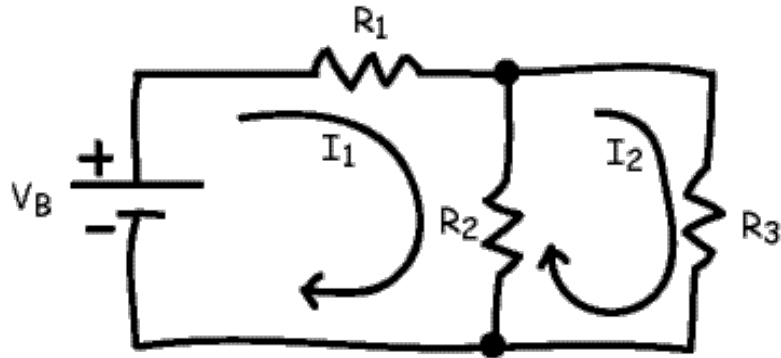


This handout covers the two-loop circuit that we discussed in class. It is important to understand this general case, as two-loop circuits form the basis of many biological problems - e.g., neuronal dendrites that are at a different potential than the soma, coupled biochemical reactions, etc. - and even practical issues like to how to run multiple electrophoresis gels when you are down to one power supply!



### Loop 1

$$-V_B + I_1 R_1 + (I_1 - I_2) R_2 = 0$$

or

$$V_B = I_1 (R_1 + R_2) - I_2 R_2 .$$

### Loop 2

$$(I_2 - I_1) R_2 + I_2 R_3 = 0$$

or

$$I_2 = I_1 \frac{R_2}{R_2 + R_3} .$$

Thus

$$V_B = I_1 (R_1 + R_2) - I_1 \frac{R_2}{R_2 + R_3} R_2$$

$$= I_1 (R_1 + R_2) \frac{R_2 + R_3}{R_2 + R_3} - I_1 \frac{R_2}{R_2 + R_3} R_2$$

$$= I_1 \frac{R_1 R_2 + R_1 R_3 + R_2^2 + R_2 R_3 - R_2^2}{R_2 + R_3}$$

$$= I_1 \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_2 + R_3} .$$

Thus

$$I_1 = V_B \frac{R_2 + R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

$$I_2 = V_B \frac{R_2 + R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} \frac{R_2}{R_2 + R_3}$$
$$= V_B \frac{R_2}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

$$I_1 - I_2 = V_B \frac{R_2 + R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} - V_B \frac{R_2}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$
$$= V_B \frac{R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} .$$

and

$$I_1 - I_2 = V_B \frac{R_2 + R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} - V_B \frac{R_2}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

### Current Division

Current through  $R_1$  is  $I_1$ , which is the total current supplied by the battery.

Current through resistor  $R_2$  is  $(I_1 - I_2) = I_1 \frac{R_3}{R_2 + R_3}$

Current through resistor  $R_3$  is  $I_2 = I_1 \frac{R_2}{R_2 + R_3}$

**For the in-class demonstration, we let  $R \equiv R_1 = R_2 = R_3$ ,**

$I_1 = \frac{V_B}{R} \frac{2}{3}$ , e.g., voltage drop across  $R_1$  is  $I_1 R_1 = \frac{2}{3} V_B$ .

$I_2 = \frac{V_B}{R} \frac{1}{3}$ , e.g., voltage drop across  $R_3$  is  $I_2 R_3 = \frac{1}{3} V_B$ ; same as the drop across  $R_2$ .

**How well do these match with the measured values?**