Quiz 5

1. ∘ $V_{Na} - I R_{Na} - I R_{Leak} - V_{Leak} = 0$ (Kirchoff's loop rule)

∘ $V_{membrane} = V_{Na} - I R_{Na}$

$$\frac{V_{Na} - V_{Leak}}{R_{Na} + R_{Leak}} = I$$

Substituting,

$$V_{membrane} = V_{Na} - \left(\frac{V_{Na} - V_{Leak}}{R_{Na} + R_{Leak}}\right) R_{Na} = \frac{V_{Na} R_{Leak} + V_{Leak} R_{Na}}{R_{Na} + R_{Leak}}$$

2. (Brackets mean "the units of")

$$\Phi_B = B_\perp A$$

$$[\Phi_B] = [B_\perp][A]$$

$$1\ Wb = 1\ T \cdot m^2$$

$$[B] = \frac{[F]}{[I][\ell]}$$

$$1\ T = 1\ N/A \cdot m$$

$$= 1\ \frac{kg \cdot m}{s^2} \cdot \frac{1}{A \cdot m}$$

$$\therefore 1\ Wb = 1\ \frac{kg}{C \cdot s} \cdot m^2$$

3. $|\mathcal{E}| = N \frac{|\Delta \Phi_B|}{\Delta t}$

$|\Delta \Phi_B| = |\Delta B_\perp| A$ since A is constant

$$|\mathcal{E}| = NA \frac{|\Delta B_\perp|}{\Delta t}$$

$$= 10 [\pi (10 \times 10^{-6} m)^2](1.0 \times 10^{-2}\ T/s)$$

$$|\mathcal{E}| = 3.1 \times 10^{-11}\ V$$

Quiz 5

4. $\Delta \Phi_B = \Delta (B_\perp A)$

$= (\Delta B_\perp) A$ because the loop's area is constant

The magnetic flux through (into) the paper increases
as the field strength increases so a current is
induced in the coil to create a flux in
the opposite direction. To create a magnetic field
out of the page (and therefore a flux because
the wire initially caused no magnetic field),
the current in the wire flows counterclockwise.

5. The current through an LR circuit
at time t is given by

$$I = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau}\right).$$

When $t=0$, $e^{-t/\tau} = 1$, so $\boxed{I = 0}$

Quiz 5

6. A long time after the switch is closed, the current is no longer changing, so $\frac{\Delta I}{\Delta t} = 0$. The voltage drop across an inductor is $\mathcal{E} = -L\frac{\Delta I}{\Delta t}$. Therefore, after a very long time, there is no voltage drop across the inductors. The voltage across the resistor equals the battery's voltage, so using Ohm's law, $\boxed{I = V/R}$.

7. Writing out the loop equation for the circuit,

$$V - L_1\frac{\Delta I}{\Delta t} - L_2\frac{\Delta I}{\Delta t} - IR = 0,$$

we can combine terms:

$$V - (L_1 + L_2)\frac{\Delta I}{\Delta t} - IR = 0$$

Thus, the effect of two inductors in series is the same as one inductor whose inductance is the sum of the original inductors.

$$\therefore \tau = L/R \longrightarrow \tau = \frac{L_1 + L_2}{R}$$

$$\tau = \frac{15 \times 10^{-6}\,\text{H}}{8\ \Omega}$$

$$\boxed{\tau = 1.9 \times 10^{-6}\ \text{s}}$$

Quiz 5

8. $\sum F = -mg + I \overset{w}{\boxtimes} B$

$I = \mathcal{E}/R$ where $\mathcal{E}$ is the magnitude of the induced emf

$$\mathcal{E} = N \frac{\Delta \Phi_B}{\Delta t}$$

$$= N \frac{Bw(\Delta d)}{\boxtimes(\Delta t)} \qquad (N=1)$$

If $a=0$, $F=0$, and the loop will move with constant (terminal) velocity.

$v = \frac{\Delta d}{\Delta t}$. Substituting,

$$-mg + \frac{Bwv}{R} \overset{w}{\boxtimes} B = 0$$

$$\boxed{v = \frac{mgR}{(wB)^2}}$$

Quiz 5

10. $B = \mu_0 n I$

$= (4\pi \times 10^{-7}\ T\cdot m/A)\left(\frac{1000\ turns}{100 \times 10^{-2}\ m}\right)(2.5\ A)$

$B = 3.1 \times 10^{-3}\ T$

$\phi_B = B_\perp A$

$= (3.1 \times 10^{-3}\ T)[\pi (5.0 \times 10^{-2}\ m)^2]$

$= 2.5 \times 10^{-5}\ Wb$

9. $\mathcal{E}_{Max} = NBA\omega$

$= (1000)(0.4\ T)(1.0 \times 10^{-2}\ m)^2 (2\pi \cdot 60\ Hz)$

$\mathcal{E}_{Max} = 15\ V$