

Arecibo, a large radio telescope in Puerto Rico, gathers electromagnetic radiation in the form of radio waves. These long wavelengths pass through obscuring dust clouds, allowing astronomers to create images of the core region of the Milky Way Galaxy, which can't be observed in the visible spectrum.

CHAPTER 21

Alternating Current Circuits and Electromagnetic Waves

Every time we turn on a television set, a stereo system, or any of a multitude of other electric appliances, we call on alternating currents (AC) to provide the power to operate them. We begin our study of AC circuits by examining the characteristics of a circuit containing a source of emf and one other circuit element: a resistor, a capacitor, or an inductor. Then we examine what happens when these elements are connected in combination with each other. Our discussion is limited to simple series configurations of the three kinds of elements.

We conclude this chapter with a discussion of **electromagnetic waves**, which are composed of fluctuating electric and magnetic fields. Electromagnetic waves in the form of visible light enable us to view the world around us; infrared waves warm our environment; radio-frequency waves carry our television and radio programs, as well as information about processes in the core of our galaxy. X-rays allow us to perceive structures hidden inside our bodies, and study properties of distant, collapsed stars. Light is key to our understanding of the universe.

21.1 RESISTORS IN AN AC CIRCUIT

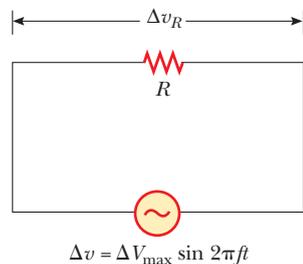
An AC circuit consists of combinations of circuit elements and an AC generator or an AC source, which provides the alternating current. We have seen that the output of an AC generator is sinusoidal and varies with time according to

$$\Delta v = \Delta V_{\max} \sin 2\pi ft \quad [21.1]$$

where Δv is the instantaneous voltage, ΔV_{\max} is the maximum voltage of the AC generator, and f is the frequency at which the voltage changes, measured in hertz (Hz). (Compare Equations 20.7 and 20.8 with Equation 21.1.) We first consider a simple

OUTLINE

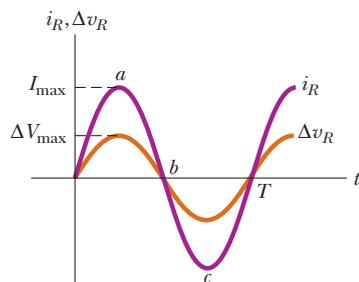
- 21.1 Resistors in an AC Circuit
- 21.2 Capacitors in an AC Circuit
- 21.3 Inductors in an AC Circuit
- 21.4 The *RLC* Series Circuit
- 21.5 Power in an AC Circuit
- 21.6 Resonance in a Series *RLC* Circuit
- 21.7 The Transformer
- 21.8 Maxwell's Predictions
- 21.9 Hertz's Confirmation of Maxwell's Predictions
- 21.10 Production of Electromagnetic Waves by an Antenna
- 21.11 Properties of Electromagnetic Waves
- 21.12 The Spectrum of Electromagnetic Waves
- 21.13 The Doppler Effect for Electromagnetic Waves

**ACTIVE FIGURE 21.1**

A series circuit consisting of a resistor R connected to an AC generator, designated by the symbol

**Physics Now™**

Log into PhysicsNow at www.cp7e.com and go to Active Figure 21.1, where you can adjust the resistance, the frequency, and the maximum voltage of the circuit shown. The results can be studied with the graph and phasor diagram in Active Figure 21.2.

**ACTIVE FIGURE 21.2**

A plot of current and voltage across a resistor versus time.

Physics Now™

Log into PhysicsNow at www.cp7e.com and go to Active Figure 21.2, where you can adjust the resistance, the frequency, and the maximum voltage of the circuit in Active Figure 21.1. The results can be studied with the graph and phasor diagram in Active Figure 21.20.

circuit consisting of a resistor and an AC source (designated by the symbol ) , as in Active Figure 21.1. The current and the voltage across the resistor are shown in Active Figure 21.2.

To explain the concept of alternating current, we begin by discussing the current-versus-time curve in Active Figure 21.2. At point a on the curve, the current has a maximum value in one direction, arbitrarily called the positive direction. Between points a and b , the current is decreasing in magnitude but is still in the positive direction. At point b , the current is momentarily zero; it then begins to increase in the opposite (negative) direction between points b and c . At point c , the current has reached its maximum value in the negative direction.

The current and voltage are in step with each other because they vary identically with time. **Because the current and the voltage reach their maximum values at the same time, they are said to be in phase.** Notice that **the average value of the current over one cycle is zero.** This is because the current is maintained in one direction (the positive direction) for the same amount of time and at the same magnitude as it is in the opposite direction (the negative direction). However, the direction of the current has no effect on the behavior of the resistor in the circuit: the collisions between electrons and the fixed atoms of the resistor result in an increase in the resistor's temperature regardless of the direction of the current.

We can quantify this discussion by recalling that the rate at which electrical energy is dissipated in a resistor, the power \mathcal{P} , is

$$\mathcal{P} = i^2 R$$

where i is the *instantaneous* current in the resistor. Because the heating effect of a current is proportional to the *square* of the current, it makes no difference whether the sign associated with the current is positive or negative. However, the heating effect produced by an alternating current with a maximum value of I_{\max} is *not the same* as that produced by a direct current of the same value. The reason is that the alternating current has this maximum value for only an instant of time during a cycle. The important quantity in an AC circuit is a special kind of average value of current, called the **rms current**—the direct current that dissipates the same amount of energy in a resistor that is dissipated by the actual alternating current. To find the rms current, we first square the current, then find its average value, and finally take the square root of this average value. Hence, the rms current is the square root of the average (*mean*) of the *square* of the current. Because i^2 varies as $\sin^2 2\pi ft$, the average value of i^2 is $\frac{1}{2} I_{\max}^2$ (Fig. 21.3b).¹ Therefore, the rms current I_{rms} is related to the maximum value of the alternating current I_{\max} by

$$I_{\text{rms}} = \frac{I_{\max}}{\sqrt{2}} = 0.707 I_{\max} \quad [21.2]$$

This equation says that an alternating current with a maximum value of 3 A produces the same heating effect in a resistor as a direct current of $(3/\sqrt{2})$ A. We can therefore say that the average power dissipated in a resistor that carries alternating current I is

$$\mathcal{P}_{\text{av}} = I_{\text{rms}}^2 R.$$

¹The fact that $(i^2)_{\text{av}} = I_{\max}^2/2$ can be shown as follows: The current in the circuit varies with time according to the expression $i = I_{\max} \sin 2\pi ft$, so $i^2 = I_{\max}^2 \sin^2 2\pi ft$. Therefore, we can find the average value of i^2 by calculating the average value of $\sin^2 2\pi ft$. Note that a graph of $\cos^2 2\pi ft$ versus time is identical to a graph of $\sin^2 2\pi ft$ versus time, except that the points are shifted on the time axis. Thus, the time average of $\sin^2 2\pi ft$ is equal to the time average of $\cos^2 2\pi ft$, taken over one or more cycles. That is,

$$(\sin^2 2\pi ft)_{\text{av}} = (\cos^2 2\pi ft)_{\text{av}}$$

With this fact and the trigonometric identity $\sin^2 \theta + \cos^2 \theta = 1$, we get

$$(\sin^2 2\pi ft)_{\text{av}} + (\cos^2 2\pi ft)_{\text{av}} = 2(\sin^2 2\pi ft)_{\text{av}} = 1$$

$$(\sin^2 2\pi ft)_{\text{av}} = \frac{1}{2}$$

When this result is substituted into the expression $i^2 = I_{\max}^2 \sin^2 2\pi ft$, we get $(i^2)_{\text{av}} = I_{\text{rms}}^2 = I_{\max}^2/2$, or $I_{\text{rms}} = I_{\max}/\sqrt{2}$, where I_{rms} is the rms current.

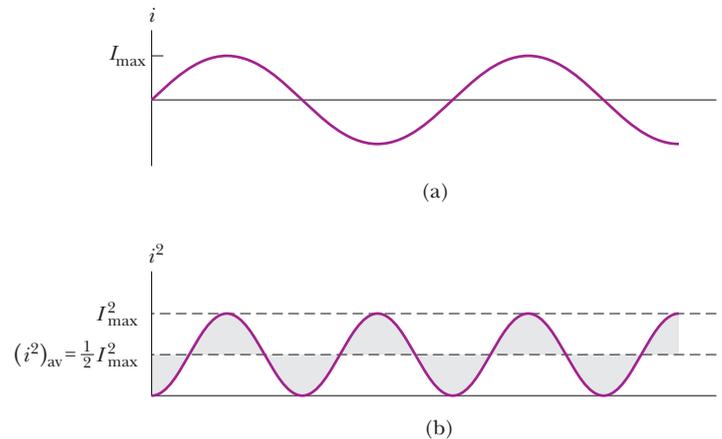


Figure 21.3 (a) Plot of the current in a resistor as a function of time. (b) Plot of the square of the current in a resistor as a function of time. Notice that the gray shaded regions *under* the curve and *above* the dashed line for $I_{\max}^2/2$ have the same area as the gray shaded regions *above* the curve and *below* the dashed line for $I_{\max}^2/2$. Thus, the average value of i^2 is $I_{\max}^2/2$.

Alternating voltages are also best discussed in terms of rms voltages, with a relationship identical to the preceding one,

$$\Delta V_{\text{rms}} = \frac{\Delta V_{\text{max}}}{\sqrt{2}} = 0.707 \Delta V_{\text{max}} \quad [21.3]$$

◀ rms voltage

where ΔV_{rms} is the rms voltage and ΔV_{max} is the maximum value of the alternating voltage.

When we speak of measuring an AC voltage of 120 V from an electric outlet, we really mean an rms voltage of 120 V. A quick calculation using Equation 21.3 shows that such an AC voltage actually has a peak value of about 170 V. In this chapter we use rms values when discussing alternating currents and voltages. One reason is that AC ammeters and voltmeters are designed to read rms values. Further, if we use rms values, many of the equations for alternating current will have the same form as those used in the study of direct-current (DC) circuits. Table 21.1 summarizes the notations used throughout the chapter.

Consider the series circuit in Figure 21.1, consisting of a resistor connected to an AC generator. A resistor impedes the current in an AC circuit, just as it does in a DC circuit. Ohm's law is therefore valid for an AC circuit, and we have

$$\Delta V_{R,\text{rms}} = I_{\text{rms}} R \quad [21.4a]$$

The rms voltage across a resistor is equal to the rms current in the circuit times the resistance. This equation is also true if maximum values of current and voltage are used:

$$\Delta V_{R,\text{max}} = I_{\text{max}} R \quad [21.4b]$$

Quick Quiz 21.1

Which of the following statements can be true for a resistor connected in a simple series circuit to an operating AC generator? (a) $\mathcal{P}_{\text{av}} = 0$ and $i_{\text{av}} = 0$ (b) $\mathcal{P}_{\text{av}} = 0$ and $i_{\text{av}} > 0$ (c) $\mathcal{P}_{\text{av}} > 0$ and $i_{\text{av}} = 0$ (d) $\mathcal{P}_{\text{av}} > 0$ and $i_{\text{av}} > 0$

EXAMPLE 21.1 What Is the rms Current?

Goal Perform basic AC circuit calculations for a purely resistive circuit.

Problem An AC voltage source has an output of $\Delta v = (2.00 \times 10^2 \text{ V}) \sin 2\pi ft$. This source is connected to a $1.00 \times 10^2\text{-}\Omega$ resistor as in Figure 21.1. Find the rms voltage and rms current in the resistor.

Strategy Compare the expression for the voltage output just given with the general form, $\Delta v = \Delta V_{\text{max}} \sin 2\pi ft$, finding the maximum voltage. Substitute this result into the expression for the rms voltage.

TABLE 21.1

Notation Used in This Chapter

	Voltage	Current
Instantaneous value	Δv	i
Maximum value	ΔV_{max}	I_{max}
rms value	ΔV_{rms}	I_{rms}

Solution

Obtain the maximum voltage by comparison of the given expression for the output with the general expression:

$$\Delta v = (2.00 \times 10^2 \text{ V}) \sin 2\pi ft \quad \Delta v = \Delta V_{\text{max}} \sin 2\pi ft$$

$$\rightarrow \Delta V_{\text{max}} = 2.00 \times 10^2 \text{ V}$$

Next, substitute into Equation 21.3 to find the rms voltage of the source:

$$\Delta V_{\text{rms}} = \frac{\Delta V_{\text{max}}}{\sqrt{2}} = \frac{2.00 \times 10^2 \text{ V}}{\sqrt{2}} = 141 \text{ V}$$

Substitute this result into Ohm's law to find the rms current:

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{R} = \frac{141 \text{ V}}{1.00 \times 10^2 \Omega} = 1.41 \text{ A}$$

Remarks Notice how the concept of rms values allows the handling of an AC circuit quantitatively in much the same way as a DC circuit.

Exercise 21.1

Find the maximum current in the circuit and the average power delivered to the circuit.

Answer 2.00 A; $2.00 \times 10^2 \text{ W}$

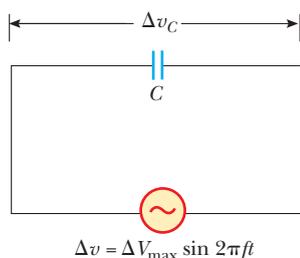


Figure 21.4 A series circuit consisting of a capacitor C connected to an AC generator.

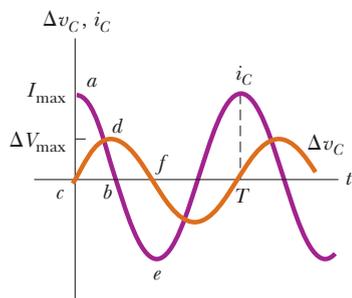


Figure 21.5 Plots of current and voltage across a capacitor versus time in an AC circuit. The voltage lags the current by 90° .

The voltage across a capacitor lags the current by 90° ▶

Capacitive reactance ▶

21.2 CAPACITORS IN AN AC CIRCUIT

To understand the effect of a capacitor on the behavior of a circuit containing an AC voltage source, we first review what happens when a capacitor is placed in a circuit containing a DC source, such as a battery. When the switch is closed in a series circuit containing a battery, a resistor, and a capacitor, the initial charge on the plates of the capacitor is zero. The motion of charge through the circuit is therefore relatively free, and there is a large current in the circuit. As more charge accumulates on the capacitor, the voltage across it increases, opposing the current. After some time interval, which depends on the time constant RC , the current approaches zero. Consequently, a capacitor in a DC circuit limits or impedes the current so that it approaches zero after a brief time.

Now consider the simple series circuit in Figure 21.4, consisting of a capacitor connected to an AC generator. We sketch curves of current versus time and voltage versus time, and then attempt to make the graphs seem reasonable. The curves are shown in Figure 21.5. First, note that the segment of the current curve from a to b indicates that the current starts out at a rather large value. This can be understood by recognizing that there is no charge on the capacitor at $t = 0$; as a consequence, there is nothing in the circuit except the resistance of the wires to hinder the flow of charge at this instant. However, the current decreases as the voltage across the capacitor increases from c to d on the voltage curve. When the voltage is at point d , the current reverses and begins to increase in the opposite direction (from b to e on the current curve). During this time, the voltage across the capacitor decreases from d to f because the plates are now losing the charge they accumulated earlier. The remainder of the cycle for both voltage and current is a repeat of what happened during the first half of the cycle. The current reaches a maximum value in the opposite direction at point e on the current curve and then decreases as the voltage across the capacitor builds up.

In a purely resistive circuit, the current and voltage are always in step with each other. This isn't the case when a capacitor is in the circuit. In Figure 21.5, when an alternating voltage is applied across a capacitor, the voltage reaches its maximum value one-quarter of a cycle after the current reaches its maximum value. We say that **the voltage across a capacitor always lags the current by 90°** .

The impeding effect of a capacitor on the current in an AC circuit is expressed in terms of a factor called the **capacitive reactance** X_C , defined as

$$X_C \equiv \frac{1}{2\pi fC} \quad [21.5]$$

When C is in farads and f is in hertz, the unit of X_C is the ohm. Notice that $2\pi f = \omega$, the angular frequency.

From Equation 21.5, as the frequency f of the voltage source increases, the capacitive reactance X_C (the impeding effect of the capacitor) decreases, so the current increases. At high frequency, there is less time available to charge the capacitor, so less charge and voltage accumulate on the capacitor, which translates into less opposition to the flow of charge and, consequently, a higher current. The analogy between capacitive reactance and resistance means that we can write an equation of the same form as Ohm's law to describe AC circuits containing capacitors. This equation relates the rms voltage and rms current in the circuit to the capacitive reactance:

$$\Delta V_{C,\text{rms}} = I_{\text{rms}} X_C \quad [21.6]$$

EXAMPLE 21.2 A Purely Capacitive AC Circuit

Goal Perform basic AC circuit calculations for a capacitive circuit.

