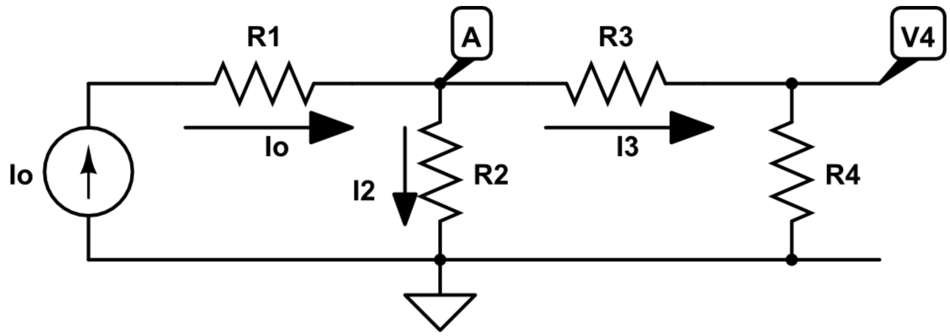
Problem 1 (SH).

Call the node joining the three resistors (R_1, R_2 and R_3) node A with voltage V_A. The voltage V_4 is a simple voltage divider of R_3 and R_4 of the voltage V_A. We can then finish solving by using Kirchoff's Current Law at node A ($I_o = I_2 + I_3$).



$$I_2 = \frac{V_A - 0}{R_2} \qquad I_3 = \frac{V_A}{R_3 + R_4}$$

$$I_o = I_2 + I_3 = \left(\frac{V_A}{R_2}\right) + \left(\frac{V_A}{R_3 + R_4}\right) = \frac{R_2 + R_3 + R_4}{R_2(R_3 + R_4)} V_A \longrightarrow V_A = \frac{(R_3 + R_4)R_2}{R_2 + R_3 + R_4} I_o$$

$$V_4 = \frac{R_4}{R_3 + R_4} V_A = \left(\frac{R_4}{R_3 + R_4}\right)\left(\frac{(R_3 + R_4)R_2}{R_2 + R_3 + R_4}\right) I_o = \frac{R_4 R_2}{R_2 + R_3 + R_4} I_o$$

$$\therefore \boxed{V_4 = \frac{R_4 R_2}{R_2 + R_3 + R_4} I_o}$$

Problem 2 (KG).

This is simply two voltage dividers. Notating the voltage V^+ as the voltage divider at the node between R_1 and R_2 ($V_+ = \frac{R_2}{R_1+R_2} V_o$) and V^- as the voltage divider of R_3 and R_4 ($V_- = \frac{R_4}{R_3+R_4} V_o$), the voltage we're interested in finding is simply $V = V^+ - V^- = \left(\frac{R_2}{R_1 + R_2} - \frac{R_4}{R_3 + R_4}\right) V_o$.

$$\therefore \boxed{V = \left(\frac{R_2}{R_1 + R_2} - \frac{R_4}{R_3 + R_4}\right) V_o}$$

Problem 3 (KG).

For a small number ϵ, we have the Taylor expansion:

$$(1 + \epsilon)^n = 1 + n\epsilon + ...$$

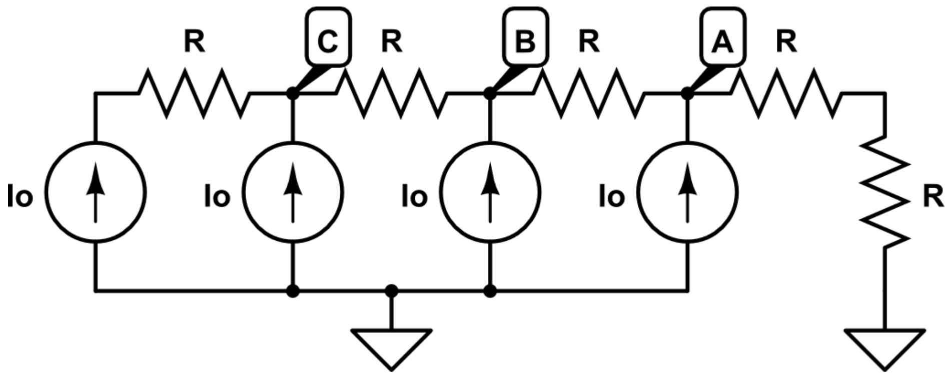
We apply this to our answer in Problem 2 where $R >> \delta R$ and we set $R = R_1 = R_2 = R_3$ and $R_4 = R + \delta R$.

