Notes on Full Wave Rectification - Physics 120

For any periodic function, $v(t)$, defined by $v(t+T) = v(t)$

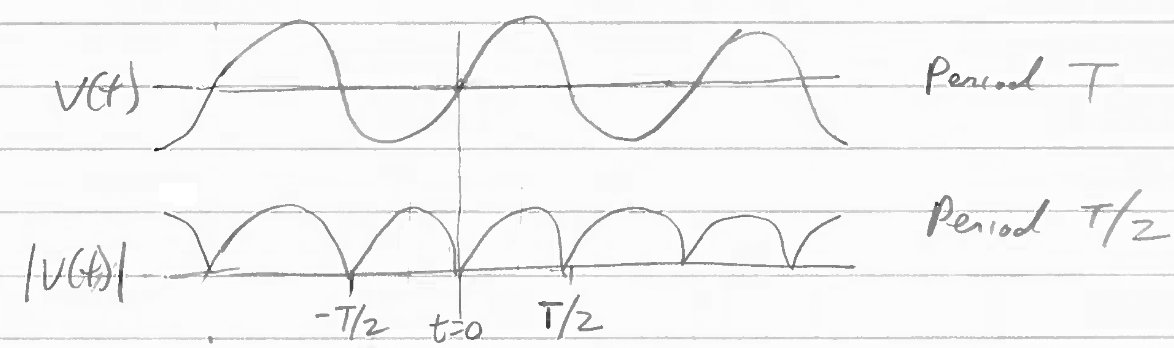
$$v(t) = \sum_{k=-\infty}^{+\infty} c_k e^{jk\omega_0 t} \quad ; \quad \omega = \frac{2\pi}{T}$$

when
$$c_k = \frac{1}{T}\int_{-T/2}^{T/2} v(t)\, e^{-jk\omega_0 t}\, dt$$

$$'' = \underbrace{\frac{1}{T}\int_{-T/2}^{T/2} v(t)\cos(k\omega_0 t)\,dt}_{\text{even part}} \; - j \underbrace{\int_{-T/2}^{T/2} v(t)\sin(k\omega_0 t)\,dt}_{\text{odd part}}$$

Note that
$$c_0 = \frac{1}{T}\int_{-T/2}^{T/2} v(t)\,dt = \langle v(t) \rangle$$

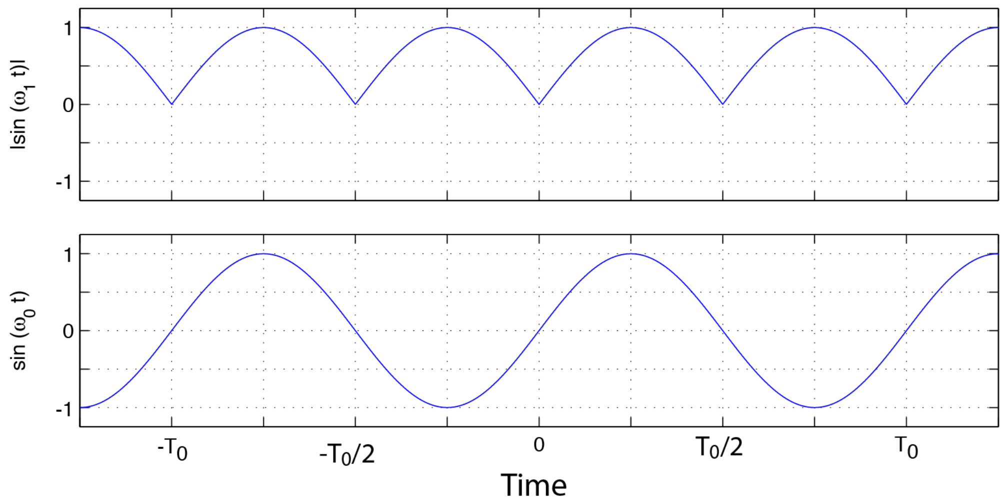
Example: Full wave rectification



$$|V(t)| = |\sin(\omega_0 t)| \Longrightarrow C_k = \frac{2}{\pi}\left(\frac{1}{1-4k^2}\right)$$

$$C_0 = 2/\pi \,;\; C_{\pm 1} = \frac{-C_0}{3} \,;\; C_{\pm 2} = \frac{-C_0}{15} \,;\; C_{\pm 3} = \frac{-C_0}{35}$$

Fourier Series for Rectified Sine Wave $|\sin(\omega_0 t)|$



The Fourier series coefficients are:

$$c_k = \frac{1}{T_o}\int_{-T_0/2}^{T_0/2} dt \left|\sin\left(\omega_o t\right)\right| e^{-ik\omega_0 t}$$

$$= -\frac{1}{T_o}\int_{-T_0/2}^{0} dt \sin\left(\omega_o t\right) e^{-ik\omega_0 t} + \frac{1}{T_o}\int_{0}^{T_0/2} dt \sin\left(\omega_o t\right) e^{-ik\omega_0 t}$$

$$= \frac{2}{T_o}\int_{0}^{T_0/2} dt \sin\left(\omega_o t\right) e^{-ik\omega_0 t}$$

$$= \frac{2}{T_0}\int_{0}^{T_0/2} dt \sin\left(\frac{2\pi t}{T_0}\right) e^{-i2\pi kt/T_0}$$

$$= \frac{2}{T_0}\int_{0}^{T_0/2} dt \left(\frac{e^{i2\pi t/T_0} - e^{-i2\pi t/T_0}}{2i}\right) e^{-i2\pi kt/T_0}$$

$$= \frac{1}{iT_0}\int_{0}^{T_0/2} dt \left(e^{i2\pi t(1-k)/T_0} - e^{-i2\pi t(1+k)/T_0}\right)$$

$$= \frac{1}{iT_0}\left[\frac{e^{i2\pi t(1-k)/T_0}}{i2\pi(1-k)/T_0} - \frac{e^{-i2\pi t(1+k)/T_0}}{i2\pi(1+k)/T_0}\right]_0^{T_0/2}$$

$$= \frac{1}{2\pi}\left[\frac{e^{i\pi(1-k)}}{1-k} + \frac{e^{-i\pi(1+k)}}{1+k} - \frac{1}{1-k} - \frac{1}{1+k}\right]$$

But $e^{i\pi(1\pm k)} = e^{i\pi} e^{\pm i\pi k} = \begin{matrix} +1; \text{ k odd} \\ -1; \text{ k even} \end{matrix}$

$$= \begin{cases} \dfrac{-1}{2\pi}\left[\dfrac{-1}{1-k} + \dfrac{-1}{1+k} - \dfrac{1}{1-k} - \dfrac{1}{1+k}\right] = \dfrac{1}{\pi}\left[\dfrac{2}{(1-k)(1+k)}\right] & \text{k even} \\[2ex] \dfrac{-1}{2\pi}\left[\dfrac{+1}{1-k} + \dfrac{+1}{1+k} - \dfrac{1}{1-k} - \dfrac{1}{1+k}\right] = 0 & \text{k odd} \end{cases}$$

$$= \begin{cases} \dfrac{2}{\pi}\left(\dfrac{1}{1-k^2}\right) & \text{k even} \\[2ex] 0 & \text{k odd} \end{cases}$$