \mu\text{m}$  in diameter? Mind the units!

- A.  $3.2 \times 10^5 \Omega$  (correct)
- B.  $3.1 \times 10^{-15} \Omega$  ( $R = \rho L/A$  w/ correct units.)
- C.  $3.2 \times 10^3 \Omega$  (converting  $\mu\text{m}$ , not  $\mu\text{m}^2$ )
- D.  $8.0 \times 10^7 \Omega$  (diameter, not radius)
- E.  $8.0 \times 10^9 \Omega$  (errors from c and d)

4. A room heater operates at  $220 \text{ Volts}$  and consumes  $2000 \text{ W}$  of power. How much current does the heater draw?

- A.  $0.11 \text{ A}$  ( $220 \text{ V} / 2000 \text{ W}$ )
- B.  $4.1 \times 10^{-3} \text{ A}$  ( $P = IV^2$ )
- C.  $3.0 \text{ A}$  ( $P = IV$ )
- D.  $9.1 \text{ A}$  (correct)
- E.  $4.4 \times 10^5 \text{ A}$  ( $220 \text{ V} \cdot 2000 \text{ W}$ )

5. In bacterial photosynthesis, the absorption of one photon causes a pair of electrons to move across the membrane. Under high light levels, pairs of electrons cross once every  $100 \mu\text{s}$ . What is the current?

- A.  $1.6 \times 10^{-21} \text{ A}$  ( $1 \text{ e}^-$  instead of  $2 \text{ e}^-$ )
- B.  $3.2 \times 10^{-21} \text{ A}$  ( $\text{s}$  instead of  $\mu\text{s}$ )
- C.  $2.0 \text{ A}$  ( $2 \text{ e}^- \dots$ )
- D.  $1.6 \times 10^{-15} \text{ A}$  ( $1 \text{ e}^-$  not  $2$ )
- E.  $3.2 \times 10^{-15} \text{ A}$  (correct)

6. A battery supplies current to a circuit with 3 resistors, as shown below in figure 2. What is the correct expression for the total current  $I$ ?

- A.  $V/R$
- B.  $V/(2R)$
- C.  $2V/R$
- D.  $5V/4R$
- E.  $3V/R$

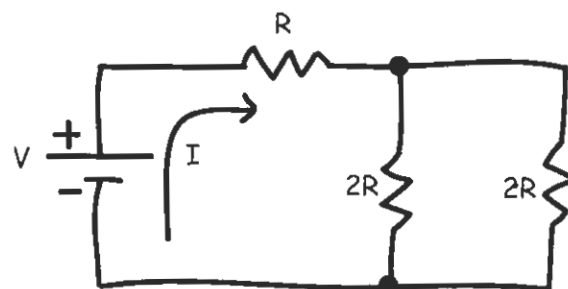


Figure 2

7. Find the equivalent resistance between points A and B for the circuit shown below in figure 3.

- A.  $50 \Omega$
- B.  $3.0 \times 10^2 \Omega$
- C.  $112 \Omega$
- D.  $71 \Omega$
- E.  $3.8 \Omega$

(correct)  
 (Req parallel =  $\frac{1}{R_1} + \frac{1}{R_2}$ )  
 (systematically confusing series & //)  
 (top in series)  
 (25  $\Omega$  in series w/ bottom)

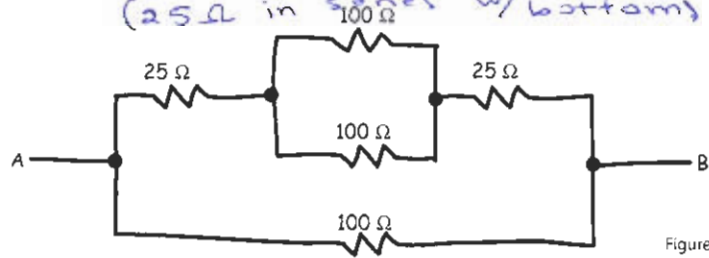


Figure 3

8. A battery supplies current to a circuit with 4 resistors, as shown below in figure 4. What is the correct expression for the potential  $V_1$ ? Hint – you do not need to keep track of  $V_2$  to find  $V_1$ .

