Problem 1 (KG). We are given the following inputs:

\[ V_{\text{mod}} = V_0 + V_1 \cos (\omega_m t) \quad V_{\text{carrier}} = V_1 \cos (\omega_c t) \]

The FET equation in Ohmic region and conductance \( g_m = \frac{1}{R_{DS}} \) is as follows

\[
I_D = \frac{2I_{DS}}{V_{GS(\text{off})}} \left( (V_{GS} - V_{GS(\text{off})})V_{DS} - \frac{V_{DS}^2}{2} \right)
\]

\[
\frac{1}{R_{DS}} = \frac{2I_{DS}}{V_{GS(\text{off})}} \left( V_{GS} - V_{GS(\text{off})} - \frac{V_{DS}}{2} \right)
\]

We see that we have a voltage divider constructed from the connections of both resistors \( R \) at the gate and drain. We handle this using KCL and see the following:

\[
0 = \frac{V_{GS} - V_{\text{mod}}}{R} + \frac{V_{GS} - V_{DS}}{R} \quad \Rightarrow \quad V_{GS} = \frac{V_{\text{mod}} + V_{DS}}{2} \quad \checkmark
\]

\[
\Rightarrow \quad \frac{1}{R_{DS}} = \frac{2I_{DS}}{V_{GS(\text{off})}} \left( \frac{V_{\text{mod}}}{2} - V_{GS(\text{off})} \right) \quad \checkmark
\]

To find our expression for \( V_{\text{out}} \), we see that the circuit can be casted as a voltage divider where the FET acts as a resistor \( R_{DS} \):

\[
V_{\text{out}} = V_{DS} = \left( \frac{R_{DS}}{R_D + R_{DS}} \right) V_{\text{carrier}}
\]

\[
= \frac{V_{\text{carrier}}}{R_{DS} + 1}
\]

\[
= \frac{2I_{DS}R_D}{V_{GS(\text{off})}} \left( \frac{V_{\text{mod}}}{2} - V_{GS(\text{off})} \right) + 1
\]

\[
= \frac{I_{DS}R_D}{V_{GS(\text{off})}} \left( V_0 + V_1 \cos (\omega_m t) - 2V_{GS(\text{off})} \right) + 1 \quad \checkmark
\]

Observe from the hint we are given that

\[
V_{\text{out}} \approx A \cos (\omega_c t) + B \frac{V_1}{V_{GS(\text{off})}} \cos (\omega_c t) \cos (\omega_m t) = A \cos (\omega_c t) \left( 1 + B \frac{V_1}{A V_{GS(\text{off})}} \cos (\omega_m t) \right)
\]
From our expression of \( V_{\text{out}} \) we see that we need to perform a Taylor expansion for a small parameter \( \epsilon \) such that \((1 + \epsilon)^n \approx 1 + n\epsilon\) where \( n = -1 \). This means that

\[
\epsilon \approx \frac{B'}{A'} \frac{V_1^m}{V_{GS(\text{off})}} \cos(\omega_m t)
\]

where I have relabeled with \( A' \) and \( B' \) and I will show that these are related to \( A \) and \( B \) by \( V_c^1 \). Now let’s rearrange the terms in the denominator of \( V_{\text{out}} \) to isolate this \( \epsilon \) term

\[
V_{out} = V_{\text{carrier}} \left(1 - 2 \frac{I_{DSS} R_D}{V_{GS(\text{off})}} + 1 \right)^{-1} V_m^0 \cos(\omega_m t) \]

\[
V_{out} \approx V_{\text{carrier}} \left(1 + \frac{A' + B' \frac{V_1^m}{V_{GS(\text{off})}} \cos(\omega_m t)}{1 + \frac{B'}{A'} \frac{V_1^m}{V_{GS(\text{off})}} \cos(\omega_m t)} \right)^{-1}
\]

Here, we see that \( A' = 1 - 2 \frac{I_{DSS} R_D}{V_{GS(\text{off})}} = 51 \) and \( B' = \frac{I_{DSS} R_D}{V_{GS(\text{off})}} = 25 \) \( \Rightarrow \frac{B'}{A'} \approx 0.5 \) from using \( I_{DSS} = 10mA \), \( V_{GS(\text{off})} = -4V \), and \( R_D = 10k\Omega \). Recall that in the actual lab we used \( V_1^m \approx 0.5V \). Then,

\[
\frac{B'}{A'} \frac{V_1^m}{V_{GS(\text{off})}} \approx -0.06 \Rightarrow \frac{B'}{A'} \frac{V_1^m}{V_{GS(\text{off})}} \cos(\omega_m t) \ll 1
\]

where we know that cosine varies between 0 and 1 and it’s clear that a small number times a small number is a smaller number (small means less than 1). This shows us what our final solution indeed matches our hint and you can infer \( A \) and \( B \) from inspection.

\[
V_{out} \approx V_1^m \cos(\omega_c t) - \frac{V_1^m B'}{A'} \frac{V_1^m}{V_{GS(\text{off})}} \cos(\omega_c t) \cos(\omega_m t)
\]

\[
\approx A \cos(\omega_c t) + B \frac{V_1^m}{V_{GS(\text{off})}} \cos(\omega_c t) \cos(\omega_m t)
\]