Problem An $8.00\text{-}\mu\text{F}$ capacitor is connected to the terminals of an AC generator with an rms voltage of $1.50 \times 10^2 \text{ V}$ and a frequency of 60.0 Hz . Find the capacitive reactance and the rms current in the circuit.

Strategy Substitute values into Equations 21.5 and 21.6.

Solution

Substitute the values of f and C into Equation 21.5:

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi (60.0 \text{ Hz})(8.00 \times 10^{-6} \text{ F})} = 332 \Omega$$

Solve Equation 21.6 for the current, and substitute X_C and the rms voltage to find the rms current:

$$I_{\text{rms}} = \frac{\Delta V_{C,\text{rms}}}{X_C} = \frac{1.50 \times 10^2 \text{ V}}{332 \Omega} = 0.452 \text{ A}$$

Remark Again, notice how similar the technique is to that of analyzing a DC circuit with a resistor.

Exercise 21.2

If the frequency is doubled, what happens to the capacitive reactance and the rms current?

Answer X_C is halved, and I_{rms} is doubled.

21.3 INDUCTORS IN AN AC CIRCUIT

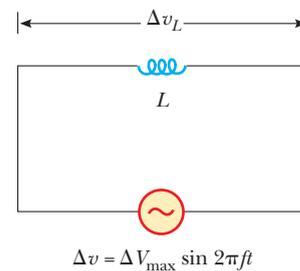
Now consider an AC circuit consisting only of an inductor connected to the terminals of an AC source, as in Active Figure 21.6. (In any real circuit, there is some resistance in the wire forming the inductive coil, but we ignore this for now.) The changing current output of the generator produces a back emf that impedes the current in the circuit. The magnitude of this back emf is

$$\Delta v_L = L \frac{\Delta I}{\Delta t} \quad [21.7]$$

The effective resistance of the coil in an AC circuit is measured by a quantity called the **inductive reactance**, X_L :

$$X_L \equiv 2\pi f L \quad [21.8]$$

When f is in hertz and L is in henries, the unit of X_L is the ohm. The inductive reactance *increases* with increasing frequency and increasing inductance. Contrast these facts with capacitors, where increasing frequency or capacitance *decreases* the capacitive reactance.

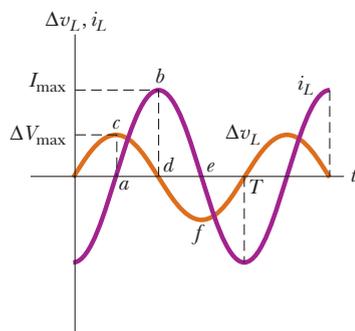


ACTIVE FIGURE 21.6

A series circuit consisting of an inductor L connected to an AC generator.

Physics Now™

Log into PhysicsNow at www.cp7e.com and go to Active Figure 21.6, where you can adjust the inductance, the frequency, and the maximum voltage. The results can be studied with the graph and phasor diagram in Active Figure 21.7.

**ACTIVE FIGURE 21.7**

Plots of current and voltage across an inductor versus time in an AC circuit. The voltage leads the current by 90° .

Physics Now™

Log into PhysicsNow at www.cp7e.com and go to Active Figure 21.6, where you can adjust the inductance, the frequency, and the maximum voltage. The results can be studied with the graph and phasor diagram in Active Figure 21.7.

To understand the meaning of inductive reactance, compare Equation 21.8 with Equation 21.7. First, note from Equation 21.8 that the inductive reactance depends on the inductance L . This is reasonable, because the back emf (Eq. 21.7) is large for large values of L . Second, note that the inductive reactance depends on the frequency f . This, too, is reasonable, because the back emf depends on $\Delta I/\Delta t$, a quantity that is large when the current changes rapidly, as it would for high frequencies.

With inductive reactance defined in this way, we can write an equation of the same form as Ohm's law for the voltage across the coil or inductor:

$$\Delta V_{L,\text{rms}} = I_{\text{rms}} X_L \quad [21.9]$$

where $\Delta V_{L,\text{rms}}$ is the rms voltage across the coil and I_{rms} is the rms current in the coil.

Active Figure 21.7 shows the instantaneous voltage and instantaneous current across the coil as functions of time. When a sinusoidal voltage is applied across an inductor, the voltage reaches its maximum value one-quarter of an oscillation period before the current reaches its maximum value. In this situation, we say that **the voltage across an inductor always leads the current by 90°** .

To see why there is a phase relationship between voltage and current, we examine a few points on the curves of Active Figure 21.7. At point a on the current curve, the current is beginning to increase in the positive direction. At this instant, the rate of change of current, $\Delta I/\Delta t$ (the slope of the current curve), is at a maximum, and we see from Equation 21.7 that the voltage across the inductor is consequently also at a maximum. As the current rises between points a and b on the curve, $\Delta I/\Delta t$ gradually decreases until it reaches zero at point b . As a result, the voltage across the inductor is decreasing during this same time interval, as the segment between c and d on the voltage curve indicates. Immediately after point b , the current begins to decrease, although it still has the same direction it had during the previous quarter cycle. As the current decreases to zero (from b to e on the curve), a voltage is again induced in the coil (from d to f), but the polarity of this voltage is opposite the polarity of the voltage induced between c and d . This occurs because back emfs always oppose the change in the current.

We could continue to examine other segments of the curves, but no new information would be gained because the current and voltage variations are repetitive.

EXAMPLE 21.3 A Purely Inductive AC Circuit

Goal Perform basic AC circuit calculations for an inductive circuit.

Problem In a purely inductive AC circuit (see Active Fig. 21.6), $L = 25.0$ mH and the rms voltage is 1.50×10^2 V. Find the inductive reactance and rms current in the circuit if the frequency is 60.0 Hz.

Solution

Substitute L and f into Equation 21.8 to get the inductive reactance:

$$X_L = 2\pi fL = 2\pi(60.0 \text{ s}^{-1})(25.0 \times 10^{-3} \text{ H}) = 9.42 \Omega$$

Solve Equation 21.9 for the rms current and substitute:

$$I_{\text{rms}} = \frac{\Delta V_{L,\text{rms}}}{X_L} = \frac{1.50 \times 10^2 \text{ V}}{9.42 \Omega} = 15.9 \text{ A}$$

Remark The analogy with DC circuits is even closer than in the capacitive case, because in the inductive equivalent of Ohm's law, the voltage across an inductor is *proportional* to the inductance L , just as the voltage across a resistor is proportional to R in Ohm's law.

Exercise 21.3

Calculate the inductive reactance and rms current in a similar circuit if the frequency is again 60.0 Hz, but the rms voltage is 85.0 V and the inductance is 47.0 mH.

Answers $X_L = 17.7 \Omega$; $I = 4.80 \text{ A}$

21.4 THE RLC SERIES CIRCUIT

In the foregoing sections, we examined the effects of an inductor, a capacitor, and a resistor when they are connected separately across an AC voltage source. We now consider what happens when these devices are combined.

Active Figure 21.8 shows a circuit containing a resistor, an inductor, and a capacitor connected in series across an AC source that supplies a total voltage Δv at some instant. The current in the circuit is the same at all points in the circuit at any instant and varies sinusoidally with time, as indicated in Active Figure 21.9a. This fact can be expressed mathematically as

$$i = I_{\max} \sin 2\pi ft$$

Earlier, we learned that the voltage across each element may or may not be in phase with the current. The instantaneous voltages across the three elements, shown in Active Figure 21.9, have the following phase relations to the instantaneous current:

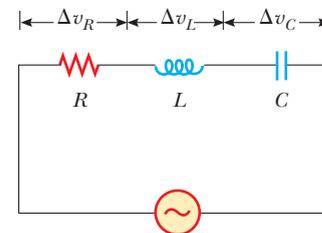
1. The instantaneous voltage Δv_R across the resistor is *in phase* with the instantaneous current. (See Active Fig. 21.9b.)
2. The instantaneous voltage Δv_L across the inductor *leads* the current by 90° . (See Active Fig. 21.9c.)
3. The instantaneous voltage Δv_C across the capacitor *lags* the current by 90° . (See Active Fig. 21.9d.)

The net instantaneous voltage Δv supplied by the AC source equals the sum of the instantaneous voltages across the separate elements: $\Delta v = \Delta v_R + \Delta v_C + \Delta v_L$. This doesn't mean, however, that the voltages measured with an AC voltmeter across R , C , and L sum to the measured source voltage! In fact, the measured voltages *don't* sum to the measured source voltage, because the voltages across R , C , and L all have different phases.

To account for the different phases of the voltage drops, we use a technique involving vectors. We represent the voltage across each element with a rotating vector, as in Figure 21.10. The rotating vectors are referred to as **phasors**, and the diagram is called a **phasor diagram**. This particular diagram represents the circuit voltage given by the expression $\Delta v = \Delta V_{\max} \sin(2\pi ft + \phi)$, where ΔV_{\max} is the maximum voltage (the magnitude or length of the rotating vector or phasor) and ϕ is the angle between the phasor and the $+x$ -axis when $t = 0$. The phasor can be viewed as a vector of magnitude ΔV_{\max} rotating at a constant frequency f so that its projection along the y -axis is the instantaneous voltage in the circuit. Because ϕ is the phase angle between the voltage and current in the circuit, the phasor for the current (not shown in Fig. 21.10) lies along the positive x -axis when $t = 0$ and is expressed by the relation $i = I_{\max} \sin(2\pi ft)$.

The phasor diagrams in Figure 21.11 (page 700) are useful for analyzing the *series RLC* circuit. Voltages in phase with the current are represented by vectors along the positive x -axis, and voltages out of phase with the current lie along other directions. ΔV_R is horizontal and to the right because it's in phase with the current. Likewise, ΔV_L is represented by a phasor along the positive y -axis because it leads the current² by 90° . Finally, ΔV_C is along the negative y -axis because it lags the current² by 90° . If the phasors are added as vector quantities in order to account for the different phases of the voltages across R , L , and C , Figure 21.11a shows that the only x -component for the voltages is ΔV_R and the net y -component is $\Delta V_L - \Delta V_C$. We now add the phasors vectorially to find the phasor ΔV_{\max} (Fig. 21.11b), which represents the maximum voltage. The right triangle in Figure 21.11b gives the following equations for the maximum voltage and the phase angle ϕ between the maximum voltage and the current:

²A mnemonic to help you remember the phase relationships in *RLC* circuits is “*ELI* the *ICE* man.” *E* represents the voltage \mathcal{E} , *I* the current, *L* the inductance, and *C* the capacitance. Thus, the name *ELI* means that, in an inductive circuit, the voltage \mathcal{E} leads the current *I*. In a capacitive circuit, *ICE* means that the current leads the voltage.

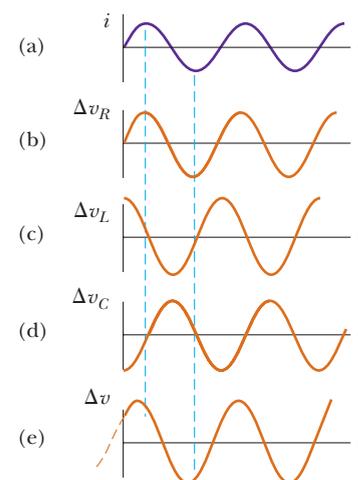


ACTIVE FIGURE 21.8

A series circuit consisting of a resistor, an inductor, and a capacitor connected to an AC generator.

PhysicsNow™

Log into PhysicsNow at www.cp7e.com and go to Active Figure 21.8, where you can adjust the resistance, the inductance, and the capacitance. The results can be studied with the graph in Active Figure 21.9 and the phasor diagram in Figure 21.10.



ACTIVE FIGURE 21.9

Phase relations in the series *RLC* circuit shown in Figure 21.8.

PhysicsNow™

Log into PhysicsNow at www.cp7e.com and go to Active Figure 21.9, where you can adjust the resistance, the inductance, and the capacitance in Active Figure 21.8. The results can be studied with the graph in this figure and the phasor diagram in Figure

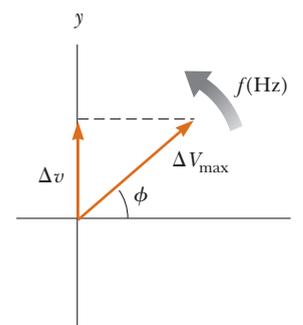
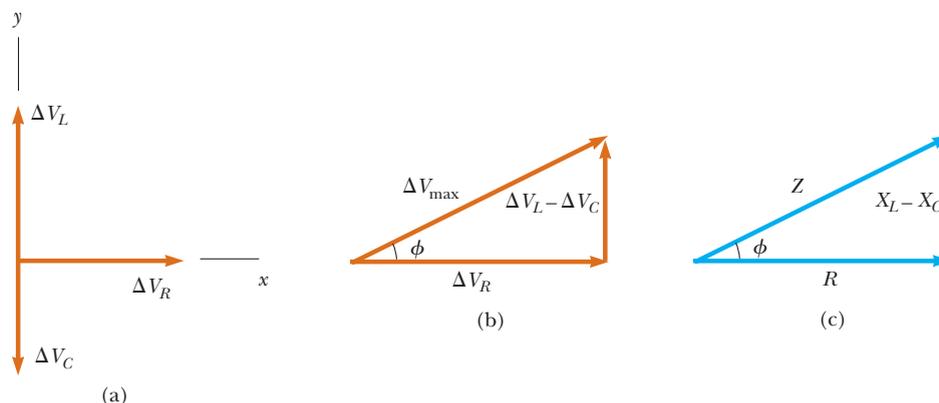


Figure 21.10 A phasor diagram for the voltage in an AC circuit, where ϕ is the phase angle between the voltage and the current and Δv is the instantaneous voltage.

Figure 21.11 (a) A phasor diagram for the RLC circuit. (b) Addition of the phasors as vectors gives $\Delta V_{\max} = \sqrt{\Delta V_R^2 + (\Delta V_L - \Delta V_C)^2}$. (c) The reactance triangle that gives the impedance relation $Z = \sqrt{R^2 + (X_L - X_C)^2}$.



$$\Delta V_{\max} = \sqrt{\Delta V_R^2 + (\Delta V_L - \Delta V_C)^2} \quad [21.10]$$

$$\tan \phi = \frac{\Delta V_L - \Delta V_C}{\Delta V_R} \quad [21.11]$$

In these equations, all voltages are maximum values. Although we choose to use maximum voltages in our analysis, the preceding equations apply equally well to rms voltages, because the two quantities are related to each other by the same factor for all circuit elements. The result for the maximum voltage ΔV_{\max} given by Equation 21.10 reinforces the fact that **the voltages across the resistor, capacitor, and inductor are not in phase, so one cannot simply add them to get the voltage across the combination of element, or the source voltage.**

Quick Quiz 21.2

For the circuit of Figure 21.8, is the instantaneous voltage of the source equal to (a) the sum of the maximum voltages across the elements, (b) the sum of the instantaneous voltages across the elements, or (c) the sum of the rms voltages across the elements?

We can write Equation 21.10 in the form of Ohm's law, using the relations $\Delta V_R = I_{\max}R$, $\Delta V_L = I_{\max}X_L$, and $\Delta V_C = I_{\max}X_C$, where I_{\max} is the maximum current in the circuit:

$$\Delta V_{\max} = I_{\max} \sqrt{R^2 + (X_L - X_C)^2} \quad [21.12]$$

It's convenient to define a parameter called the **impedance** Z of the circuit as

Impedance ►

$$Z \equiv \sqrt{R^2 + (X_L - X_C)^2} \quad [21.13]$$

so that Equation 21.12 becomes

$$\Delta V_{\max} = I_{\max} Z \quad [21.14]$$

Equation 21.14 is in the form of Ohm's law, $\Delta V = IR$, with R replaced by the impedance in ohms. Indeed, Equation 21.14 can be regarded as a generalized form of Ohm's law applied to a series AC circuit. Both the impedance and, therefore, the current in an AC circuit depend on the resistance, the inductance, the capacitance, and the frequency (because the reactances are frequency dependent).

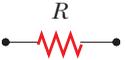
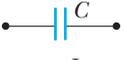
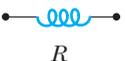
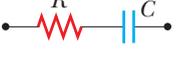
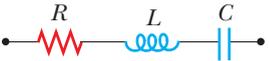
It's useful to represent the impedance Z with a vector diagram such as the one depicted in Figure 21.11c. A right triangle is constructed with right side $X_L - X_C$, base R , and hypotenuse Z . Applying the Pythagorean theorem to this triangle, we see that

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

which is Equation 21.13. Furthermore, we see from the vector diagram in Figure 21.11c that the phase angle ϕ between the current and the voltage obeys the

TABLE 21.2

Impedance Values and Phase Angles for Various Combinations of Circuit Elements^a

Circuit Elements	Impedance Z	Phase Angle ϕ
	R	0°
	X_C	-90°
	X_L	$+90^\circ$
	$\sqrt{R^2 + X_C^2}$	Negative, between -90° and 0°
	$\sqrt{R^2 + X_L^2}$	Positive, between 0° and 90°
	$\sqrt{R^2 + (X_L - X_C)^2}$	Negative if $X_C > X_L$ Positive if $X_C < X_L$

^aIn each case, an AC voltage (not shown) is applied across the combination of elements (that is, across the dots).

relationship

$$\tan \phi = \frac{X_L - X_C}{R}$$

[21.15]

◀ Phase angle ϕ

The physical significance of the phase angle will become apparent in Section 21.5.