- A.  $1/3 V_B$
- B.  $1/2 V_B$
- C.  $2/3 V_B$
- D.  $4/5 V_B$
- E.  $2 V_B$

9. What is the correct expression for the potential  $V_2$  in Figure 4? Think carefully before you start!

- A.  $1/5 V_B$
- B.  $1/4 V_B$
- C.  $1/3 V_B$
- D.  $1/2 V_B$
- E.  $2/5 V_B$

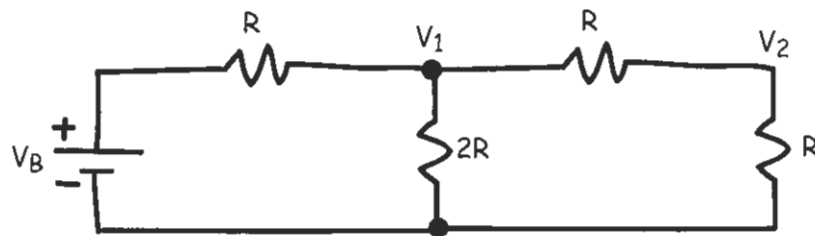


Figure 4

10. In a real battery with interval resistance (figure 5), the voltage drop across an external load resistor is

- A. always less than the open circuit voltage,  $V_{\text{battery}}$
- B. equal to the open circuit voltage
- C. unconstrained
- D. zero
- E. always greater than the open circuit voltage

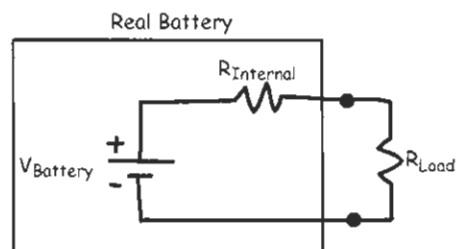


Figure 5

### 1B Quiz 3

1. Voltage is proportional to current and resistance ( $V=IR$ ), and current is proportional to drift velocity

( $I = nq v_d A$ ). Substituting,

$$V = (nq v_d A) R$$

$$v_d = \frac{V}{nqAR}$$

∴ charge carrier velocity is proportional to the voltage drop across the resistor.

1B Quiz 3

$$\begin{aligned} 2. E &= \frac{1}{2} C (\Delta V)^2 \\ &= \frac{1}{2} (4 \text{ F}) (150 \text{ V})^2 \\ &= 4.5 \text{ J} \end{aligned}$$

$$\begin{aligned} 3. \rho &= 100 \Omega \cdot \text{cm} = 1.00 \Omega \cdot \text{m} \\ l &= 1.0 \text{ mm} = 1.0 \times 10^{-3} \text{ m} \\ d &= 2.0 \mu\text{m} \\ \hookrightarrow A &= \pi r^2 = \pi (1.0 \times 10^{-6} \text{ m})^2 = 3.14 \times 10^{-12} \text{ m}^2 \end{aligned}$$

$$\begin{aligned} R &= \rho \frac{l}{A} \\ &= (1 \Omega \cdot \text{m}) \frac{(1.0 \times 10^{-3} \text{ m})}{(3.14 \times 10^{-12} \text{ m}^2)} \end{aligned}$$

$$\boxed{R = 3.18 \times 10^8 \Omega}$$

$$\begin{aligned} 4. V &= 220 \text{ V} \\ P &= 2000 \text{ W} \end{aligned}$$

$$P = IV$$

$$I = \frac{P}{V}$$

$$= \frac{2000 \text{ W}}{220 \text{ V}}$$

$$\boxed{I = 9.1 \text{ A}}$$

### 1B Quiz 3

5. Current ( $I$ )  $\equiv \frac{\text{charge } (Q)}{\text{time } (t)}$

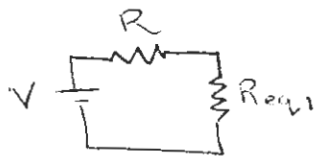
$$Q = 2e = 2 \times 1.60 \times 10^{-19} \text{ C}$$