$$\begin{aligned} V &= \left(\frac{R_2}{R_1 + R_2} - \frac{R_4}{R_3 + R_4}\right) V_o && \text{(substitution)} \\ &= \left(\frac{R}{2R} - \frac{R + \delta R}{2R + \delta R}\right) V_o \\ &= \frac{2R^2 + R\delta R - 2R^2 - 2R\delta R}{4R^2 + 2R\delta R} V_o \\ &= -\frac{R\delta R}{4R^2 + 2R\delta R} V_o \\ &= -\frac{\delta R}{4R + 2\delta R} V_o \end{aligned}$$

$$
\begin{aligned}
&= -\frac{\delta R}{4R}\left(\frac{1}{1+\frac{\delta R}{2R}}\right)V_o \\
&= -\frac{\delta R}{4R}\left(1+\frac{\delta R}{2R}\right)^{-1}V_o && \text{(apply Taylor expansion)} \\
&\approx -\frac{\delta R}{4R}\left(1-\frac{\delta R}{2R}+...\right)V_o && \text{(higher order terms vanish: } \mathcal{O}(\delta R^2)\approx 0) \\
&\approx \boxed{-\frac{V_o}{4R}\delta R}
\end{aligned}
$$

Problem 4 (SH).

Using Kirchoff's Current Law at nodes A, B, and C as labeled below, and the notation I_{ij} as the current from node i to j, we can quickly find the requested I.



@ Node A: $I = I_o + I_{BA}$ @ Node B: $I_{BA} = I_o + I_{CB}$ @ Node C: $I_{CB} = I_o + I_o = 2I_o$

$I_{BA} = I_o + I_{CB} = I_o + 2I_o = 3I_o$ $I = I_o + I_{BA} = I_o + 3I_o = \boxed{4I_o}$

Problem 5 (BW).

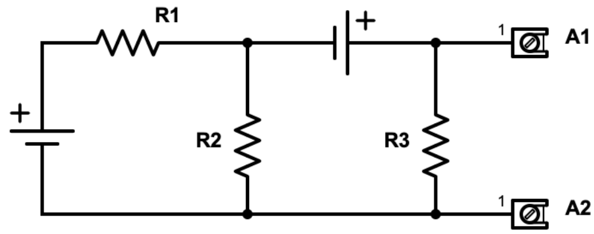
A Thevenin Equivalent circuit consists of a voltage source in series with a resistor. A and A' serve as terminal leads across both the expanded and the equivalent circuits.

The general method to deriving Thevenin Equivalent circuits are as follows:

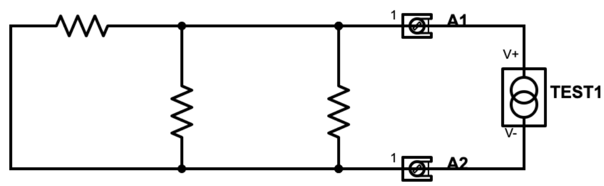
1. Choose a ground/reference point. This can be any node in your circuit but you want to minimize the work you need to do to solve your circuit. In this circuit, setting A' (and the entire node to its left) to ground is very convenient.
2. Set all independent sources to zero and solve for Thevenin Resistance (R_T) across A ($A1$ in the graphic below) and A' ($A2$). In other words, turn all voltage sources to shorting wires and all current sources to "open circuits" and find the equivalent resistance
3. Solve for the open circuit voltage across A and A'. In this particular case, solve for voltage across the rightmost resistor, R_3.

Mind that all voltage sources drive V_0 across their two connecting nodes and all current sources drive I_0 into the adjacent node. All resistors have resistance, R.