Table 21.2 provides impedance values and phase angles for some series circuits containing different combinations of circuit elements.

Parallel alternating current circuits are also useful in everyday applications. We won't discuss them here, however, because their analysis is beyond the scope of this book.

Quick Quiz 21.3

The switch in the circuit shown in Figure 21.12 is closed and the lightbulb glows steadily. The inductor is a simple air-core solenoid. As an iron rod is being inserted into the interior of the solenoid, the brightness of the lightbulb (a) increases, (b) decreases, or (c) remains the same.

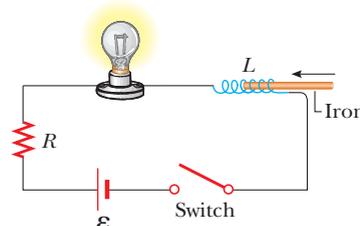
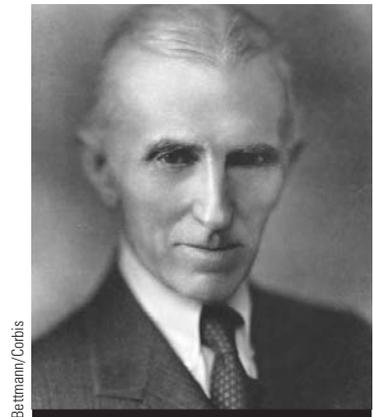


Figure 21.12 (Quick Quiz 21.3)



Bettmann/Corbis

NIKOLA TESLA (1856–1943)

Tesla was born in Croatia, but spent most of his professional life as an inventor in the United States. He was a key figure in the development of alternating-current electricity, high-voltage transformers, and the transport of electrical power via AC transmission lines. Tesla's viewpoint was at odds with the ideas of Edison, who committed himself to the use of direct current in power transmission. Tesla's AC approach won out.

Problem-Solving Strategy Alternating Current

The following procedure is recommended for solving alternating-current problems:

1. Calculate as many of the unknown quantities, such as X_L and X_C , as possible.
2. Apply the equation $\Delta V_{\max} = I_{\max} Z$ to the portion of the circuit of interest. For example, if you want to know the voltage drop across the combination of an inductor and a resistor, the equation for the voltage drop reduces to $\Delta V_{\max} = I_{\max} \sqrt{R^2 + X_L^2}$.

EXAMPLE 21.4 An *RLC* Circuit

Goal Analyze a series *RLC* AC circuit and find the phase angle.

Problem A series *RLC* AC circuit has resistance $R = 2.50 \times 10^2 \Omega$, inductance $L = 0.600 \text{ H}$, capacitance $C = 3.50 \mu\text{F}$, frequency $f = 60.0 \text{ Hz}$, and maximum voltage $\Delta V_{\max} = 1.50 \times 10^2 \text{ V}$. Find (a) the impedance, (b) the maximum current in the circuit, (c) the phase angle, and (d) the maximum voltages across the elements.

Strategy Calculate the inductive and capacitive reactances, then substitute them and given quantities into the appropriate equations.

Solution

(a) Find the impedance of the circuit.

First, calculate the inductive and capacitive reactances:

$$X_L = 2\pi fL = 226 \Omega \quad X_C = 1/2\pi fC = 758 \Omega$$

Substitute these results and the resistance R into Equation 21.13 to obtain the impedance of the circuit:

$$\begin{aligned} Z &= \sqrt{R^2 + (X_L - X_C)^2} \\ &= \sqrt{(2.50 \times 10^2 \Omega)^2 + (226 \Omega - 758 \Omega)^2} = 588 \Omega \end{aligned}$$

(b) Find the maximum current.

Use Equation 21.12, the equivalent of Ohm's law, to find the maximum current:

$$I_{\max} = \frac{\Delta V_{\max}}{Z} = \frac{1.50 \times 10^2 \text{ V}}{588 \Omega} = 0.255 \text{ A}$$

(c) Find the phase angle.

Calculate the phase angle between the current and the voltage with Equation 21.15:

$$\phi = \tan^{-1} \frac{X_L - X_C}{R} = \tan^{-1} \left(\frac{226 \Omega - 758 \Omega}{2.50 \times 10^2 \Omega} \right) = -64.8^\circ$$

(d) Find the maximum voltages across the elements.

Substitute into the "Ohm's law" expressions for each individual type of current element:

$$\Delta V_{R,\max} = I_{\max} R = (0.255 \text{ A})(2.50 \times 10^2 \Omega) = 63.8 \text{ V}$$

$$\Delta V_{L,\max} = I_{\max} X_L = (0.255 \text{ A})(226 \Omega) = 57.6 \text{ V}$$

$$\Delta V_{C,\max} = I_{\max} X_C = (0.255 \text{ A})(758 \Omega) = 193 \text{ V}$$

Remarks Because the circuit is more capacitive than inductive ($X_C > X_L$), ϕ is negative. A negative phase angle means that the current leads the applied voltage. Notice also that the sum of the maximum voltages across the elements is $\Delta V_R + \Delta V_L + \Delta V_C = 314 \text{ V}$, which is much greater than the maximum voltage of the generator, 150 V. As we saw in Quick Quiz 21.2, the sum of the maximum voltages is a meaningless quantity because when alternating voltages are added, *both their amplitudes and their phases* must be taken into account. We know that the maximum voltages across the various elements occur at different times, so it doesn't make sense to add all the maximum values. The correct way to "add" the voltages is through Equation 21.10.

Exercise 21.4

Analyze a series RLC AC circuit for which $R = 175 \Omega$, $L = 0.500 \text{ H}$, $C = 22.5 \mu\text{F}$, $f = 60.0 \text{ Hz}$, and $\Delta V_{\max} = 325 \text{ V}$. Find (a) the impedance, (b) the maximum current, (c) the phase angle, and (d) the maximum voltages across the elements.

Answers (a) 189Ω (b) 1.72 A (c) 22.0° (d) $\Delta V_{R,\max} = 301 \text{ V}$, $\Delta V_{L,\max} = 324 \text{ V}$, $\Delta V_{C,\max} = 203 \text{ V}$

21.5 POWER IN AN AC CIRCUIT

No power losses are associated with pure capacitors and pure inductors in an AC circuit. A pure capacitor, by definition, has no resistance or inductance, while a pure inductor has no resistance or capacitance. (These are idealizations: in a real capacitor, for example, inductive effects could become important at high frequencies.) We begin by analyzing the power dissipated in an AC circuit that contains only a generator and a capacitor.

When the current increases in one direction in an AC circuit, charge accumulates on the capacitor and a voltage drop appears across it. When the voltage reaches its maximum value, the energy stored in the capacitor is

$$PE_C = \frac{1}{2} C (\Delta V_{\max})^2$$

However, this energy storage is only momentary: When the current reverses direction, the charge leaves the capacitor plates and returns to the voltage source. During one-half of each cycle the capacitor is being charged, and during the other half

the charge is being returned to the voltage source. Therefore, the average power supplied by the source is zero. In other words, **no power losses occur in a capacitor in an AC circuit.**

Similarly, the source must do work against the back emf of an inductor that is carrying a current. When the current reaches its maximum value, the energy stored in the inductor is a maximum and is given by

$$PE_L = \frac{1}{2}LI_{\max}^2$$

When the current begins to decrease in the circuit, this stored energy is returned to the source as the inductor attempts to maintain the current in the circuit. The average power delivered to a resistor in an *RLC* circuit is

$$\mathcal{P}_{\text{av}} = I_{\text{rms}}^2 R \quad [21.16]$$

The average power delivered by the generator is converted to internal energy in the resistor. No power loss occurs in an ideal capacitor or inductor.

An alternate equation for the average power loss in an AC circuit can be found by substituting (from Ohm's law) $R = \Delta V_R / I_{\text{rms}}$ into Equation 21.16:

$$\mathcal{P}_{\text{av}} = I_{\text{rms}} \Delta V_R$$

It's convenient to refer to a voltage triangle that shows the relationship among ΔV_{rms} , ΔV_R , and $\Delta V_L - \Delta V_C$, such as Figure 21.11b. (Remember that Fig. 21.11 applies to *both* maximum and rms voltages.) From this figure, we see that the voltage drop across a resistor can be written in terms of the voltage of the source, ΔV_{rms} :

$$\Delta V_R = \Delta V_{\text{rms}} \cos \phi$$

Hence, the average power delivered by a generator in an AC circuit is

$$\mathcal{P}_{\text{av}} = I_{\text{rms}} \Delta V_{\text{rms}} \cos \phi \quad [21.17]$$

◀ Average power

where the quantity $\cos \phi$ is called the **power factor**.

Equation 21.17 shows that the power delivered by an AC source to any circuit depends on the phase difference between the source voltage and the resulting current. This fact has many interesting applications. For example, factories often use devices such as large motors in machines, generators, and transformers that have a large inductive load due to all the windings. To deliver greater power to such devices without using excessively high voltages, factory technicians introduce capacitance in the circuits to shift the phase.

APPLICATION

Shifting Phase to Deliver More Power

EXAMPLE 21.5 Average Power in an *RLC* Series Circuit

Goal Understand power in *RLC* series circuits.

Problem Calculate the average power delivered to the series *RLC* circuit described in Example 21.4.

Strategy After finding the rms current and rms voltage with Equations 21.2 and 21.3, substitute into Equation 21.17, using the phase angle found in Example 21.4.

Solution

First, use Equations 21.2 and 21.3 to calculate the rms current and rms voltage:

$$I_{\text{rms}} = \frac{I_{\max}}{\sqrt{2}} = \frac{0.255 \text{ A}}{\sqrt{2}} = 0.180 \text{ A}$$

$$\Delta V_{\text{rms}} = \frac{\Delta V_{\max}}{\sqrt{2}} = \frac{1.50 \times 10^2 \text{ V}}{\sqrt{2}} = 106 \text{ V}$$

Substitute these results and the phase angle $\phi = -64.8^\circ$ into Equation 21.17 to find the average power:

$$\mathcal{P}_{\text{av}} = I_{\text{rms}} \Delta V_{\text{rms}} \cos \phi = (0.180 \text{ A})(106 \text{ V}) \cos (-64.8^\circ)$$

$$= 8.12 \text{ W}$$

Remark The same result can be obtained from Equation 21.16, $\mathcal{P}_{av} = I_{rms}^2 R$.

Exercise 21.5

Repeat this problem, using the system described in Exercise 21.4.

Answer 259 W

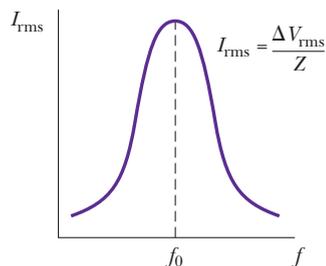


Figure 21.13 A plot of current amplitude in a series RLC circuit versus frequency of the generator voltage. Note that the current reaches its maximum value at the resonance frequency f_0 .

Resonance frequency ►

21.6 RESONANCE IN A SERIES RLC CIRCUIT

In general, the rms current in a series RLC circuit can be written

$$I_{rms} = \frac{\Delta V_{rms}}{Z} = \frac{\Delta V_{rms}}{\sqrt{R^2 + (X_L - X_C)^2}} \quad [21.18]$$

From this equation, we see that if the frequency is varied, the current has its *maximum* value when the impedance has its *minimum* value. This occurs when $X_L = X_C$. In such a circumstance, the impedance of the circuit reduces to $Z = R$. The frequency f_0 at which this happens is called the **resonance frequency** of the circuit. To find f_0 , we set $X_L = X_C$, which gives, from Equations 21.5 and 21.8,

$$2\pi f_0 L = \frac{1}{2\pi f_0 C}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \quad [21.19]$$

Figure 21.13 is a plot of current as a function of frequency for a circuit containing a fixed value for both the capacitance and the inductance. From Equation 21.18, it must be concluded that the current would become infinite at resonance when $R = 0$. Although Equation 21.18 predicts this result, real circuits always have some resistance, which limits the value of the current.

The tuning circuit of a radio is an important application of a series resonance circuit. The radio is tuned to a particular station (which transmits a specific radio-frequency signal) by varying a capacitor, which changes the resonance frequency of the tuning circuit. When this resonance frequency matches that of the incoming radio wave, the current in the tuning circuit increases.

APPLICATION

Tuning Your Radio

Applying Physics 21.1 Metal Detectors in Airports

When you walk through the doorway of an airport metal detector, as the person in Figure 21.14 is doing, you are really walking through a coil of many turns. How might the metal detector work?

Explanation The metal detector is essentially a resonant circuit. The portal you step through is an inductor (a large loop of conducting wire) that is part of the circuit. The frequency of the circuit is tuned to the resonant frequency of the circuit when there is no metal in the inductor. When you walk through with metal in your pocket, you change the effective inductance of the resonance circuit, resulting in a change in the current in the circuit. This change in current is detected, and an electronic circuit causes a sound to be emitted as an alarm.



Figure 21.14 (Applying Physics 21.1) An airport metal detector.

EXAMPLE 21.6 A Circuit in Resonance

Goal Understand resonance frequency and its relation to inductance, capacitance, and the rms current.

Problem Consider a series RLC circuit for which $R = 1.50 \times 10^2 \Omega$, $L = 20.0 \text{ mH}$, $\Delta V_{\text{rms}} = 20.0 \text{ V}$, and $f = 796 \text{ s}^{-1}$. (a) Determine the value of the capacitance for which the rms current is a maximum. (b) Find the maximum rms current in the circuit.

Strategy The current is a maximum at the resonance frequency f_0 , which should be set equal to the driving frequency, 796 s^{-1} . The resulting equation can be solved for C . For part (b), substitute into Equation 21.18 to get the maximum rms current.

Solution

(a) Find the capacitance giving the maximum current in the circuit (the resonance condition).

Solve the resonance frequency for the capacitance:

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \rightarrow \sqrt{LC} = \frac{1}{2\pi f_0} \rightarrow LC = \frac{1}{4\pi^2 f_0^2}$$

$$C = \frac{1}{4\pi^2 f_0^2 L}$$

Insert the given values, substituting the source frequency for the resonance frequency, f_0 :

$$C = \frac{1}{4\pi^2 (796 \text{ Hz})^2 (20.0 \times 10^{-3} \text{ H})} = 2.00 \times 10^{-6} \text{ F}$$

(b) Find the maximum rms current in the circuit.

The capacitive and inductive reactances are equal, so $Z = R = 1.50 \times 10^2 \Omega$. Substitute into Equation 21.18 to find the rms current:

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{Z} = \frac{20.0 \text{ V}}{1.50 \times 10^2 \Omega} = 0.133 \text{ A}$$

Remark Because the impedance Z is in the denominator of Equation 21.18, the maximum current will always occur when $X_L = X_C$, since that yields the minimum value of Z .

Exercise 21.6

Consider a series RLC circuit for which $R = 1.20 \times 10^2 \Omega$, $C = 3.10 \times 10^{-5} \text{ F}$, $\Delta V_{\text{rms}} = 35.0 \text{ V}$, and $f = 60.0 \text{ s}^{-1}$. (a) Determine the value of the inductance for which the rms current is a maximum. (b) Find the maximum rms current in the circuit.

Answers (a) 0.227 H (b) 0.292 A

21.7 THE TRANSFORMER

It's often necessary to change a small AC voltage to a larger one or vice versa. Such changes are effected with a device called a transformer.

In its simplest form, the **AC transformer** consists of two coils of wire wound around a core of soft iron, as shown in Figure 21.15. The coil on the left, which is connected to the input AC voltage source and has N_1 turns, is called the primary winding, or the *primary*. The coil on the right, which is connected to a resistor R and consists of N_2 turns, is the *secondary*. The purpose of the common iron core is to increase the magnetic flux and to provide a medium in which nearly all the flux through one coil passes through the other.