$$t = 100 \mu\text{s} = 1.00 \times 10^{-4} \text{ s}$$

$$I = \frac{3.2 \times 10^{-19} \text{ C}}{1.0 \times 10^{-4} \text{ s}}$$

$$I = 3.2 \times 10^{-15} \text{ A}$$

6. Find  $R_{eq}$



$$\frac{1}{R_{eq1}} = \frac{1}{2R} + \frac{1}{2R}$$

$$R_{eq1} = R$$



$$R_{eq} = R + R_{eq1}$$

$$= 2R$$

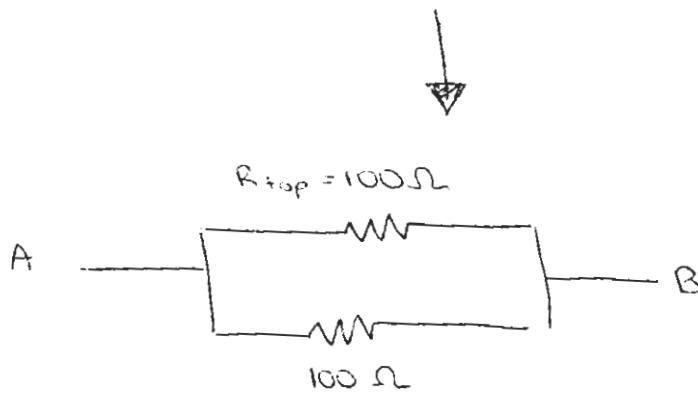
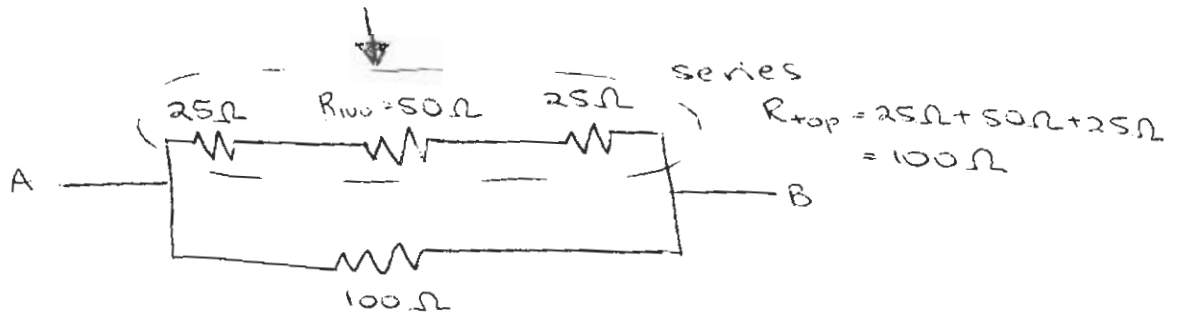
Using  $V = IR$ ,  $I = \frac{V}{R}$  where resistance =  $2R$

$$\therefore I = \frac{V}{2R}$$

1B Quiz 3

7. Find  $R_{eq}$

parallel  $\frac{1}{R_{100}} = \frac{1}{100\Omega} + \frac{1}{100\Omega} \rightarrow R_{100} = 50\Omega$



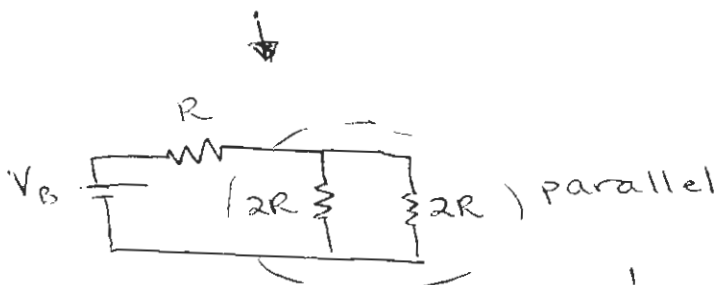
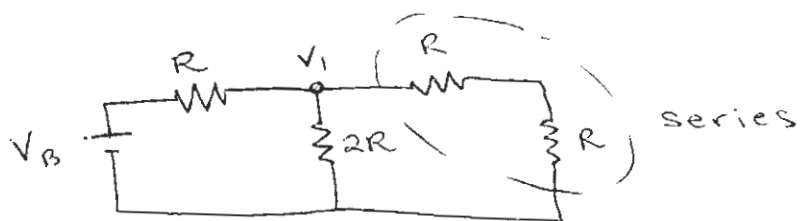
$$\frac{1}{R_{eq}} = \frac{1}{R_{top}} + \frac{1}{100\Omega}$$

$R_{eq} = 50\Omega$



# IB Quiz 3

8.



$$\frac{1}{R_{\text{par}}} = \frac{1}{2R} + \frac{1}{2R}$$

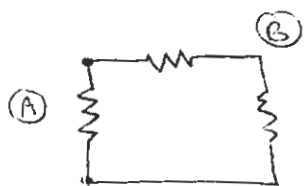
$$R_{\text{par}} = R$$



by symmetry  
 same resistance, so  $V$  same voltage drop  
 across both:  $\frac{V_B}{2}$ .

$$\therefore \boxed{V_1 = \frac{V_B}{2}}$$

9.



Branches (A) and (B) are in parallel, so the same voltage drops across both branches. In #7, we found that this

was  $\frac{V_B}{2}$ . The two resistors in series in branch B have the same resistance, so half of the voltage in branch B drops across each.

$$\therefore \boxed{V_2 = \frac{V_B}{4}}$$

## 1B Quiz 3

10. Any time the circuit is closed, current from the battery will flow across the battery's internal resistance. Therefore there is a voltage drop across the internal resistance, leaving less voltage to drop across the load resistance than the open circuit voltage. Therefore the voltage drop across the external load is

always less than  $V_{\text{battery}}$ .