Supporting Text

Signal Conditioning. Electrode impedances in physiological saline were typically 1 MΩ at 10 Hz for both reactive and resistive components. All electrical signals, *i.e.*, those for the mystacial electromyogram (EMG), the cortical local field potential (LFP), and the EMG reference signals from the nose and the LFP reference signal from an unrelated region of cortex, were impedance-buffered at the head of the animal (Fig. 7). We used n-channel field effect transistors (FETs) (SST4118; Vishay Siliconix) with matched transconductances (g_m = 180 μS) that were configured as common source transconductance amplifiers. The FETs were built into each of two 10-pin headstage connectors (nos. SMS-110-01-G-S and TMS-110-01-G-S-RA, Samtec); 16 pins for EMG or LFP signals and 2 pins for a reference signal, plus a system ground that was tied to bolts in the animals skull. The buffered signals traversed approximately 1 m of cable (no. NEF34-1646; Cooner Wire) to a precision load resistor (39.2 kΩ), followed by a 1 Hz high-pass single-pole filter. We measured the difference between each filtered channel and the filtered reference signals with a FET-input instrumentation amplifier with a fixed gain of 100 (V/V) (no. INA101, Burr Brown, Texas Instruments). The appropriate reference was chosen for each channel through a set of switches. Although we later compute differences between pairs of EMG signals and pairs of LFP signals, which should cancel the value of any reference signals, we employ separate EMG and cortical references as an additional precaution against possible unbalanced common input to the two sets of signals.

The low-impedance differential EMG and LFP signals pass through a commutator and 5 m of cable to high-pass filters, amplifiers, and antialias filters. The EMG signals were high-pass filtered at 200 Hz (no. UAF42 configured as a 4-pole Bessel filter; Burr Brown) and amplified between 32- and 128-times. The LFP signals were high-pass filtered at 1 Hz (single pole filter) and amplified between 32- and 128-times. All signals were then band-limited to 10 kHz with an 8-pole constant-phase low-pass filter (no. D68L8D – 10.0kHz, Frequency Devices) and digitized at 25 kHz with a 12-bit digital-to-analog converter (no. AT-MIO-16E-1, National Instruments). The difference between any two brain signals, rectification for the EMG signals, low-pass filtering of the difference, and subsampling of the difference were performed numerically (MatLab; The MathWorks).

Spectral estimates. Spectral methods provide a means to characterize the response and the variability in that response as an average across trials. Spectra power densities of individual epochs, denoted S(f), were calculated with the direct multi-taper spectral estimation techniques of Thomson (1, 2); this procedure minimizes the leakage between neighboring frequency bands. We denote the measured time-series for the i-*th* trial, either ∇EMG or ∇LFP, as $\mathbf{V}^{(i)} \equiv \left\{V^{(i)}(t)\right\}_{t=0}^{T}$ where T is the duration of the epoch. The mean value is removed from $\mathbf{V}^{(i)}$ to form $\delta\mathbf{V}^{(i)} = \mathbf{V}^{(i)} - \int_{0}^{T} dt\,\mathbf{V}^{(i)}$. We compute the spectral power in terms of an average over all instances and tapers, *i.e.,*

$$S(f) \equiv \left\langle \tilde{\mathbf{V}}^{(i,k)}\tilde{\mathbf{V}}^{*(i,k)} \right\rangle$$

where $\left\langle \bullet\bullet\bullet \right\rangle$ refers to averaging over independent realizations, *i.e.*,

$$\left\langle \bullet\bullet\bullet \right\rangle \equiv \frac{1}{NK}\sum_{i=0}^{N}\sum_{k=1}^{K} \quad \text{and}$$

$$\tilde{\mathbf{V}}^{(i,k)} \equiv \left\{\tilde{V}^{(i,k)}(f)\right\}_{f=0}^{f_s/2} = \frac{1}{f_s}\sum_{t=0}^{T} e^{i2\pi ft}\delta V^{(i)}(t)w^{(k)}(t)$$

is the discrete Fourier transform of the product of the mean-subtracted time-series, $\delta\mathbf{V}^{(i)}$, with the k-th Slepian taper, $\mathbf{w}^{(k)} \equiv \left\{w^{(k)}(t)\right\}_{t=0}^{T}$. The parameter N is the number of instances of the waveform and K is the number of tapers in a given spectral estimate. The spectral coherence between pairs of time-series, denoted C(f), is defined analogously to the spectral power, with

$$C(f) \equiv \frac{\left\langle \tilde{\mathbf{V}}^{(i,k)}\tilde{\mathbf{U}}^{*(i,k)} \right\rangle}{\sqrt{S_V(f)S_U(f)}}$$

where $\tilde{\mathbf{V}}^{(i,k)}$ and $\tilde{\mathbf{U}}^{(i,k)}$ are the Fourier transforms of the two respective time-series. In these estimates, the spectrum is averaged over a half-bandwidth of $\Delta f = (K+1)/(2T)$; this value ranged from 2.5 to 3 Hz in the present studies. Numerically, $\tilde{\mathbf{V}}^{(i,k)}$ and $\tilde{\mathbf{U}}^{(i,k)}$ are computed as the fast Fourier transforms of the respective product after it is padded to at least 4-times the initial length.