When an input AC voltage ΔV_1 is applied to the primary, the induced voltage across it is given by

$$\Delta V_1 = -N_1 \frac{\Delta \Phi_B}{\Delta t} \quad [21.20]$$

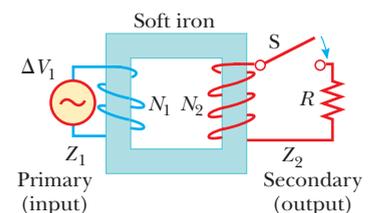


Figure 21.15 An ideal transformer consists of two coils wound on the same soft iron core. An AC voltage ΔV_1 is applied to the primary coil, and the output voltage ΔV_2 is observed across the load resistance R after the switch is closed.

where Φ_B is the magnetic flux through each turn. If we assume that no flux leaks from the iron core, then the flux through each turn of the primary equals the flux through each turn of the secondary. Hence, the voltage across the secondary coil is

$$\Delta V_2 = -N_2 \frac{\Delta \Phi_B}{\Delta t} \quad [21.21]$$

The term $\Delta \Phi_B / \Delta t$ is common to Equations 21.20 and 21.21 and can be algebraically eliminated, giving

$$\Delta V_2 = \frac{N_2}{N_1} \Delta V_1 \quad [21.22]$$

When N_2 is greater than N_1 , ΔV_2 exceeds ΔV_1 and the transformer is referred to as a *step-up transformer*. When N_2 is less than N_1 , making ΔV_2 less than ΔV_1 , we have a *step-down transformer*.

By Faraday's law, a voltage is generated across the secondary only when there is a *change* in the number of flux lines passing through the secondary. The input current in the primary must therefore change with time, which is what happens when an alternating current is used. When the input at the primary is a direct current, however, a voltage output occurs at the secondary only at the instant a switch in the primary circuit is opened or closed. Once the current in the primary reaches a steady value, the output voltage at the secondary is zero.

It may seem that a transformer is a device in which it is possible to get something for nothing. For example, a step-up transformer can change an input voltage from, say, 10 V to 100 V. This means that each coulomb of charge leaving the secondary has 100 J of energy, whereas each coulomb of charge entering the primary has only 10 J of energy. That is not the case, however, because **the power input to the primary equals the power output at the secondary**:

$$I_1 \Delta V_1 = I_2 \Delta V_2 \quad [21.23]$$

While the *voltage* at the secondary may be, say, ten times greater than the voltage at the primary, the *current* in the secondary will be smaller than the primary's current by a factor of ten. Equation 21.23 assumes an **ideal transformer**, in which there are no power losses between the primary and the secondary. Real transformers typically have power efficiencies ranging from 90% to 99%. Power losses occur because of such factors as eddy currents induced in the iron core of the transformer, which dissipate energy in the form of I^2R losses.

When electric power is transmitted over large distances, it's economical to use a high voltage and a low current because the power lost via resistive heating in the transmission lines varies as I^2R . This means that if a utility company can reduce the current by a factor of ten, for example, the power loss is reduced by a factor of one hundred. In practice, the voltage is stepped up to around 230 000 V at the generating station, then stepped down to around 20 000 V at a distribution station, and finally stepped down to 120 V at the customer's utility pole.

In an ideal transformer, the input power equals the output power. ►

APPLICATION

Long-Distance Electric Power Transmission

EXAMPLE 21.7 Distributing Power to a City

Goal Understand transformers and their role in reducing power loss.

Problem A generator at a utility company produces 1.00×10^2 A of current at 4.00×10^3 V. The voltage is stepped up to 2.40×10^5 V by a transformer before being sent on a high-voltage transmission line across a rural area to a city. Assume that the effective resistance of the power line is 30.0Ω and that the transformers are ideal. **(a)** Determine the percentage of power lost in the transmission line. **(b)** What percentage of the original power would be lost in the transmission line if the voltage were not stepped up?

Strategy Solving this problem is just a matter of substitution into the equation for transformers and the equation for power loss. To obtain the fraction of power lost, it's also necessary to compute the power output of the generator—the current times the potential difference created by the generator.

Solution

(a) Determine the percentage of power lost in the line.

Substitute into Equation 21.23 to find the current in the transmission line:

$$I_2 = \frac{I_1 \Delta V_1}{\Delta V_2} = \frac{(1.00 \times 10^2 \text{ A})(4.00 \times 10^3 \text{ V})}{2.40 \times 10^5 \text{ V}} = 1.67 \text{ A}$$

Now use Equation 21.16 to find the power lost in the transmission line:

$$(1) \quad \mathcal{P}_{\text{lost}} = I_2^2 R = (1.67 \text{ A})^2 (30.0 \Omega) = 83.7 \text{ W}$$

Calculate the power output of the generator:

$$\mathcal{P} = I_1 \Delta V_1 = (1.00 \times 10^2 \text{ A})(4.00 \times 10^3 \text{ V}) = 4.00 \times 10^5 \text{ W}$$

Finally, divide $\mathcal{P}_{\text{lost}}$ by the power output and multiply by 100 to find the percentage of power lost:

$$\% \text{ power lost} = \left(\frac{83.7 \text{ W}}{4.00 \times 10^5 \text{ W}} \right) \times 100 = 0.0209\%$$

(b) What percentage of the original power would be lost in the transmission line if the voltage were not stepped up?

Replace the stepped-up current in equation (1) by the original current of $1.00 \times 10^2 \text{ A}$.

$$\mathcal{P}_{\text{lost}} = I^2 R = (1.00 \times 10^2 \text{ A})^2 (30.0 \Omega) = 3.00 \times 10^5 \text{ W}$$

Calculate the percentage loss, as before:

$$\% \text{ power lost} = \left(\frac{3.00 \times 10^5 \text{ W}}{4.00 \times 10^5 \text{ W}} \right) \times 100 = 75\%$$

Remarks This example illustrates the advantage of high-voltage transmission lines. At the city, a transformer at a substation steps the voltage back down to about 4 000 V, and this voltage is maintained across utility lines throughout the city. When the power is to be used at a home or business, a transformer on a utility pole near the establishment reduces the voltage to 240 V or 120 V.

Exercise 21.7

Suppose the same generator has the voltage stepped up to only $7.50 \times 10^4 \text{ V}$ and the resistance of the line is 85.0Ω . Find the percentage of power lost in this case.

Answer 0.604%



This cylindrical step-down transformer drops the voltage from 4 000 V to 220 V for delivery to a group of residences.

21.8 MAXWELL'S PREDICTIONS

During the early stages of their study and development, electric and magnetic phenomena were thought to be unrelated. In 1865, however, James Clerk Maxwell (1831–1879) provided a mathematical theory that showed a close relationship between all electric and magnetic phenomena. In addition to unifying the formerly separate fields of electricity and magnetism, his brilliant theory predicted that electric and magnetic fields can move through space as waves. The theory he developed is based on the following four pieces of information:

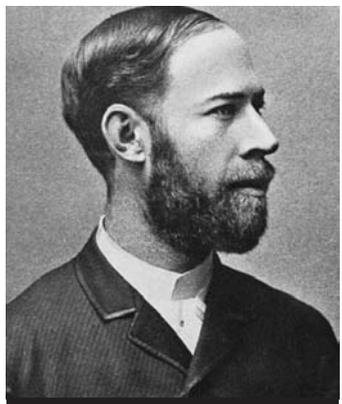
1. Electric field lines originate on positive charges and terminate on negative charges.
2. Magnetic field lines always form closed loops—they don't begin or end anywhere.
3. A varying magnetic field induces an emf and hence an electric field. This is a statement of Faraday's law (Chapter 20).
4. Magnetic fields are generated by moving charges (or currents), as summarized in Ampère's law (Chapter 19).



North Wind Photo Archives

JAMES CLERK MAXWELL, Scottish Theoretical Physicist (1831–1879)

Maxwell developed the electromagnetic theory of light, the kinetic theory of gases, and explained the nature of Saturn's rings and color vision. Maxwell's successful interpretation of the electromagnetic field resulted in the equations that bear his name. Formidable mathematical ability combined with great insight enabled him to lead the way in the study of electromagnetism and kinetic theory.



Bettmann/Corbis

HEINRICH RUDOLF HERTZ, German Physicist (1857–1894)

Hertz made his most important discovery of radio waves in 1887. After finding that the speed of a radio wave was the same as that of light, Hertz showed that radio waves, like light waves, could be reflected, refracted, and diffracted. Hertz died of blood poisoning at the age of 36. During his short life, he made many contributions to science. The hertz, equal to one complete vibration or cycle per second, is named after him.

The first statement is a consequence of the nature of the electrostatic force between charged particles, given by Coulomb's law. It embodies the fact that **free charges (electric monopoles) exist in nature**.

The second statement—that magnetic fields form continuous loops—is exemplified by the magnetic field lines around a long, straight wire, which are closed circles, and the magnetic field lines of a bar magnet, which form closed loops. It says, in contrast to the first statement, that **free magnetic charges (magnetic monopoles) don't exist in nature**.

The third statement is equivalent to Faraday's law of induction, and the fourth is equivalent to Ampère's law.

In one of the greatest theoretical developments of the 19th century, Maxwell used these four statements within a corresponding mathematical framework to prove that electric and magnetic fields play symmetric roles in nature. It was already known from experiments that a changing magnetic field produced an electric field according to Faraday's law. Maxwell believed that nature was symmetric, and he therefore hypothesized that a changing electric field should produce a magnetic field. This hypothesis could not be proven experimentally at the time it was developed, because the magnetic fields generated by changing electric fields are generally very weak and therefore difficult to detect.

To justify his hypothesis, Maxwell searched for other phenomena that might be explained by it. He turned his attention to the motion of rapidly oscillating (accelerating) charges, such as those in a conducting rod connected to an alternating voltage. Such charges are accelerated and, according to Maxwell's predictions, generate changing electric and magnetic fields. The changing fields cause electromagnetic disturbances that travel through space as waves, similar to the spreading water waves created by a pebble thrown into a pool. The waves sent out by the oscillating charges are fluctuating electric and magnetic fields, so they are called *electromagnetic waves*. From Faraday's law and from Maxwell's own generalization of Ampère's law, Maxwell calculated the speed of the waves to be equal to the speed of light, $c = 3 \times 10^8$ m/s. He concluded that visible light and other electromagnetic waves consist of fluctuating electric and magnetic fields traveling through empty space, with each varying field inducing the other! This was truly one of the greatest discoveries of science, on a par with Newton's discovery of the laws of motion. Like Newton's laws, it had a profound influence on later scientific developments.

21.9 HERTZ'S CONFIRMATION OF MAXWELL'S PREDICTIONS

In 1887, after Maxwell's death, Heinrich Hertz (1857–1894) was the first to generate and detect electromagnetic waves in a laboratory setting, using *LC* circuits. In such a circuit, a charged capacitor is connected to an inductor, as in Figure 21.16. When the switch is closed, oscillations occur in the current in the circuit and in the charge on the capacitor. If the resistance of the circuit is neglected, no energy is dissipated and the oscillations continue.

In the following analysis, we neglect the resistance in the circuit. We assume that the capacitor has an initial charge of Q_{\max} and that the switch is closed at $t = 0$. When the capacitor is fully charged, the total energy in the circuit is stored in the electric field of the capacitor and is equal to $Q_{\max}^2/2C$. At this time, the current is zero, so no energy is stored in the inductor. As the capacitor begins to discharge, the energy stored in its electric field decreases. At the same time, the current increases and energy equal to $LI^2/2$ is now stored in the magnetic field of the inductor. Thus, energy is transferred from the electric field of the capacitor to the magnetic field of the inductor. When the capacitor is fully discharged, it stores no energy. At this time, the current reaches its maximum value and all of the energy is stored in the inductor. The process then repeats in the reverse direction. The energy continues to transfer between the inductor and the capacitor, corresponding to oscillations in the current and charge.

As we saw in Section 21.6, the frequency of oscillation of an LC circuit is called the *resonance frequency* of the circuit and is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

The circuit Hertz used in his investigations of electromagnetic waves is similar to that just discussed and is shown schematically in Figure 21.17. An induction coil (a large coil of wire) is connected to two metal spheres with a narrow gap between them to form a capacitor. Oscillations are initiated in the circuit by short voltage pulses sent via the coil to the spheres, charging one positive, the other negative. Because L and C are quite small in this circuit, the frequency of oscillation is quite high, $f \approx 100$ MHz. This circuit is called a transmitter because it produces electromagnetic waves.

Several meters from the transmitter circuit, Hertz placed a second circuit, the receiver, which consisted of a single loop of wire connected to two spheres. It had its own effective inductance, capacitance, and natural frequency of oscillation. Hertz found that energy was being sent from the transmitter to the receiver when the resonance frequency of the receiver was adjusted to match that of the transmitter. The energy transfer was detected when the voltage across the spheres in the receiver circuit became high enough to produce ionization in the air, which caused sparks to appear in the air gap separating the spheres. Hertz's experiment is analogous to the mechanical phenomenon in which a tuning fork picks up the vibrations from another, identical tuning fork.

Hertz hypothesized that the energy transferred from the transmitter to the receiver is carried in the form of waves, now recognized as electromagnetic waves. In a series of experiments, he also showed that the radiation generated by the transmitter exhibits wave properties: interference, diffraction, reflection, refraction, and polarization. As you will see shortly, all of these properties are exhibited by light. It became evident that Hertz's electromagnetic waves had the same known properties of light waves and differed only in frequency and wavelength. Hertz effectively confirmed Maxwell's theory by showing that Maxwell's mysterious electromagnetic waves existed and had all the properties of light waves.

Perhaps the most convincing experiment Hertz performed was the measurement of the speed of waves from the transmitter, accomplished as follows: waves of known frequency from the transmitter were reflected from a metal sheet so that an interference pattern was set up, much like the standing-wave pattern on a stretched string. As we learned in our discussion of standing waves, the distance between nodes is $\lambda/2$, so Hertz was able to determine the wavelength λ . Using the relationship $v = \lambda f$, he found that v was close to 3×10^8 m/s, the known speed of visible light. Hertz's experiments thus provided the first evidence in support of Maxwell's theory.

21.10 PRODUCTION OF ELECTROMAGNETIC WAVES BY AN ANTENNA

In the previous section, we found that the energy stored in an LC circuit is continually transferred between the electric field of the capacitor and the magnetic field of the inductor. However, this energy transfer continues for prolonged periods of time only when the changes occur slowly. If the current alternates rapidly, the circuit loses some of its energy in the form of electromagnetic waves. In fact, electromagnetic waves are radiated by *any* circuit carrying an alternating current. The fundamental mechanism responsible for this radiation is the acceleration of a charged particle. **Whenever a charged particle accelerates it radiates energy.**

An alternating voltage applied to the wires of an antenna forces electric charges in the antenna to oscillate. This is a common technique for accelerating charged particles and is the source of the radio waves emitted by the broadcast antenna of a radio station.

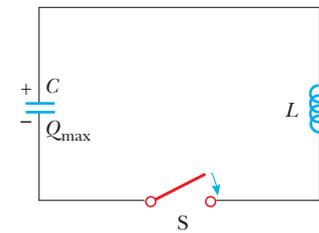


Figure 21.16 A simple LC circuit. The capacitor has an initial charge of Q_{\max} and the switch is closed at $t = 0$.

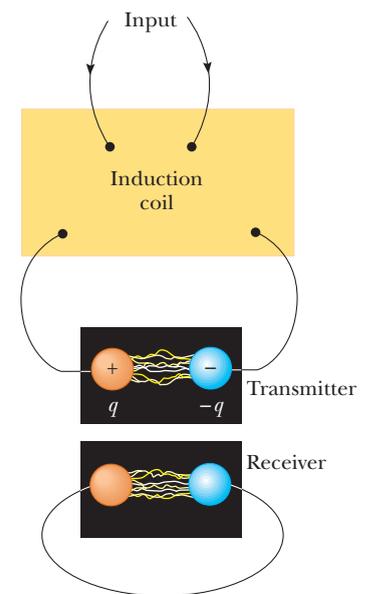


Figure 21.17 A schematic diagram of Hertz's apparatus for generating and detecting electromagnetic waves. The transmitter consists of two spherical electrodes connected to an induction coil, which provides short voltage surges to the spheres, setting up oscillations in the discharge. The receiver is a nearby single loop of wire containing a second spark gap.

APPLICATION

Radio-Wave Transmission

Figure 21.18 An electric field set up by oscillating charges in an antenna. The field moves away from the antenna at the speed of light.

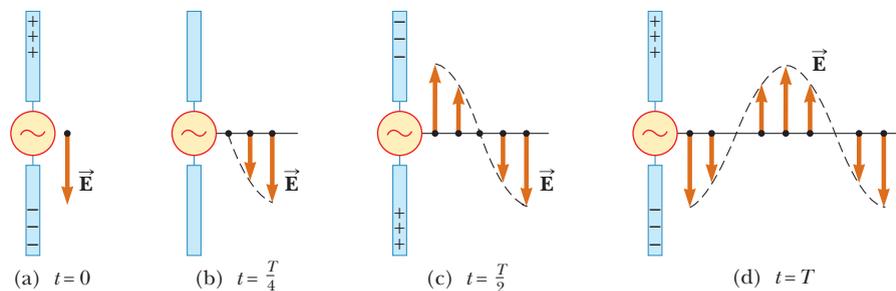


Figure 21.18 illustrates the production of an electromagnetic wave by oscillating electric charges in an antenna. Two metal rods are connected to an AC source, which causes charges to oscillate between the rods. The output voltage of the generator is sinusoidal. At $t = 0$, the upper rod is given a maximum positive charge and the bottom rod an equal negative charge, as in Figure 21.18a. The electric field near the antenna at this instant is also shown in the figure. As the charges oscillate, the rods become less charged, the field near the rods decreases in strength, and the downward-directed maximum electric field produced at $t = 0$ moves away from the rod. When the charges are neutralized, as in Figure 21.18b, the electric field has dropped to zero, after an interval equal to one-quarter of the period of oscillation. Continuing in this fashion, the upper rod soon obtains a maximum negative charge and the lower rod becomes positive, as in Figure 21.18c, resulting in an electric field directed upward. This occurs after an interval equal to one-half the period of oscillation. The oscillations continue as indicated in Figure 21.18d. Note that the electric field near the antenna oscillates in phase with the charge distribution: the field points down when the upper rod is positive and up when the upper rod is negative. Further, the magnitude of the field at any instant depends on the amount of charge on the rods at that instant.

