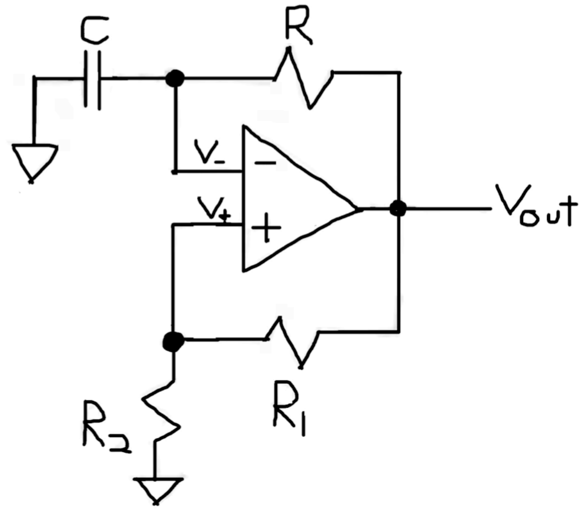
Notes on Relaxation Oscillators

Physics 120, David Kleinfeld, Spring 2018

This is a basic oscillator circuit in which the voltage across a capacitor relaxes toward a time varying target voltage. Consider the circuit below, using a high gain op-amp as a comparator:



The time-varying voltage at V_+ serves as a target level that is a fraction $R_2/(R_1+R_2)$ of the output voltage V_{out}. The time-varying voltage at V_- heads toward this value and when it exceeds the value, the output of the amplifier changes sign, thus the target changes sign, and the voltage at V_- heads toward this new value.

We know that $V_{out} = A(V_+ - V_-)$ where A >> 1.

Since V_{out} is bounded by V_{supply}. this means that

$$V_{out} = \begin{cases} +V_{supply} & V_+ > V_-. \\ -V_{supply} & V_- < V_+. \end{cases}$$

The voltage divider assures that the value of V_-, which can asymptote at $V_- = V_{out}$, will be compared to a value for V_+ that is smaller than V_{out}, *i.e.*, $V_+ = \frac{R_2}{R_1+R_2} V_{out}$.

$V_-(t)$ will evolve in time according to:

$$C\frac{dV_-}{dt} + \frac{V_- - V_{out}}{R} = 0$$

with $\tau \equiv RC$. Thus:

$$\frac{dV_-}{dt} + \frac{1}{\tau}V_- = \frac{1}{\tau}V_{out}$$

The solution to this homogeneous part of this equation is:

$$V_-(t) = V_-(0)e^{-t/\tau}.$$

so that the full solution is:

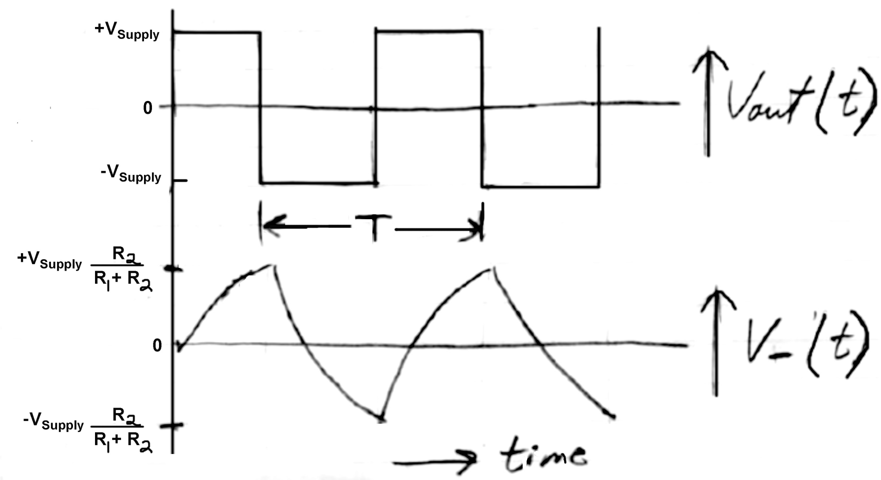
$$V_-(t) = V_-(0)e^{-t/\tau} + \int_o^t \left(\frac{1}{\tau}V_{out}\right) e^{-(t-x)/\tau}\, dx.$$

We take t = 0 as the time of the last transition and consider the interval of time up to the next transition, i.e., t = T/2, so that V_{out} is a constant. The value of V_(0) just after the transition, i.e., $V_-(0^+)$, is opposite in sign to that of V_{out}, so

$$V_-(0) = -\frac{R_2}{R_1+R_2} V_{out}.$$

Thus:

$$V_-(t) = -\frac{R_2}{R_1+R_2}V_{out}\, e^{-t/\tau} + V_{out}\left(1 - e^{-t/\tau}\right)$$



At t = T/2 the value of V_(t) will reach the threshold level

$$V_-(\frac{T}{2}) = +\frac{R_2}{R_1+R_2} V_{out}$$

so

$$\frac{R_2}{R_1+R_2} V_{out} = -\frac{R_2}{R_1+R_2}V_{out}\, e^{-\frac{T}{2}/\tau} + V_{out}\left(1 - e^{-\frac{T}{2}/\tau}\right)$$

or

$$T = 2\,\tau\, log\left(1 + 2\, {}^{R_2}/_{R_1}\right).$$

As a partial check, when $R_2 \longrightarrow \infty$ we have $T \longrightarrow \infty$.
Also, when $R_2 \longrightarrow 0$ we have $T \longrightarrow 4\,\tau\left({}^{R_2}/_{R_1}\right)$.