Starting with the first circuit that contains two voltage sources:

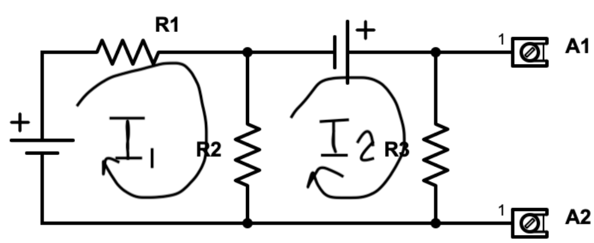


Set the two voltage sources to 0; short them out, leaving only the terminals and the resistors in their original locations.



With respect to the terminals $A1$ and $A2$, R_1 and R_2 are in parallel with each other. $R_1||R_2$ is in parallel with R_3. The equivalent resistance between $A1$ and $A2$ is $R_T = [(\frac{1}{R} + \frac{1}{R})^{-1} + \frac{1}{R}]^{-1}$; $R_T = R/3$. The test current is there for theoretical completeness (as you cannot measure voltage without a current); it is not directly used to solve for equivalent resistance.

Use the loop current method to solve for open circuit voltage, aka the voltage over R_3. The current loops have to rotate in the same direction.

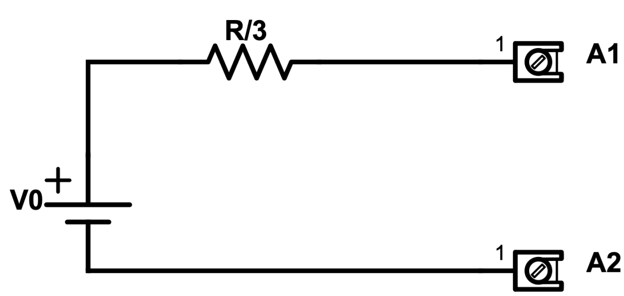


By Kirchoff's laws, the voltage added around each loop (two loops) should total to zero. Voltage sources contribute positively to the loop while resistors contribute negatively. R_2 is special because it receives contributions from both I_1 and I_2. Just keep the signs consistent with respect to current direction.

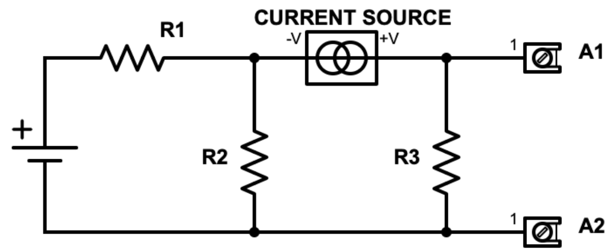
$V_0 - I_2 * R - I_2 * R + I_1 * R = 0$

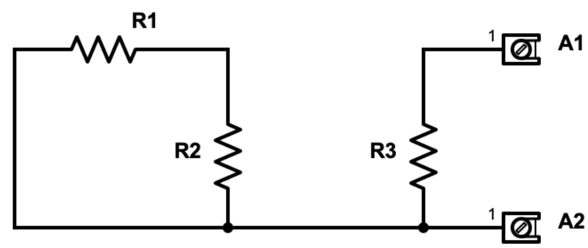
$V_0 - I_1 * R - I_1 * R + I_2 * R = 0$

Solve the system of two equations for I_1 and I_2. We are interested in I_2, which turns out to be $I_2 = \frac{V_0}{R}$. Thevenin voltage, V_T, is $V_T = I_2 * R_3 = V_0$



The second current can be solved using the exact same method. For brevity, the steps are briefly listed below:





Notice that R_1 and R_2 are not subject to any voltage potential. This means they are under equilibrium, which implies no current. If there is no current, there is no voltage drop. We can omit R_1 and R_2 from the Thevenin resistance. Thus, $R_T = R_3 = R$.

The current source drives current through R_3 and into the "ground" at $A2$ by KCL (what goes into $A1$ must come out of $A1$). Therefore, the Thevenin Voltage, $V_T = I_0 * R$. You can verify using Node Voltage analysis that the current coming from the voltage source cancels with the current going through R_2 into "ground". This is consistent with the assumption that R_1 and R_2 do not contribute to the Thevenin Equivalent circuits from when we were solving for R_T.