As the charges continue to oscillate (and accelerate) between the rods, the electric field set up by the charges moves away from the antenna in all directions at the speed of light. Figure 21.18 shows the electric field pattern on one side of the antenna at certain times during the oscillation cycle. As you can see, one cycle of charge oscillation produces one full wavelength in the electric field pattern.

Because the oscillating charges create a current in the rods, a magnetic field is also generated when the current in the rods is upward, as shown in Figure 21.19. The magnetic field lines circle the antenna (recall right-hand rule number 2) and are perpendicular to the electric field at all points. As the current changes with time, the magnetic field lines spread out from the antenna. At great distances from the antenna, the strengths of the electric and magnetic fields become very weak. At these distances, however, it is necessary to take into account the facts that (1) a changing magnetic field produces an electric field and (2) a changing electric field produces a magnetic field, as predicted by Maxwell. These induced electric and magnetic fields are in phase: at any point, the two fields reach their maximum values at the same instant. This synchrony is illustrated at one instant of time in Active Figure 21.20. Note that (1) the \vec{E} and \vec{B} fields are perpendicular to each other, and (2) both fields are perpendicular to the direction of motion of the wave. This second property is characteristic of transverse waves. Hence, we see that **an electromagnetic wave is a transverse wave**.

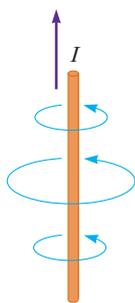


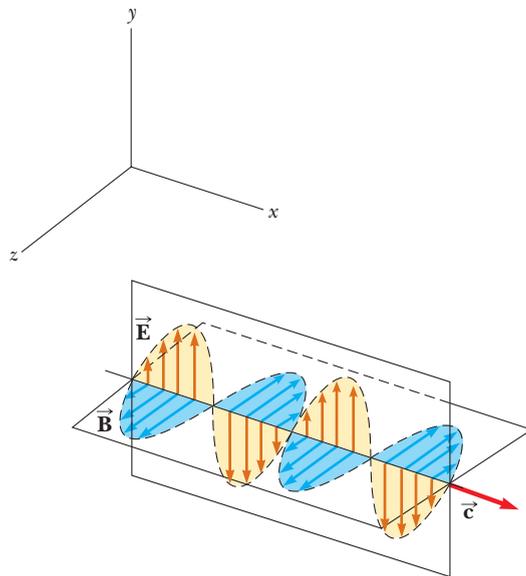
Figure 21.19 Magnetic field lines around an antenna carrying a changing current.

TIP 21.1 Accelerated Charges Produce Electromagnetic Waves

Stationary charges produce only electric fields, while charges in uniform motion (i.e., constant velocity) produce electric and magnetic fields, but no electromagnetic waves. In contrast, accelerated charges produce electromagnetic waves as well as electric and magnetic fields. An accelerating charge also radiates energy.

21.11 PROPERTIES OF ELECTROMAGNETIC WAVES

We have seen that Maxwell's detailed analysis predicted the existence and properties of electromagnetic waves. In this section we summarize what we know about electromagnetic waves thus far and consider some additional properties. In our discussion here and in future sections, we will often make reference to a type of wave called a **plane wave**. A plane electromagnetic wave is a wave traveling from a very distant source. Active Figure 21.20 pictures such a wave at a given instant of time. In

**ACTIVE FIGURE 21.20**

An electromagnetic wave sent out by oscillating charges in an antenna, represented at one instant of time and far from the antenna, moving in the positive x -direction with speed c . Note that the electric field is perpendicular to the magnetic field, and both are perpendicular to the direction of wave propagation. The variations of E and B with time are sinusoidal.

PhysicsNow™

Log into PhysicsNow at www.cp7e.com and go to Active Figure 21.20, where you can observe the wave and the variations of the fields. In addition, you can take a “snapshot” of the wave at an instant of time and investigate the electric and magnetic fields at that instant.

this case, the oscillations of the electric and magnetic fields take place in planes perpendicular to the x -axis and are therefore perpendicular to the direction of travel of the wave. Because of the latter property, electromagnetic waves are transverse waves. In the figure, the electric field \vec{E} is in the y -direction and the magnetic field \vec{B} is in the z -direction. Light propagates in a direction perpendicular to these two fields. That direction is determined by yet another right-hand rule: (1) point the fingers of your right hand in the direction of \vec{E} , (2) curl them in the direction of \vec{B} , (3) the right thumb then points in the direction of propagation of the wave.

Electromagnetic waves travel with the speed of light. In fact, it can be shown that the speed of an electromagnetic wave is related to the permeability and permittivity of the medium through which it travels. Maxwell found this relationship for free space to be

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \quad [21.24] \quad \leftarrow \text{Speed of light.}$$

where c is the speed of light, $\mu_0 = 4\pi \times 10^{-7} \text{ N} \cdot \text{s}^2/\text{C}^2$ is the permeability constant of vacuum, and $\epsilon_0 = 8.854\,19 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2$ is the permittivity of free space. Substituting these values into Equation 21.24, we find that

$$c = 2.997\,92 \times 10^8 \text{ m/s} \quad [21.25]$$

The fact that electromagnetic waves travel at the same speed as light in vacuum led scientists to conclude (correctly) that **light is an electromagnetic wave**.

Maxwell also proved the following relationship for electromagnetic waves:

$$\frac{E}{B} = c \quad [21.26]$$

TIP 21.2 *E Stronger Than B?*

The relationship $E = Bc$ makes it appear that the electric fields associated with light are much larger than the magnetic fields. This is not the case: The units are different, so the quantities can't be directly compared. The two fields contribute equally to the energy of a light wave.

Light is an electromagnetic wave and transports energy and momentum. ►

which states that the ratio of the magnitude of the electric field to the magnitude of the magnetic field equals the speed of light.

Electromagnetic waves carry energy as they travel through space, and this energy can be transferred to objects placed in their paths. The average rate at which energy passes through an area perpendicular to the direction of travel of a wave, or the average power per unit area, is called the **intensity** I of the wave, and is given by

$$I = \frac{E_{\max} B_{\max}}{2\mu_0} \quad [21.27]$$

where E_{\max} and B_{\max} are the *maximum* values of E and B . The quantity I is analogous to the intensity of sound waves introduced in Chapter 14. From Equation 21.26, we see that $E_{\max} = cB_{\max} = B_{\max}/\sqrt{\mu_0\epsilon_0}$. Equation 21.27 can therefore also be expressed as

$$I = \frac{E_{\max}^2}{2\mu_0 c} = \frac{c}{2\mu_0} B_{\max}^2 \quad [21.28]$$

Note that in these expressions we use the *average* power per unit area. A detailed analysis would show that the energy carried by an electromagnetic wave is shared equally by the electric and magnetic fields.

Electromagnetic waves have an average intensity given by Equation 21.28. When the waves strike an area A of an object's surface for a given time Δt , energy $U = IA\Delta t$ is transferred to the surface. Momentum is transferred, as well. Hence, pressure is exerted on a surface when an electromagnetic wave impinges on it. In what follows, we assume that the electromagnetic wave transports a total energy U to a surface in a time Δt . If the surface absorbs all the incident energy U in this time, Maxwell showed that the total momentum \vec{p} delivered to this surface has a magnitude

$$p = \frac{U}{c} \quad (\text{complete absorption}) \quad [21.29]$$

If the surface is a perfect reflector, then the momentum transferred in a time Δt for normal incidence is twice that given by Equation 21.29. This is analogous to a molecule of gas bouncing off the wall of a container in a perfectly elastic collision. If the molecule is initially traveling in the positive x -direction at velocity v , and after the collision is traveling in the negative x -direction at velocity $-v$, then its change in momentum is given by $\Delta p = mv - (-mv) = 2mv$. Light bouncing off a perfect reflector is a similar process, so for complete reflection,

$$p = \frac{2U}{c} \quad (\text{complete reflection}) \quad [21.30]$$

Although radiation pressures are very small (about $5 \times 10^{-6} \text{ N/m}^2$ for direct sunlight), they have been measured with a device such as the one shown in Figure 21.21. Light is allowed to strike a mirror and a black disk that are connected to each other by a horizontal bar suspended from a fine fiber. Light striking the black disk is completely absorbed, so *all* of the momentum of the light is transferred to the disk. Light striking the mirror head-on is totally reflected; hence, the momentum transfer to the mirror is twice that transmitted to the disk. As a result, the horizontal bar supporting the disks twists counterclockwise as seen from above. The bar comes to equilibrium at some angle under the action of the torques caused by radiation pressure and the twisting of the fiber. The radiation pressure can be determined by measuring the angle at which equilibrium occurs. The apparatus must be placed in a high vacuum to eliminate the effects of air currents. It's interesting that similar experiments demonstrate that electromagnetic waves carry angular momentum, as well.

In summary, electromagnetic waves traveling through free space have the following properties:

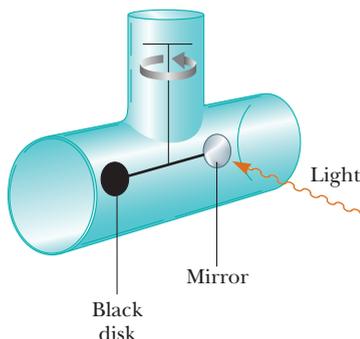


Figure 21.21 An apparatus for measuring the radiation pressure of light. In practice, the system is contained in a high vacuum.

1. Electromagnetic waves travel at the speed of light.
2. Electromagnetic waves are transverse waves, because the electric and magnetic fields are perpendicular to the direction of propagation of the wave and to each other.
3. The ratio of the electric field to the magnetic field in an electromagnetic wave equals the speed of light.
4. Electromagnetic waves carry both energy and momentum, which can be delivered to a surface.

◀ Some properties of electromagnetic waves

Applying Physics 21.2 Solar System Dust

In the interplanetary space in the Solar System, there is a large amount of dust. Although interplanetary dust can in theory have a variety of sizes—from molecular size upward—why are there very few dust particles smaller than about $0.2 \mu\text{m}$ in the Solar System? [*Hint:* The Solar System originally contained dust particles of all sizes.]

Explanation Dust particles in the Solar System are subject to two forces: the gravitational force toward the Sun and the force from radiation pressure, which is directed

away from the Sun. The gravitational force is proportional to the cube of the radius of a spherical dust particle, because it is proportional to the mass (ρV) of the particle. The radiation pressure is proportional to the square of the radius, because it depends on the cross-sectional area of the particle. For large particles, the gravitational force is larger than the force of radiation pressure, and the weak attraction to the Sun causes such particles to move slowly towards it. For small particles, less than about $0.2 \mu\text{m}$, the larger force from radiation pressure sweeps them out of the Solar System.

Quick Quiz 21.4

In an apparatus such as that in Figure 21.21, suppose the black disk is replaced by one with half the radius. Which of the following are different after the disk is replaced? (a) radiation pressure on the disk; (b) radiation force on the disk; (c) radiation momentum delivered to the disk in a given time interval.

EXAMPLE 21.8 A Hot Tin Roof (Solar-Powered Homes)

Goal Calculate some basic properties of light and relate them to thermal radiation.

Problem Assume that the Sun delivers an average power per unit area of about $1.00 \times 10^3 \text{ W/m}^2$ to Earth's surface. (a) Calculate the total power incident on a flat tin roof 8.00 m by 20.0 m . Assume that the radiation is incident *normal* (perpendicular) to the roof. (b) Calculate the peak electric field of the light. (c) Compute the peak magnetic field of the light. (d) The tin roof reflects some light, and convection, conduction, and radiation transport the rest of the thermal energy away, until some equilibrium temperature is established. If the roof is a perfect blackbody and rids itself of one-half of the incident radiation through thermal radiation, what's its equilibrium temperature? Assume the ambient temperature is 298 K .

Solution

(a) Calculate the power delivered to the roof.

Multiply the intensity by the area to get the power:

$$\begin{aligned} \mathcal{P} &= IA = (1.00 \times 10^3 \text{ W/m}^2)(8.00 \text{ m} \times 20.0 \text{ m}) \\ &= 1.60 \times 10^5 \text{ W} \end{aligned}$$



(Example 21.8) A solar home in Oregon.

John Neal/Photo Researchers, Inc.

(b) Calculate the peak electric field of the light.

Solve Equation 21.28 for E_{\max} :

$$I = \frac{E_{\max}^2}{2\mu_0 c} \rightarrow E_{\max} = \sqrt{2\mu_0 c I}$$

$$E_{\max} = \sqrt{2(4\pi \times 10^{-7} \text{ N}\cdot\text{s}^2/\text{C}^2)(3.00 \times 10^8 \text{ m/s})(1.00 \times 10^3 \text{ W/m}^2)}$$

$$= 868 \text{ V/m}$$

(c) Compute the peak magnetic field of the light.

Obtain B_{\max} using Equation 21.26:

$$B_{\max} = \frac{E_{\max}}{c} = \frac{868 \text{ V/m}}{3.00 \times 10^8 \text{ m/s}} = 2.89 \times 10^{-6} \text{ T}$$

(d) Find the equilibrium temperature of the roof.

Substitute into Stefan's law. Only one-half the incident power should be substituted, and twice the area of the roof (both the top and the underside of the roof count).

$$\mathcal{P} = \sigma e A (T^4 - T_0^4)$$

$$T^4 = T_0^4 + \frac{\mathcal{P}}{\sigma e A}$$

$$= (298 \text{ K})^4 + \frac{(0.500)(1.60 \times 10^5 \text{ W/m}^2)}{(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(1)(3.20 \times 10^2 \text{ m}^2)}$$

$$T = 333 \text{ K} = 60.0^\circ \text{ C}$$

Remarks If the incident power could *all* be converted to electric power, it would be more than enough for the average home. Unfortunately, solar energy isn't easily harnessed, and the prospects for large-scale conversion are not as bright as they may appear from this simple calculation. For example, the conversion efficiency from solar to electrical energy is far less than 100%; 10% is typical for photovoltaic cells. Roof systems for using solar energy to raise the temperature of water with efficiencies of around 50% have been built. Other practical problems must be considered, however, such as overcast days, geographic location, and energy storage.

Exercise 21.8

A spherical satellite orbiting Earth is lighted on one side by the Sun, with intensity $1\,340 \text{ W/m}^2$. (a) If the radius of the satellite is 1.00 m , what power is incident upon it? [*Note:* The satellite effectively intercepts radiation only over a cross section—an area equal to that of a disk, πr^2 .) (b) Calculate the peak electric field. (c) Calculate the peak magnetic field.

Answer (a) $4.21 \times 10^3 \text{ W}$ (b) $1.01 \times 10^3 \text{ V/m}$ (c) $3.35 \times 10^{-6} \text{ T}$

EXAMPLE 21.9 Clipper Ships of Space

Goal Relate the intensity of light to its mechanical effect on matter.

Problem Aluminized mylar film is a highly reflective, lightweight material that could be used to make sails for spacecraft driven by the light of the sun. Suppose a sail with area 1.00 km^2 is orbiting the Sun at a distance of $1.50 \times 10^{11} \text{ m}$. The sail has a mass of $5.00 \times 10^3 \text{ kg}$ and is tethered to a payload of mass $2.00 \times 10^4 \text{ kg}$. (a) If the intensity of sunlight is $1.34 \times 10^3 \text{ W}$ and the sail is oriented perpendicular to the incident light, what radial force is exerted on the sail? (b) About how long would it take to change the radial speed of the sail by 1.00 km/s ? Assume that the sail is perfectly reflecting.

Strategy Equation 21.30 gives the momentum imparted when light strikes an object and is totally reflected. The change in this momentum with time is a force. For part (b), use Newton's second law to obtain the acceleration. The velocity kinematics equation then yields the necessary time to achieve the desired change in speed.

Solution

(a) Find the force exerted on the sail.

