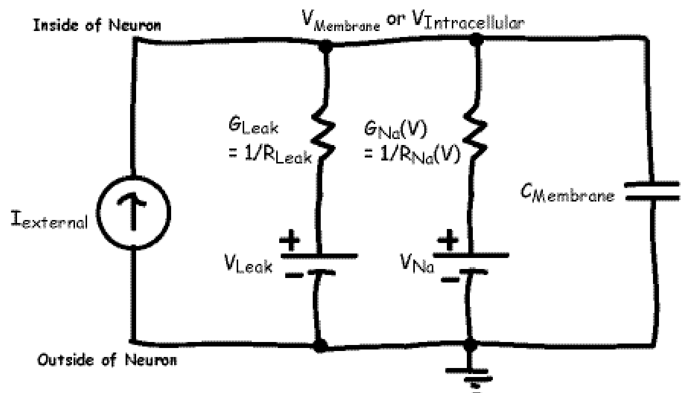
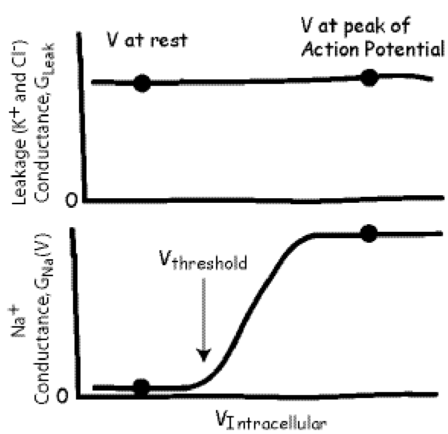
This handout pertains to the operation of a neuron. The neuron does three things. First, it integrates inputs, in the form of both positive and negative currents, from select neighboring cells. Second, it generates a pulse if the voltage caused by the sum of those currents exceeds a threshold value. Third, the pulse is used as a signal to trigger injection of current to select neighboring cells. In this way, the neuron functions as a logic element – the correct pattern of input, which can come from sensing the environment, is turned into an output, which can drive further processing or a motor task.

The kernel of pulse generation may be understood in terms of a minimal neuron that has a steady-state potential in the absence of input and a voltage-dependent Na^{+}-ion conductance that turns on if a voltage threshold is crossed. This leads to a form of bistability that will explain the rising edge of a neuronal pulse (action potential) – which lasts 0.1 ms – and the rising edge of a cardiac pulse as well. We ignore the slow recovery phase for the moment, which is ~1 ms or 10-times the rise time for neurons and 100 ms or longer for cardiac cells.





For the circuit above, and noting that I = GV where $G \equiv 1/R$, we can use Kirchoff's node law to write

$$G_{Leak}\left[V_m - V_{Leak}\right] + G_{Na}(V)\left[V_m - V_{Na}\right] + C_m \frac{dV_m}{dt} + I_{external} = 0$$

where

$$I_{external} = \sum_{\text{all synapses}} I_{synaptic} + I_{applied}\,.$$

In steady state, the capacitive current is zero, and:

$$G_{Leak}\left[V_m - V_{Leak}\right] + G_{Na}(V)\left[V_m - V_{Na}\right] + I_{external} = 0\,.$$

There are two limits:

1. The potential is below threshold, so $G_{Na}(V) = 0$ and:

$$G_{Leak}\left[V_m - V_{Leak}\right] + I_{external} = 0.$$

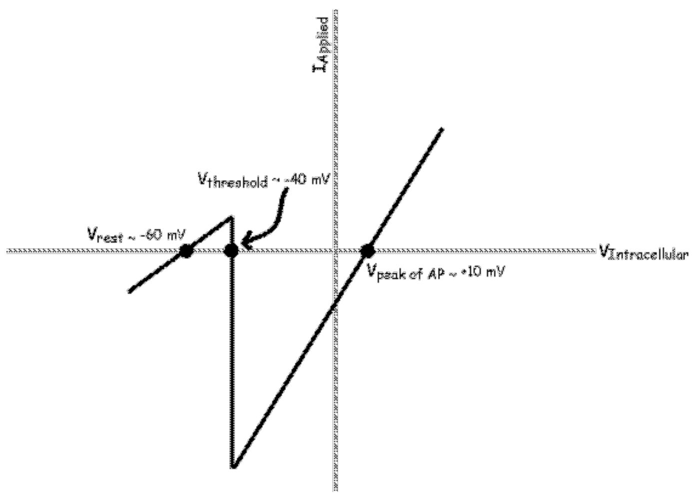
A little rearranging gives:

$$V_m = V_{Leak} - \frac{I_{external}}{G_{Leak}}.$$

The input conductance is just

$$\frac{\Delta I_{applied}}{\Delta V_m} = G_{Leak}.$$

When there are no input currents, $I_{external} = 0$ and $V_m = V_{Leak}$. We refer to this value of V_m as the resting potential, V_{rest}.



2. When the potential is above threshold, $G_{Na}(V) = G_{Na}^{max}$ and:

$$G_{Leak}\left[V_m - V_{Leak}\right] + G_{Na}^{max}\left[V_m - V_{Na}\right] + I_{external} = 0.$$

A little rearranging gives:

$$V_m = \frac{G_{Leak}V_{Leak} + G_{Na}^{max}V_{Na}}{G_{Leak} + G_{Na}^{max}} - \frac{I_{external}}{G_{Leak} + G_{Na}^{max}}.$$

The input conductance is now increased to

$$\frac{\Delta I_{applied}}{\Delta V_m} = G_{Leak} + G_{Na}^{max}.$$

When there are no input currents, $V_m = \frac{G_{Leak}V_{Leak} + G_{Na}^{max}V_{Na}}{G_{Leak} + G_{Na}^{max}}$ is the potential at the top of the action potential. We refer to this value as $V_{peak\ of\ AP}$.