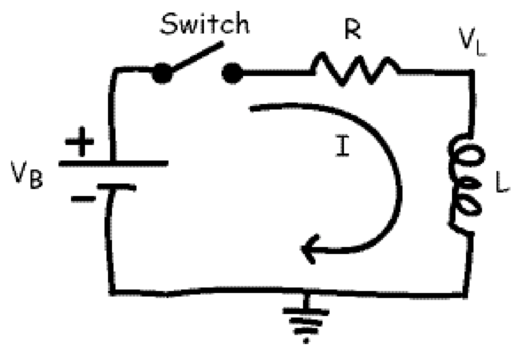
This handout covers current and voltage changes in a single-loop circuit with a resistor and an inductor.



For the circuit above the initial current is zero and the switch is closed at time $t = 0^+$. We apply Kirchoff's Law, i.e., the sum of voltage drops around a loop is zero, and get:

$$0 = -V_B + IR + L\frac{\Delta I}{\Delta t} .$$

A little rearranging to place each term in the form of a current gives:

$$\frac{V_B}{R} = I + \tau\frac{\Delta I}{\Delta t}$$

where:

$$\tau \equiv L/R.$$

To solve this equation, we take the limit that $\Delta t \rightarrow 0$ and write:

$$\frac{V_B}{R} = I + \tau\frac{dI}{dt}.$$

The solution of this equation - recall that $\int_a^b \frac{dx}{x} = \log_e(b) - \log_e(a)$ and try to solve it yourself – is:

$$I(t) = \frac{V_B}{R}\left(1 - e^{-t/\tau}\right)$$

so that $I(t = 0^+) = 0$ and $I(t \rightarrow \infty) = V_B/R$. The voltage drop across the inductor is:

$$V_L(t) = L\frac{dI}{dt} = L\frac{V_B}{R}\frac{d}{dt}\left(1 - e^{-t/\tau}\right) = \frac{LV_B}{R\tau}e^{-t/\tau} = \frac{LV_B}{R}\frac{R}{L}e^{-t/\tau}$$

or:

$$V_L(t) = V_B e^{-t/\tau}$$

so that $V_L(t = 0^+) = V_B$ and $V_L(t \rightarrow \infty) = 0$.