Write Equation 21.30, and substitute $U = \mathcal{P} \Delta t = IA \Delta t$ for the energy delivered to the sail:

$$\Delta p = \frac{2U}{c} = \frac{2\mathcal{P}\Delta t}{c} = \frac{2IA\Delta t}{c}$$

Divide both sides by Δt , obtaining the force $\Delta p/\Delta t$ exerted by the light on the sail:

$$F = \frac{\Delta p}{\Delta t} = \frac{2IA}{c} = \frac{2(1340 \text{ W/m}^2)(1.00 \times 10^6 \text{ m}^2)}{3.00 \times 10^8 \text{ m/s}} = 8.93 \text{ N}$$

(b) Find the time it takes to change the radial speed by 1.00 km/s.

Substitute the force into Newton's second law and solve for the acceleration of the sail:

$$a = \frac{F}{m} = \frac{8.93 \text{ N}}{2.50 \times 10^4 \text{ kg}} = 3.57 \times 10^{-4} \text{ m/s}^2$$

Apply the kinematics velocity equation:

$$v = at + v_0$$

Solve for t :

$$t = \frac{v - v_0}{a} = \frac{1.00 \times 10^3 \text{ m/s}}{3.57 \times 10^{-4} \text{ m/s}^2} = 2.80 \times 10^6 \text{ s}$$

Remarks The answer is a little over a month. While the acceleration is very low, there are no fuel costs, and within a few months the velocity can change sufficiently to allow the spacecraft to reach any planet in the solar system. Such spacecraft may be useful for certain purposes and are highly economical, but require a considerable amount of patience.

Exercise 21.9

A laser has a power of 22.0 W and a beam radius of 0.500 mm. (a) Find the intensity of the laser. (b) Suppose you were floating in space and pointed the laser beam away from you. What would your acceleration be? Assume your total mass, including equipment is 72.0 kg and that the force is directed through your center of mass. (*Hint:* The change in momentum is the same as in the nonreflective case.) (c) Compare the acceleration found in part (b) with the acceleration of gravity of a space station of mass 1.00×10^6 kg, if the station's center of mass is 100.0 m away.

Answers (a) $2.80 \times 10^7 \text{ W/m}^2$ (b) $1.02 \times 10^{-9} \text{ m/s}^2$ (c) $6.67 \times 10^{-9} \text{ m/s}^2$. If you were planning to use your laser welding torch as a thruster to get you back to the station, don't bother—the force of gravity is stronger. Better yet, get somebody to toss you a line.

21.12 THE SPECTRUM OF ELECTROMAGNETIC WAVES

All electromagnetic waves travel in a vacuum with the speed of light, c . These waves transport energy and momentum from some source to a receiver. In 1887, Hertz successfully generated and detected the radio-frequency electromagnetic waves predicted by Maxwell. Maxwell himself had recognized as electromagnetic waves both visible light and the infrared radiation discovered in 1800 by William Herschel. It is now known that other forms of electromagnetic waves exist that are distinguished by their frequencies and wavelengths.

Because all electromagnetic waves travel through free space with a speed c , their frequency f and wavelength λ are related by the important expression

$$c = f\lambda \quad [21.31]$$

The various types of electromagnetic waves are presented in Figure 21.22 (page 716). Note the wide and overlapping range of frequencies and wavelengths. For instance, an AM radio wave with a frequency of 5.00 MHz (a typical value) has a wavelength of

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{5.00 \times 10^6 \text{ s}^{-1}} = 60.0 \text{ m}$$

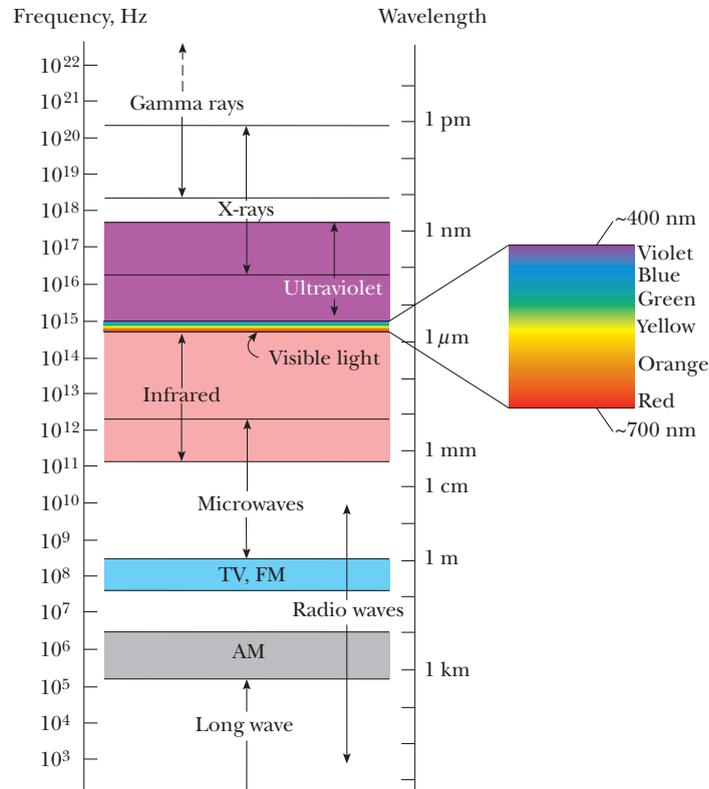
The following abbreviations are often used to designate short wavelengths and distances:



Ron Chapple/Getty Images

Wearing sunglasses lacking ultraviolet (UV) protection is worse for your eyes than wearing no sunglasses at all. Sunglasses without protection absorb some visible light, causing the pupils to dilate. This allows more UV light to enter the eye, increasing the damage to the lens of the eye over time. Without the sunglasses, the pupils constrict, reducing both visible and dangerous UV radiation. Be cool: wear sunglasses with UV protection.

Figure 21.22 The electromagnetic spectrum. Note the overlap between adjacent types of waves. The expanded view to the right shows details of the visible spectrum.



$$1 \text{ micrometer } (\mu\text{m}) = 10^{-6} \text{ m}$$

$$1 \text{ nanometer (nm)} = 10^{-9} \text{ m}$$

$$1 \text{ angstrom } (\text{\AA}) = 10^{-10} \text{ m}$$

The wavelengths of visible light, for example, range from $0.4 \mu\text{m}$ to $0.7 \mu\text{m}$, or 400 nm to 700 nm , or $4\,000 \text{ \AA}$ to $7\,000 \text{ \AA}$.

Quick Quiz 21.5

Which of the following statements are true about light waves? (a) The higher the frequency, the longer the wavelength. (b) The lower the frequency, the longer the wavelength. (c) Higher frequency light travels faster than lower frequency light. (d) The shorter the wavelength, the higher the frequency. (e) The lower the frequency, the shorter the wavelength.

Brief descriptions of the wave types follow, in order of decreasing wavelength. There is no sharp division between one kind of electromagnetic wave and the next. All forms of electromagnetic radiation are produced by accelerating charges.

Radio waves, which were discussed in Section 21.10, are the result of charges accelerating through conducting wires. They are, of course, used in radio and television communication systems.

Microwaves (short-wavelength radio waves) have wavelengths ranging between about 1 mm and 30 cm and are generated by electronic devices. Their short wavelengths make them well suited for the radar systems used in aircraft navigation and for the study of atomic and molecular properties of matter. Microwave ovens are an interesting domestic application of these waves. It has been suggested that solar energy might be harnessed by beaming microwaves to Earth from a solar collector in space.

Infrared waves (sometimes incorrectly called “heat waves”), produced by hot objects and molecules, have wavelengths ranging from about 1 mm to the longest wavelength of visible light, $7 \times 10^{-7} \text{ m}$. They are readily absorbed by most materials. The infrared energy absorbed by a substance causes it to get warmer because the energy agitates the atoms of the object, increasing their vibrational or translational

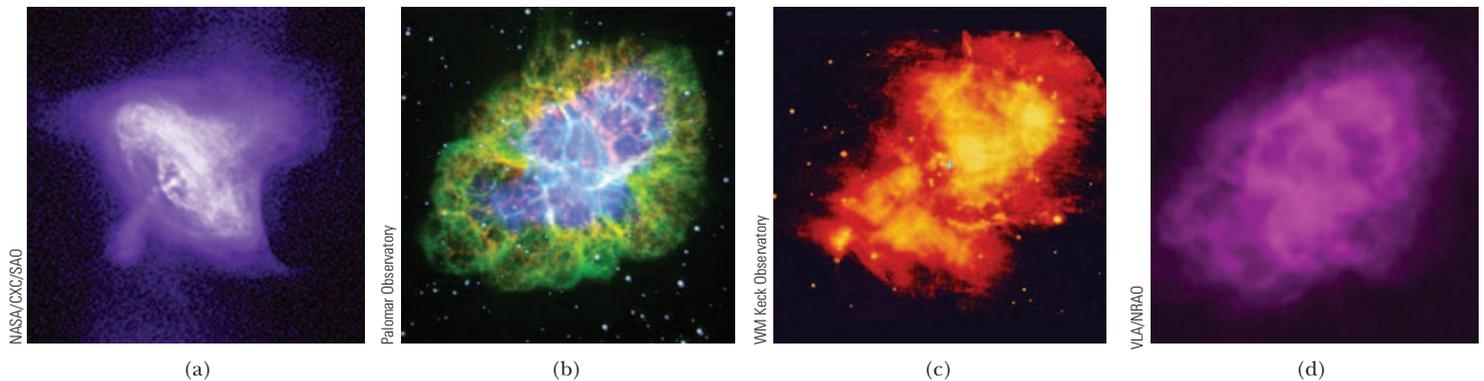


Figure 21.23 Observations in different parts of the electromagnetic spectrum show different features of the Crab Nebula. (a) X-ray image. (b) Optical image. (c) Infrared image. (d) Radio image.

motion. The result is a rise in temperature. Infrared radiation has many practical and scientific applications, including physical therapy, infrared photography, and the study of the vibrations of atoms.

Visible light, the most familiar form of electromagnetic waves, may be defined as the part of the spectrum that is detected by the human eye. Light is produced by the rearrangement of electrons in atoms and molecules. The wavelengths of visible light are classified as colors ranging from violet ($\lambda \approx 4 \times 10^{-7}$ m) to red ($\lambda \approx 7 \times 10^{-7}$ m). The eye's sensitivity is a function of wavelength and is greatest at a wavelength of about 5.6×10^{-7} m (yellow green).

Ultraviolet (UV) light covers wavelengths ranging from about 4×10^{-7} m (400 nm) down to 6×10^{-10} m (0.6 nm). The Sun is an important source of ultraviolet light (which is the main cause of suntans). Most of the ultraviolet light from the Sun is absorbed by atoms in the upper atmosphere, or stratosphere. This is fortunate, because UV light in large quantities has harmful effects on humans. One important constituent of the stratosphere is ozone (O_3), produced from reactions of oxygen with ultraviolet radiation. The resulting ozone shield causes lethal high-energy ultraviolet radiation to warm the stratosphere.

X-rays are electromagnetic waves with wavelengths from about 10^{-8} m (10 nm) down to 10^{-13} m (10^{-4} nm). The most common source of x-rays is the acceleration of high-energy electrons bombarding a metal target. X-rays are used as a diagnostic tool in medicine and as a treatment for certain forms of cancer. Because x-rays easily penetrate and damage or destroy living tissues and organisms, care must be taken to avoid unnecessary exposure and overexposure.

Gamma rays—electromagnetic waves emitted by radioactive nuclei—have wavelengths ranging from about 10^{-10} m to less than 10^{-14} m. They are highly penetrating and cause serious damage when absorbed by living tissues. Accordingly, those working near such radiation must be protected by garments containing heavily absorbing materials, such as layers of lead.

When astronomers observe the same celestial object using detectors sensitive to different regions of the electromagnetic spectrum, striking variations in the object's features can be seen. Figure 21.23 shows images of the Crab Nebula made in four different wavelength ranges. The Crab Nebula is the remnant of a supernova explosion that was seen on the Earth in 1054 A.D. (Compare with Fig. 8.28).

Applying Physics 21.3 The Sun and the Evolution of the Eye

The center of sensitivity of our eyes coincides with the center of the wavelength distribution of the Sun. Is this an amazing coincidence?

Explanation This is not a coincidence; rather it's the result of biological evolution. Humans have evolved

with vision most sensitive to wavelengths that are strongest from the Sun. If aliens from another planet ever arrived at Earth, their eyes would have the center of sensitivity at wavelengths different from ours. If their sun were a red dwarf, for example, they'd be most sensitive to red light.

21.13 THE DOPPLER EFFECT FOR ELECTROMAGNETIC WAVES

As we saw in Section 14.6, sound waves exhibit the Doppler effect when the observer, the source, or both are moving relative to the medium of propagation. Recall that in the Doppler effect, the observed frequency of the wave is larger or smaller than the frequency emitted by the source of the wave.

A Doppler effect also occurs for electromagnetic waves, but it differs from the Doppler effect for sound waves in two ways. First, in the Doppler effect for sound waves, motion relative to the medium is most important, because sound waves require a medium in which to propagate. In contrast, the medium of propagation plays no role in the Doppler effect for electromagnetic waves, because the waves require no medium in which to propagate. Second, the speed of sound that appears in the equation for the Doppler effect for sound depends on the reference frame in which it is measured. In contrast, as we shall see in Chapter 26, the speed of electromagnetic waves has the same value in all coordinate systems that are either at rest or moving at constant velocity with respect to one another.

The single equation that describes the Doppler effect for electromagnetic waves is given by the approximate expression

$$f_o \approx f_s \left(1 \pm \frac{u}{c} \right) \quad \text{if } u \ll c \quad [21.32]$$

where f_o is the observed frequency, f_s is the frequency emitted by the source, c is the speed of light in a vacuum, and u is the *relative* speed of the observer and source. Note that Equation 21.32 is valid only if u is much smaller than c . Further, it can also be used for sound as long as the relative velocity of the source and observer is much less than the velocity of sound. The positive sign in the equation must be used when the source and observer are moving toward one another, while the negative sign must be used when they are moving away from each other. Thus, we anticipate an increase in the observed frequency if the source and observer are approaching each other and a decrease if the source and observer recede from each other.

Astronomers have made important discoveries using Doppler observations on light reaching Earth from distant stars and galaxies. Such measurements have shown that most distant galaxies are moving away from the Earth. Thus, the Universe is expanding. This Doppler shift is called a *red shift* because the observed wavelengths are shifted towards the red portion (longer wavelengths) of the visible spectrum. Further, measurements show that the speed of a galaxy increases with increasing distance from the Earth. More recent Doppler effect measurements made with the Hubble Space Telescope have shown that a galaxy labeled M87 is rotating, with one edge moving toward us and the other moving away. Its measured speed of rotation was used to identify a supermassive black hole located at its center.

SUMMARY

Physics Now™ Take a practice test by logging into Physics-Now at www.cp7e.com and clicking on the Pre-Test link for this chapter.

21.1 Resistors in an AC Circuit

If an AC circuit consists of a generator and a resistor, the current in the circuit is in phase with the voltage, which means the current and voltage reach their maximum values at the same time.

In discussions of voltages and currents in AC circuits, **rms values** of voltages are usually used. One reason is that

AC ammeters and voltmeters are designed to read rms values. The rms values of currents and voltage (I_{rms} and ΔV_{rms}), are related to the maximum values of these quantities (I_{max} and ΔV_{max}) as follows:

$$I_{\text{rms}} = \frac{I_{\text{max}}}{\sqrt{2}} \quad \text{and} \quad \Delta V_{\text{rms}} = \frac{\Delta V_{\text{max}}}{\sqrt{2}} \quad [21.2, 21.3]$$

The rms voltage across a resistor is related to the rms current in the resistor by **Ohm's law**:

$$\Delta V_{R,\text{rms}} = I_{\text{rms}} R \quad [21.4]$$

21.2 Capacitors in an AC Circuit

If an AC circuit consists of a generator and a capacitor, the voltage lags behind the current by 90° . This means that the voltage reaches its maximum value one-quarter of a period after the current reaches its maximum value.

The impeding effect of a capacitor on current in an AC circuit is given by the **capacitive reactance** X_C , defined as

$$X_C \equiv \frac{1}{2\pi fC} \quad [21.5]$$

where f is the frequency of the AC voltage source.

The rms voltage across and the rms current in a capacitor are related by

$$\Delta V_{C,\text{rms}} = I_{\text{rms}} X_C \quad [21.6]$$

21.3 Inductors in an AC Circuit

If an AC circuit consists of a generator and an inductor, the voltage leads the current by 90° . This means the voltage reaches its maximum value one-quarter of a period before the current reaches its maximum value.