Standard deviations of the spectra are reported as jackknife estimates across all degrees of freedom (3).

Interpretation of |C(f)|. We consider a model that relates a signal y(t) to an underlying variable x(t) by the convolution y(t) = $\int_{-\infty}^{t} d\tau\, T(t-\tau)x(\tau)$ + η(t), where T(t) is the linear transfer function, x(t) is considered as a random variable, and η(t) is an underlying noise source, with zero mean, that is uncorrelated with x(t). In the frequency domain, $\tilde{y}(f)$ = $\tilde{T}(f)\tilde{x}(f)$ + $\tilde{\sigma}(f)$. The spectral power in the signal is $S_{yy}(f)$ = $\left\langle |\tilde{y}(f)|^2 \right\rangle$ = $|\tilde{T}(f)|^2 S_{xx}(f) + \Sigma(f)$ where $\langle \cdots \rangle$ refers to averaging over all degrees of freedom, $S_{xx}(f)$ = $\left\langle |\tilde{x}(f)|^2 \right\rangle$ is the power in the underlying variable and $\Sigma(f)$ = $\left\langle |\tilde{\sigma}(f)|^2 \right\rangle$ is the noise power; we make no assumption about the spectral content of the noise. Similarly, the cross spectral power between $\tilde{y}(f)$ and $\tilde{x}(f)$ is $S_{yy}(f)$ = $\left\langle \tilde{y}(f)\tilde{x}^*(f) \right\rangle$ = $\tilde{T}(f)S_{xx}(f)$. Recalling that the coherence between $\tilde{y}(f)$ and $\tilde{x}(f)$ is defined as C(f) = $S_{yx}(f) \big/ \sqrt{S_{yy}(f)S_{xx}(f)}$, we have $S_{yy}(f)$ = $|C(f)|^2 S_{yy}(f) + \Sigma(f)$. Thus the ratio of the variability in the measured signal to that in the underlying noise is $S_{yy}(f)\big/\Sigma(f)$ = $\left(1 - |C(f)|^2\right)^{-1}$. This expression is analogous to the variability captured by the square of the correlation coefficient, r, in regression analysis (4), where $|C(f)|^2$ within a defined spectral band plays the role of r^2.

The spectral power in the underlying variable beyond that in the noise, which forms a signal power, can be used to define a "Signal-to-Noise" ratio in a given frequency band for the above model of noise, *i.e.*,

$$\text{Signal-to-Noise} \equiv \sqrt{\frac{S_{yy}(f) - \Sigma(f)}{\Sigma(f)}} = \sqrt{\frac{|C(f)|^2}{1 - |C(f)|^2}} \; .$$

When $|C(f)|^2 \ll 1$, as occurs for the measured data in this study (Figs. 3C and 5C), the above expression simplifies to

$$\text{Signal-to-Noise} \xleftarrow{\;|C(f)| \ll 1\;} |C(f)| + O\left(|C(f)|^3\right).$$

Thus the reported magnitude of the coherence may be interpreted as a signal-to-noise ratio for predicting vibrissa position, *i.e.*, the ∇EMG, from the cortical signal, *i.e.*, the ∇LFP.

Caption

Figure 7. Abbreviated schematic of the recording electronics.

References

1.   Percival, D. B. & Walden, A. T. (1993) *Spectral Analysis for Physical Applications: Multitaper and Conventional Univariate Techniques* (Cambridge University Press, Cambridge).
2.   Thomson, D. J. (1982) *Proceedings of the IEEE* **70,** 1055-1096.
3.   Thomson, D. J. & Chave, A. D. (1991) in *Advances in Spectrum Analysis and Array Processing*, ed. Shykin, S. (Prentice Hall, pp. 58-113.
4.   Mardia, K. V., Kent, J. T. & Bibby, J. M. (1979) *Multivariate Analysis* (Academic Press, San Diego).

Headstage **Cable** **Differential Preamplifier** **Commutator** **Conditioning Electronics**
$(f_{HP} = 0.3\ Hz;\ Gain \approx 88)$ **and Cable** **(Variable HP and Gain)**

Signal Channel 1

D

G S

Siliconix SST4118

Signal Channel 2

Signal Channel 16

Reference Channel 1

Reference Channel 2

Animal Ground

0.05 µF

40 kΩ   10 MΩ

+ V

- V

Burr-Brown INA101

SPDT

High Pass Filter | Amplifier | 10 kHz Antialias Filter → To ADC Channel 1

High Pass Filter | Amplifier | 10 kHz Antialias Filter → To ADC Channel 2

High Pass Filter | Amplifier | 10 kHz Antialias Filter → To ADC Channel 16

System Ground