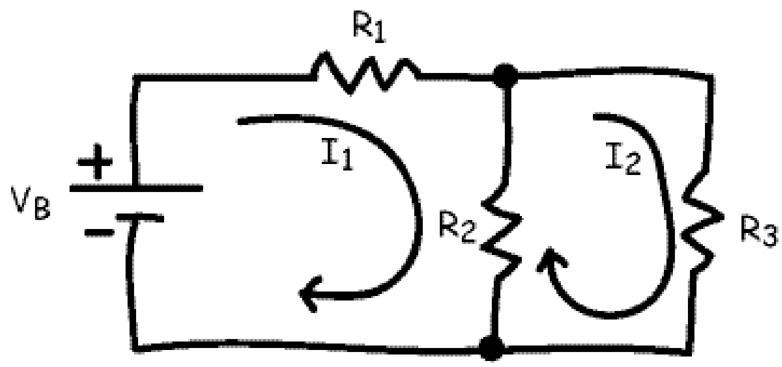
This handout covers the two-loop circuit that we discussed in class. It is important to understand this general case, as two-loop circuits form the basis of many biological problems – e.g., neuronal dendrites that are at a different potential than the soma, coupled biochemical reactions, etc. – and even practical issues like to how to run multiple electrophoresis gels when you are down to one power supply!



Loop 1

$-V_B + I_1R_1 + (I_1 - I_2)\,R_2 = 0$

or

$V_B = I_1\,(R_1 + R_2) - I_2\,R_2$.

Loop 2

$(I_2 - I_1)\,R_2 + I_2\,R_3 = 0$

or

$I_2 = I_1\,\dfrac{R_2}{R_2 + R_3}$.

Thus

$$V_B = I_1\,(R_1 + R_2) - I_1\,\frac{R_2}{R_2+R_3}R_2$$

$$= I_1\,(R_1 + R_2)\,\frac{R_2+R_3}{R_2+R_3} - I_1\,\frac{R_2}{R_2+R_3}R_2$$

$$= I_1\frac{R_1R_2 + R_1R_3 + R_2^2 + R_2R_3 - R_2^2}{R_2+R_3}$$

$$= I_1\frac{R_1R_2 + R_1R_3 + R_2R_3}{R_2+R_3}\ .$$

Thus

$$I_1 = V_B \frac{R_2 + R_3}{R_1R_2 + R_1R_3 + R_2R_3}$$

$$I_2 = V_B \frac{R_2 + R_3}{R_1R_2 + R_1R_3 + R_2R_3} \frac{R_2}{R_2 + R_3}$$

$$= V_B \frac{R_2}{R_1R_2 + R_1R_3 + R_2R_3}$$

$$I_1 - I_2 = V_B \frac{R_2 + R_3}{R_1R_2 + R_1R_3 + R_2R_3} - V_B \frac{R_2}{R_1R_2 + R_1R_3 + R_2R_3}$$

$$= V_B \frac{R_3}{R_1R_2 + R_1R_3 + R_2R_3} .$$

and

$$I_1 - I_2 = V_B \frac{R_2 + R_3}{R_1R_2 + R_1R_3 + R_2R_3} - V_B \frac{R_2}{R_1R_2 + R_1R_3 + R_2R_3}$$

Current Division

Current through R_1 is I_1, which is the total current supplied by the battery.

Current through resistor R_2 is $(I_1 - I_2) = I_1 \frac{R_3}{R_2 + R_3}$

Current through resistor R_3 is $I_2 = I_1 \frac{R_2}{R_2 + R_3}$

For the in-class demonstration, we let $R \equiv R_1 = R_2 = R_3$,

$I_1 = \frac{V_B}{R}\frac{2}{3}$, e.g., voltage drop across R_1 is $I_1R_1 = \frac{2}{3}V_B$.

$I_2 = \frac{V_B}{R}\frac{1}{3}$, e.g., voltage drop across R_3 is $I_2R_3 = \frac{1}{3}V_B$; same as the drop across R_2.

How well do these match with the measured values?