The effective impedance of a coil in an AC circuit is measured by a quantity called the **inductive reactance** X_L , defined as

$$X_L \equiv 2\pi fL \quad [21.8]$$

The rms voltage across a coil is related to the rms current in the coil by

$$\Delta V_{L,\text{rms}} = I_{\text{rms}} X_L \quad [21.9]$$

21.4 The RLC Series Circuit

In an *RLC* series AC circuit, the maximum applied voltage ΔV is related to the maximum voltages across the resistor (ΔV_R), capacitor (ΔV_C), and inductor (ΔV_L) by

$$\Delta V_{\text{max}} = \sqrt{\Delta V_R^2 + (\Delta V_L - \Delta V_C)^2} \quad [21.10]$$

If an AC circuit contains a resistor, an inductor, and a capacitor connected in series, the limit they place on the current is given by the **impedance** Z of the circuit, defined as

$$Z \equiv \sqrt{R^2 + (X_L - X_C)^2} \quad [21.13]$$

The relationship between the maximum voltage supplied to an *RLC* series AC circuit and the maximum current in the circuit, which is the same in every element, is

$$\Delta V_{\text{max}} = I_{\text{max}} Z \quad [21.14]$$

In an *RLC* series AC circuit, the applied rms voltage and current are out of phase. The **phase angle** ϕ between the current and voltage is given by

$$\tan \phi = \frac{X_L - X_C}{R} \quad [21.15]$$

21.5 Power in an AC Circuit

The **average power** delivered by the voltage source in an *RLC* series AC circuit is

$$\mathcal{P}_{\text{av}} = I_{\text{rms}} \Delta V_{\text{rms}} \cos \phi \quad [21.17]$$

where the constant $\cos \phi$ is called the **power factor**.

21.6 Resonance in a Series RLC Circuit

In general, the rms current in a series *RLC* circuit can be written

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{Z} = \frac{\Delta V_{\text{rms}}}{\sqrt{R^2 + (X_L - X_C)^2}} \quad [21.18]$$

The current has its *maximum* value when the impedance has its *minimum* value, corresponding to $X_L = X_C$ and $Z = R$. The frequency f_0 at which this happens is called the **resonance frequency** of the circuit, given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \quad [21.19]$$

21.7 The Transformer

If the primary winding of a transformer has N_1 turns and the secondary winding consists of N_2 turns, then if an input AC voltage ΔV_1 is applied to the primary, the induced voltage in the secondary winding is given by

$$\Delta V_2 = \frac{N_2}{N_1} \Delta V_1 \quad [21.22]$$

When N_2 is greater than N_1 , ΔV_2 exceeds ΔV_1 and the transformer is referred to as a *step-up transformer*. When N_2 is less than N_1 , making ΔV_2 less than ΔV_1 , we have a *step-down transformer*. In an ideal transformer, the power output equals the power input.

21.8–21.13 Electromagnetic Waves and their Properties

Electromagnetic waves were predicted by James Clerk Maxwell and experimentally confirmed by Heinrich Hertz. These waves are created by accelerating electric charges, and have the following properties:

1. Electromagnetic waves are transverse waves, because the electric and magnetic fields are perpendicular to the direction of propagation of the waves.
2. Electromagnetic waves travel at the speed of light.
3. The ratio of the electric field to the magnetic field at a given point in an electromagnetic wave equals the speed of light:

$$\frac{E}{B} = c \quad [21.26]$$

4. Electromagnetic waves carry energy as they travel through space. The average power per unit area is the intensity I , given by

$$I = \frac{E_{\text{max}} B_{\text{max}}}{2\mu_0} = \frac{E_{\text{max}}^2}{2\mu_0 c} = \frac{c}{2\mu_0} B_{\text{max}}^2 \quad [21.27, 21.28]$$

where E_{max} and B_{max} are the maximum values of the electric and magnetic fields.

5. Electromagnetic waves transport linear and angular momentum as well as energy. The momentum p delivered in time Δt at normal incidence to an object that completely absorbs light energy U is given by

$$p = \frac{U}{c} \quad (\text{complete absorption}) \quad [21.29]$$

If the surface is a perfect reflector, then the momentum delivered in time Δt at normal incidence is twice that given by Equation 21.29:

$$p = \frac{2U}{c} \quad (\text{complete reflection}) \quad [21.30]$$

6. The speed c , frequency f , and wavelength λ of an electromagnetic wave are related by

$$c = f\lambda \quad [21.31]$$

The **electromagnetic spectrum** includes waves covering a broad range of frequencies and wavelengths. These waves have a variety of applications and characteristics, depend-

ing on their frequencies or wavelengths. The frequency of a given wave can be shifted by the relative velocity of observer and source, with the observed frequency f_o given by

$$f_o \approx f_s \left(1 \pm \frac{u}{c} \right) \quad \text{if } u \ll c \quad [21.32]$$

where f_s is the frequency of the source, c is the speed of light in a vacuum, and u is the *relative* speed of the observer and source. The positive sign is used when the source and observer approach each other, the negative sign when they recede from each other.

CONCEPTUAL QUESTIONS

- Before the advent of cable television and satellite dishes, homeowners either mounted a television antenna on the roof or used “rabbit ears” atop their sets. (See Fig. Q21.1.) Certain orientations of the receiving antenna on a television set gave better reception than others. Furthermore, the best orientation varied from station to station. Explain.



Figure Q21.1

- What is the impedance of an RLC circuit at the resonance frequency?
- When a DC voltage is applied to a transformer, the primary coil sometimes will overheat and burn. Why?
- Why are the primary and secondary coils of a transformer wrapped on an iron core that passes through both coils?
- Receiving radio antennas can be in the form of conducting lines or loops. What should the orientation of each of these antennas be relative to a broadcasting antenna that is vertical?
- If the fundamental source of a sound wave is a vibrating object, what is the fundamental source of an electromagnetic wave?
- In radio transmission, a radio wave serves as a carrier wave, and the sound signal is superimposed on the carrier wave. In amplitude modulation (AM) radio, the amplitude of the carrier wave varies according to the sound wave. The Navy sometimes uses flashing lights to send Morse code between neighboring ships, a process that has similarities to radio broadcasting. Is this process AM or FM? What is the carrier frequency? What is the signal frequency? What is the broadcasting antenna? What is the receiving antenna?
- When light (or other electromagnetic radiation) travels across a given region, what is it that oscillates? What is it that is transported?
- In space sailing, which is a proposed alternative for transport to the planets, a spacecraft carries a very large sail. Sunlight striking the sail exerts a force, accelerating the spacecraft. Should the sail be absorptive or reflective to be most effective?
- How can the average value of an alternating current be zero, yet the square root of the average squared value not be zero?
- Suppose a creature from another planet had eyes that were sensitive to infrared radiation. Describe what it would see if it looked around the room that you are now in. That is, what would be bright and what would be dim?
- Why should an infrared photograph of a person look different from a photograph taken using visible light?
- Radio stations often advertise “instant news.” If what they mean is that you hear the news at the instant they speak it, is their claim true? About how long would it take for a message to travel across the United States by radio waves, assuming that the waves could travel that great distance and still be detected?
- Would an inductor and a capacitor used together in an AC circuit dissipate any energy?
- Does a wire connected to a battery emit an electromagnetic wave?
- If a high-frequency current is passed through a solenoid containing a metallic core, the core becomes warm due to induction. Explain why the temperature of the material rises in this situation.
- If the resistance in an RLC circuit remains the same, but the capacitance and inductance are each doubled, how will the resonance frequency change?
- Why is the sum of the maximum voltages across each of the elements in a series RLC circuit usually greater than the maximum applied voltage? Doesn't this violate Kirchhoff's loop rule?
- What is the advantage of transmitting power at high voltages?
- What determines the maximum voltage that can be used on a transmission line?
- Will a transformer operate if a battery is used for the input voltage across the primary? Explain.

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging □ = full solution available in *Student Solutions Manual/Study Guide*
Physics Now™ = coached problem with hints available at www.cp7e.com  = biomedical application

Section 21.1 Resistors in an AC Circuit

- An rms voltage of 100 V is applied to a purely resistive load of $5.00\ \Omega$. Find (a) the maximum voltage applied, (b) the rms current supplied, (c) the maximum current supplied, and (d) the power dissipated.
- (a) What is the resistance of a lightbulb that uses an average power of 75.0 W when connected to a 60-Hz power source with an peak voltage of 170 V? (b) What is the resistance of a 100-W bulb?
- An AC power supply that produces a maximum voltage of $\Delta V_{\max} = 100\ \text{V}$ is connected to a $24.0\text{-}\Omega$ resistor. The current and the resistor voltage are respectively measured with an ideal AC ammeter and an ideal AC voltmeter, as shown in Figure P21.3. What does each meter read? Note that an ideal ammeter has zero resistance and an ideal voltmeter has infinite resistance.

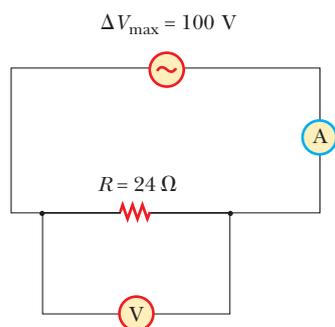


Figure P21.3

- Figure P21.4 shows three lamps connected to a 120-V AC (rms) household supply voltage. Lamps 1 and 2 have 150-W bulbs; lamp 3 has a 100-W bulb. Find the rms current and the resistance of each bulb.

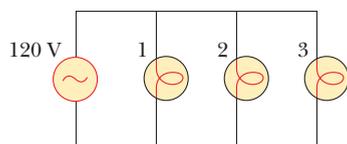


Figure P21.4

- An audio amplifier, represented by the AC source and the resistor R in Figure P21.5, delivers alternating voltages at audio frequencies to the speaker. If the source puts out an alternating voltage of 15.0 V (rms), the resistance R is $8.20\ \Omega$, and the speaker is equivalent to a resistance of $10.4\ \Omega$, what is the time-averaged power delivered to the speaker?

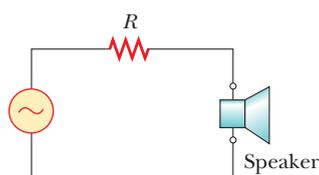


Figure P21.5

- An AC voltage source has an output voltage given by $\Delta v = (150\ \text{V}) \sin 377t$. Find (a) the rms voltage output, (b) the frequency of the source, and (c) the voltage at $t = 1/120\ \text{s}$. (d) Find the maximum current in the circuit when the voltage source is connected to a $50.0\text{-}\Omega$ resistor.

Section 21.2 Capacitors in an AC Circuit

- Show that the SI unit of capacitive reactance X_c is the ohm.
- What is the maximum current delivered to a circuit containing a $2.20\text{-}\mu\text{F}$ capacitor when it is connected across (a) a North American outlet having $\Delta V_{\text{rms}} = 120\ \text{V}$ and $f = 60.0\ \text{Hz}$ and (b) a European outlet having $\Delta V_{\text{rms}} = 240\ \text{V}$ and $f = 50.0\ \text{Hz}$?
- Physics Now**™ When a $4.0\text{-}\mu\text{F}$ capacitor is connected to a generator whose rms output is 30 V, the current in the circuit is observed to be 0.30 A. What is the frequency of the source?
- What maximum current is delivered by an AC generator with a maximum voltage of $\Delta V_{\max} = 48.0\ \text{V}$ and a frequency $f = 90.0\ \text{Hz}$ when it is connected across a $3.70\text{-}\mu\text{F}$ capacitor?
- What must be the capacitance of a capacitor inserted in a 60-Hz circuit in series with a generator of 170 V maximum output voltage to produce an rms current output of 0.75 A?
- The generator in a purely capacitive AC circuit has an angular frequency of $120\pi\ \text{rad/s}$. If $\Delta V_{\max} = 140\ \text{V}$ and $C = 6.00\ \mu\text{F}$, what is the rms current in the circuit?

Section 21.3 Inductors in an AC Circuit

- Show that the inductive reactance X_L has SI units of ohms.
- The generator in a purely inductive AC circuit has an angular frequency of $120\pi\ \text{rad/s}$. If $V_{\max} = 140\ \text{V}$ and $L = 0.100\ \text{H}$, what is the rms current in the circuit?
- An inductor has a $54.0\text{-}\Omega$ reactance at 60.0 Hz. What will be the *maximum* current if this inductor is connected to a 50.0-Hz source that produces a 100-V rms voltage?
- An inductor is connected to a 20.0-Hz power supply that produces a 50.0-V rms voltage. What inductance is needed to keep the maximum current in the circuit below 80.0 mA?
- Determine the maximum magnetic flux through an inductor connected to a standard outlet ($\Delta V_{\text{rms}} = 120\ \text{V}$, $f = 60.0\ \text{Hz}$).

Section 21.4 The RLC Series Circuit

- An inductor ($L = 400\ \text{mH}$), a capacitor ($C = 4.43\ \mu\text{F}$), and a resistor ($R = 500\ \Omega$) are connected in series. A 50.0-Hz AC generator connected in series to these elements produces a maximum current of 250 mA in the circuit. (a) Calculate the required maximum voltage ΔV_{\max} . (b) Determine the phase angle by which the current leads or lags the applied voltage.
- A $40.0\text{-}\mu\text{F}$ capacitor is connected to a $50.0\text{-}\Omega$ resistor and a generator whose rms output is 30.0 V at 60.0 Hz. Find

- (a) the rms current in the circuit, (b) the rms voltage drop across the resistor, (c) the rms voltage drop across the capacitor, and (d) the phase angle for the circuit.
20. A $50.0\text{-}\Omega$ resistor, a 0.100-H inductor, and a $10.0\text{-}\mu\text{F}$ capacitor are connected in series to a 60.0-Hz source. The rms current in the circuit is 2.75 A . Find the rms voltages across (a) the resistor, (b) the inductor, (c) the capacitor, and (d) the RLC combination. (e) Sketch the phasor diagram for this circuit.
21. A resistor ($R = 900\ \Omega$), a capacitor ($C = 0.25\ \mu\text{F}$), and an inductor ($L = 2.5\ \text{H}$) are connected in series across a 240-Hz AC source for which $\Delta V_{\text{max}} = 140\ \text{V}$. Calculate (a) the impedance of the circuit, (b) the maximum current delivered by the source, and (c) the phase angle between the current and voltage. (d) Is the current leading or lagging the voltage?
22. An AC source operating at $60\ \text{Hz}$ with a maximum voltage of $170\ \text{V}$ is connected in series with a resistor ($R = 1.2\ \text{k}\Omega$) and a capacitor ($C = 2.5\ \mu\text{F}$). (a) What is the maximum value of the current in the circuit? (b) What are the maximum values of the potential difference across the resistor and the capacitor? (c) When the current is zero, what are the magnitudes of the potential difference across the resistor, the capacitor, and the AC source? How much charge is on the capacitor at this instant? (d) When the current is at a maximum, what are the magnitudes of the potential differences across the resistor, the capacitor, and the AC source? How much charge is on the capacitor at this instant?
23. A $60.0\text{-}\Omega$ resistor, a $3.00\text{-}\mu\text{F}$ capacitor, and a 0.400-H inductor are connected in series to a 90.0-V (rms), 60.0-Hz source. Find (a) the voltage drop across the LC combination and (b) the voltage drop across the RC combination.
24. An AC source operating at $60\ \text{Hz}$ with a maximum voltage of $170\ \text{V}$ is connected in series with a resistor ($R = 1.2\ \text{k}\Omega$) and an inductor ($L = 2.8\ \text{H}$). (a) What is the maximum value of the current in the circuit? (b) What are the maximum values of the potential difference across the resistor and the inductor? (c) When the current is at a maximum, what are the magnitudes of the potential differences across the resistor, the inductor, and the AC source? (d) When the current is zero, what are the magnitudes of the potential difference across the resistor, the inductor, and the AC source?
25. A person is working near the secondary of a transformer, as shown in Figure P21.25. The primary voltage is $120\ \text{V}$ (rms) at $60.0\ \text{Hz}$. The capacitance C_s , which is the stray capacitance between the hand and the secondary winding, is $20.0\ \text{pF}$. Assuming that the person has a body resistance to ground of $R_b = 50.0\ \text{k}\Omega$, determine the rms voltage across the body. (*Hint*: Redraw the circuit with the secondary of the transformer as a simple AC source.)

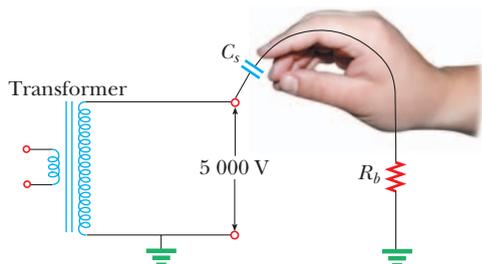


Figure P21.25

26. A coil of resistance $35.0\ \Omega$ and inductance $20.5\ \text{H}$ is in series with a capacitor and a 200-V (rms), 100-Hz source. The rms current in the circuit is $4.00\ \text{A}$. (a) Calculate the capacitance in the circuit. (b) What is ΔV_{rms} across the coil?
27. **PhysicsNow**™ An AC source with a maximum voltage of $150\ \text{V}$ and $f = 50.0\ \text{Hz}$ is connected between points a and d in Figure P21.27. Calculate the rms voltages between points (a) a and b , (b) b and c , (c) c and d , and (d) b and d .

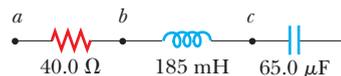


Figure P21.27

Section 21.5 Power in an AC Circuit

28. A $50.0\text{-}\Omega$ resistor is connected to a $30.0\text{-}\mu\text{F}$ capacitor and to a 60.0-Hz , 100-V (rms) source. (a) Find the power factor and the average power delivered to the circuit. (b) Repeat part (a) when the capacitor is replaced with a 0.300-H inductor.
29. A multimeter in an RL circuit records an rms current of $0.500\ \text{A}$ and a 60.0-Hz rms generator voltage of $104\ \text{V}$. A wattmeter shows that the average power delivered to the resistor is $10.0\ \text{W}$. Determine (a) the impedance in the circuit, (b) the resistance R , and (c) the inductance L .
30. An AC voltage with an amplitude of $100\ \text{V}$ is applied to a series combination of a $200\text{-}\mu\text{F}$ capacitor, a 100-mH inductor, and a $20.0\text{-}\Omega$ resistor. Calculate the power dissipation and the power factor for frequencies of (a) $60.0\ \text{Hz}$ and (b) $50.0\ \text{Hz}$.
31. An inductor and a resistor are connected in series. When connected to a 60-Hz , 90-V (rms) source, the voltage drop across the resistor is found to be $50\ \text{V}$ (rms) and the power delivered to the circuit is $14\ \text{W}$. Find (a) the value of the resistance and (b) the value of the inductance.
32. Consider a series RLC circuit with $R = 25\ \Omega$, $L = 6.0\ \text{mH}$, and $C = 25\ \mu\text{F}$. The circuit is connected to a 10-V (rms), 600-Hz AC source. (a) Is the sum of the voltage drops across R , L , and C equal to $10\ \text{V}$ (rms)? (b) Which is greatest, the power delivered to the resistor, to the capacitor, or to the inductor? (c) Find the average power delivered to the circuit.

Section 21.6 Resonance in a Series RLC circuit

33. An RLC circuit is used to tune a radio to an FM station broadcasting at $88.9\ \text{MHz}$. The resistance in the circuit is $12.0\ \Omega$ and the capacitance is $1.40\ \text{pF}$. What inductance should be present in the circuit?
34. Consider a series RLC circuit with $R = 15\ \Omega$, $L = 200\ \text{mH}$, $C = 75\ \mu\text{F}$, and a maximum voltage of $150\ \text{V}$. (a) What is the impedance of the circuit at resonance? (b) What is the resonance frequency of the circuit? (c) When will the current be greatest—at resonance, at ten percent below the resonant frequency, or at ten percent above the resonant frequency? (d) What is the rms current in the circuit at a frequency of $60\ \text{Hz}$?
35. The AM band extends from approximately $500\ \text{kHz}$ to $1\ 600\ \text{kHz}$. If a $2.0\text{-}\mu\text{H}$ inductor is used in a tuning circuit for a radio, what are the extremes that a capacitor

must reach in order to cover the complete band of frequencies?

36. A series circuit contains a 3.00-H inductor, a 3.00- μF capacitor, and a 30.0- Ω resistor connected to a 120-V (rms) source of variable frequency. Find the power delivered to the circuit when the frequency of the source is (a) the resonance frequency, (b) one-half the resonance frequency, (c) one-fourth the resonance frequency, (d) two times the resonance frequency, and (e) four times the resonance frequency. From your calculations, can you draw a conclusion about the frequency at which the maximum power is delivered to the circuit?
37. **Physics Now™** A 10.0- Ω resistor, a 10.0-mH inductor, and a 100- μF capacitor are connected in series to a 50.0-V (rms) source having variable frequency. Find the energy delivered to the circuit during one period if the operating frequency is twice the resonance frequency.

Section 21.7 The Transformer

38. An AC adapter for a telephone-answering unit uses a transformer to reduce the line voltage of 120 V (rms) to a voltage of 9.0 V. The rms current delivered to the answering system is 400 mA. (a) If the primary (input) coil in the transformer in the adapter has 240 turns, how many turns are there on the secondary (output) coil? (b) What is the rms power delivered to the transformer? Assume an ideal transformer.
39. An AC power generator produces 50 A (rms) at 3 600 V. The voltage is stepped up to 100 000 V by an ideal transformer, and the energy is transmitted through a long-distance power line that has a resistance of 100 Ω . What percentage of the power delivered by the generator is dissipated as heat in the power line?
40. A transformer is to be used to provide power for a computer disk drive that needs 6.0 V (rms) instead of the 120 V (rms) from the wall outlet. The number of turns in the primary is 400, and it delivers 500 mA (the secondary current) at an output voltage of 6.0 V (rms). (a) Should the transformer have more turns in the secondary compared with the primary, or fewer turns? (b) Find the current in the primary. (c) Find the number of turns in the secondary.
41. A transformer on a pole near a factory steps the voltage down from 3 600 V (rms) to 120 V (rms). The transformer is to deliver 1 000 kW to the factory at 90% efficiency. Find (a) the power delivered to the primary, (b) the current in the primary, and (c) the current in the secondary.
42. A transmission line that has a resistance per unit length of $4.50 \times 10^{-4} \Omega/\text{m}$ is to be used to transmit 5.00 MW over 400 miles ($6.44 \times 10^5 \text{ m}$). The output voltage of the generator is 4.50 kV (rms). (a) What is the line loss if a transformer is used to step up the voltage to 500 kV (rms)? (b) What fraction of the input power is lost to the line under these circumstances? (c) What difficulties would be encountered on attempting to transmit the 5.00 MW at the generator voltage of 4.50 kV (rms)?
- systems; such waves could penetrate the oceans to reach distant submarines. Calculate the length of a quarter-wavelength antenna for a transmitter generating ELF waves of frequency 75 Hz. How practical is this antenna?
44. Experimenters at the National Institute of Standards and Technology have made precise measurements of the speed of light using the fact that, in vacuum, the speed of electromagnetic waves is $c = 1/\sqrt{\mu_0\epsilon_0}$, where the constants $\mu_0 = 4\pi \times 10^{-7} \text{ N}\cdot\text{s}^2/\text{C}^2$ and $\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2$. What value (to four significant figures) does this formula give for the speed of light in vacuum?
45. Oxygenated hemoglobin absorbs weakly in the red (hence its red color) and strongly in the near infrared, while deoxygenated hemoglobin has the opposite absorption. This fact is used in a “pulse oximeter” to measure oxygen saturation in arterial blood. The device clips onto the end of a person’s finger and has two light-emitting diodes [a red (660 nm) and an infrared (940 nm)] and a photocell that detects the amount of light transmitted through the finger at each wavelength. (a) Determine the frequency of each of these light sources. (b) If 67% of the energy of the red source is absorbed in the blood, by what factor does the amplitude of the electromagnetic wave change? [*Hint:* The intensity of the wave is equal to the average power per unit area as given by Equation 21.28.]
46. **Operation of the pulse oximeter (see previous problem).** The transmission of light energy as it passes through a solution of light-absorbing molecules is described by the Beer–Lambert law

$$I = I_0 10^{-\epsilon CL} \quad \text{or} \quad \log_{10} \left(\frac{I}{I_0} \right) = -\epsilon CL$$

which gives the decrease in intensity I in terms of the distance L the light has traveled through a fluid with a concentration C of the light-absorbing molecule. The quantity ϵ is called the extinction coefficient, and its value depends on the frequency of the light. (It has units of m^2/mol .) Assume that the extinction coefficient for 660-nm light passing through a solution of oxygenated hemoglobin is identical to the coefficient for 940-nm light passing through deoxygenated hemoglobin. Assume also that 940-nm light has zero absorption ($\epsilon = 0$) in oxygenated hemoglobin and 660-nm light has zero absorption in deoxygenated hemoglobin. If 33% of the energy of the red source and 76% of the infrared energy is transmitted through the blood, determine the fraction of hemoglobin that is oxygenated.

Section 21.10 Production of Electromagnetic Waves by an Antenna

Section 21.11 Properties of Electromagnetic Waves

43. The U.S. Navy has long proposed the construction of extremely low frequency (ELF waves) communications
47. A microwave oven is powered by an electron tube called a magnetron that generates electromagnetic waves of frequency 2.45 GHz. The microwaves enter the oven and are reflected by the walls. The standing-wave pattern produced in the oven can cook food unevenly, with hot spots in the food at antinodes and cool spots at nodes, so a turntable is often used to rotate the food and distribute the energy. If a microwave oven is used with a cooking dish in a fixed position, the antinodes can appear as burn marks on foods such as carrot strips or cheese. The separation distance between the burns is measured to be 6.00 cm. Calculate the speed of the microwaves from these data.
48. Assume that the solar radiation incident on Earth is 1 340 W/m^2 (at the top of Earth’s atmosphere). Calculate

the total power radiated by the Sun, taking the average separation between Earth and the Sun to be 1.49×10^{11} m.

49. **PhysicsNow™** The Sun delivers an average power of $1\,340\text{ W/m}^2$ to the top of Earth's atmosphere. Find the magnitudes of \vec{E}_{max} and \vec{B}_{max} for the electromagnetic waves at the top of the atmosphere.

Section 21.12 The Spectrum of Electromagnetic Waves

50. A diathermy machine, used in physiotherapy, generates electromagnetic radiation that gives the effect of "deep heat" when absorbed in tissue. One assigned frequency for diathermy is 27.33 MHz. What is the wavelength of this radiation?
51. What are the wavelength ranges in (a) the AM radio band (540–1 600 kHz) and (b) the FM radio band (88–108 MHz)?
52. An important news announcement is transmitted by radio waves to people who are 100 km away, sitting next to their radios, and by sound waves to people sitting across the newsroom, 3.0 m from the newscaster. Who receives the news first? Explain. Take the speed of sound in air to be 343 m/s.
53. Infrared spectra are used by chemists to help identify an unknown substance. Atoms in a molecule that are bound together by a particular bond vibrate at a predictable frequency, and light at that frequency is absorbed strongly by the atom. In the case of the C=O double bond, for example, the oxygen atom is bound to the carbon by a bond that has an effective spring constant of 2 800 N/m. If we assume that the carbon atom remains stationary (it is attached to other atoms in the molecule), determine the resonant frequency of this bond and the wavelength of light that matches that frequency. Verify that this wavelength lies in the infrared region of the spectrum. (The mass of an oxygen atom is 2.66×10^{-26} kg.)

21.13 The Doppler Effect for Electromagnetic Waves

54. A spaceship is approaching a space station at a speed of 1.8×10^5 m/s. The space station has a beacon that emits green light with a frequency of 6.0×10^{14} Hz. What is the frequency of the beacon observed on the spaceship? What is the change in frequency? (Carry five digits in these calculations.)
55. While driving at a constant speed of 80 km/h, you are passed by a car traveling at 120 km/h. If the frequency of light emitted by the taillights of the car that passes you is 4.3×10^{14} Hz, what frequency will you observe? What is the change in frequency?
56. A speeder tries to explain to the police that the yellow warning lights on the side of the road looked green to her because of the Doppler shift. How fast would she have been traveling if yellow light of wavelength 580 nm had been shifted to green with a wavelength of 560 nm? (Note that, for speeds less than $0.03c$, Equation 21.32 will lead to a value for the change of frequency accurate to approximately two significant digits.)

ADDITIONAL PROBLEMS

57. As a way of determining the inductance of a coil used in a research project, a student first connects the coil to a 12.0-V battery and measures a current of 0.630 A. The student

then connects the coil to a 24.0-V (rms), 60.0-Hz generator and measures an rms current of 0.570 A. What is the inductance?

58. The intensity of solar radiation at the top of Earth's atmosphere is $1\,340\text{ W/m}^2$. Assuming that 60% of the incoming solar energy reaches Earth's surface, and assuming that you absorb 50% of the incident energy, make an order-of-magnitude estimate of the amount of solar energy you absorb in a 60-minute sunbath.
59. A $200\text{-}\Omega$ resistor is connected in series with a $5.0\text{-}\mu\text{F}$ capacitor and a 60-Hz, 120-V rms line. If electrical energy costs $\$0.080/\text{kWh}$, how much does it cost to leave this circuit connected for 24 h?
60. A series RLC circuit has a resonance frequency of $2\,000/\pi$ Hz. When it is operating at a frequency of $\omega > \omega_0$, $X_L = 12\ \Omega$ and $X_C = 8.0\ \Omega$. Calculate the values of L and C for the circuit.
61. Two connections allow contact with two circuit elements in series inside a box, but it is not known whether the circuit elements are R , L , or C . In an attempt to find what is inside the box, you make some measurements, with the following results: when a 3.0-V DC power supply is connected across the terminals, a maximum direct current of 300 mA is measured in the circuit after a suitably long time. When a 60-Hz source with maximum voltage of 3.0 V is connected instead, the maximum current is measured as 200 mA. (a) What are the two elements in the box? (b) What are their values of R , L , or C ?
62. (a) What capacitance will resonate with a one-turn loop of inductance 400 pH to give a radar wave of wavelength 3.0 cm? (b) If the capacitor has square parallel plates separated by 1.0 mm of air, what should the edge length of the plates be? (c) What is the common reactance of the loop and capacitor at resonance?
63. A dish antenna with a diameter of 20.0 m receives (at normal incidence) a radio signal from a distant source, as shown in Figure P21.63. The radio signal is a continuous sinusoidal wave with amplitude $E_{\text{max}} = 0.20\ \mu\text{V/m}$. Assume that the antenna absorbs all the radiation that falls on the dish. (a) What is the amplitude of the magnetic field in this wave? (b) What is the intensity of the radiation received by the antenna? (c) What is the power received by the antenna?

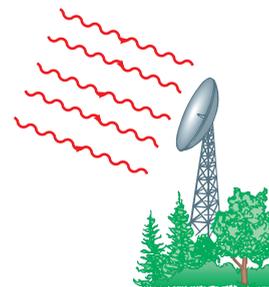


Figure P21.63

64. A particular inductor has appreciable resistance. When the inductor is connected to a 12-V battery, the current in the inductor is 3.0 A. When it is connected to an AC source with an rms output of 12 V and a frequency of 60 Hz, the current drops to 2.0 A. What are (a) the impedance at 60 Hz and (b) the inductance of the inductor?

65. One possible means of achieving space flight is to place a perfectly reflecting aluminized sheet into Earth's orbit and to use the light from the Sun to push this solar sail. Suppose such a sail, of area $6.00 \times 10^4 \text{ m}^2$ and mass 6 000 kg, is placed in orbit facing the Sun. (a) What force is exerted on the sail? (b) What is the sail's acceleration? (c) How long does it take for this sail to reach the Moon, $3.84 \times 10^8 \text{ m}$ away? Ignore all gravitational effects, and assume a solar intensity of $1\,340 \text{ W/m}^2$. [*Hint*: The radiation pressure by a reflected wave is given by $2(\text{average power per unit area})/c$.]
66. Suppose you wish to use a transformer as an impedance-matching device between an audio amplifier that has an output impedance of $8.0 \text{ k}\Omega$ and a speaker that has an input impedance of 8.0Ω . What should be the ratio of primary to secondary turns on the transformer?
67. Compute the average energy content of a liter of sunlight as it reaches the top of Earth's atmosphere, where its intensity is $1\,340 \text{ W/m}^2$.
68. In an *RLC* series circuit that includes a source of alternating current operating at fixed frequency and voltage, the resistance R is equal to the inductive reactance. If the plate separation of the capacitor is reduced to one-half of its original value, the current in the circuit doubles. Find the initial capacitive reactance in terms of R .

ACTIVITIES

- For this observation, you will need some items that can be found at many electronics stores. You will need a bicolored light-emitting diode (LED), a resistor of about 100Ω , 2 m of flexible wire, and a step-down transformer with an output of 3 to 6 V. Use the wire to connect the LED and the resistor in series with the transformer. A bicolored LED is designed such that it emits a red color when the current in the LED is in one direction and green when the current reverses. When connected to an AC source, the LED is yellow. Why?

Hold the wires and whirl the LED in a circular path. In a darkened room, you will see red and green bars at equally spaced intervals along the path of the LED. Why?

As you continue to whirl the LED in a circular path, have your partner count the number of green bars in the circle, then measure the time it takes for the LED to travel ten times around the circular path. Based on this information, determine the time it takes for the color of the LED to change from green to red to green. You should obtain an answer of $(1/60) \text{ s}$. Why?
- Rotate a portable radio (with a telescoping antenna) about a horizontal axis while it is tuned to a weak station. Such an antenna detects the varying electric field produced by the station. What can you determine about the direction of the electric field produced by the transmitter?

Now turn on your radio to a nearby station and experiment with shielding the radio from incoming waves. Is the reception affected by surrounding the radio by aluminum foil? By plastic wrap? Use any other material you have available. What kinds of material block the signal? Why?