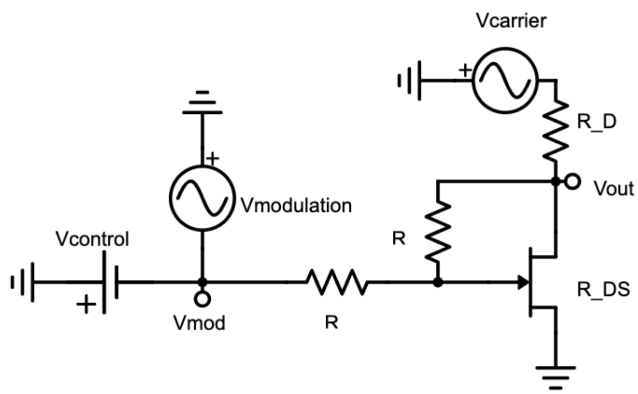

Figure 1: Circuit for lab exercise 6.3

Problem 1 (KG). We are given the following inputs:

$$V_{mod} = V_m^0 + V_m^1 \cos(\omega_m t) \qquad V_{carrier} = V_c^1 \cos(\omega_c t)$$

The FET equation in Ohmic region and conductance $g_m = \frac{1}{R_{DS}}$ is as follows

$$I_D = \frac{2I_{DSS}}{V_{GS}^2(\text{off})}\left[(V_{GS} - V_{GS}(\text{off}))V_{DS} - \frac{V_{DS}^2}{2}\right]$$
$$\frac{1}{R_{DS}} = \frac{I_D}{V_{DS}} = \frac{2I_{DSS}}{V_{GS}^2(\text{off})}\left[V_{GS} - V_{GS}(\text{off}) - \frac{V_{DS}}{2}\right]$$

We see that we have a voltage divider constructed from the connections of both resistors (R) at the gate and drain. We handle this using KCL and see the following:

$$0 = \frac{V_{GS} - V_{mod}}{R} + \frac{V_{GS} - V_{DS}}{R}$$
$$V_{GS} = \frac{V_{mod} + V_{DS}}{2} \qquad \checkmark$$
$$\implies \quad \frac{1}{R_{DS}} = \frac{2I_{DSS}}{V_{GS}^2(\text{off})}\left(\frac{V_{mod}}{2} - V_{GS}(\text{off})\right) \qquad \checkmark$$

To find our expression for V_{out}, we see that the circuit can be casted as a voltage divider where the FET acts as a resistor (R_{DS}):

$$\begin{aligned} V_{out} &= V_{DS} \\ &= \left(\frac{R_{DS}}{R_D + R_{DS}}\right) V_{carrier} \\ &= \frac{V_{carrier}}{\frac{R_D}{R_{DS}} + 1} \\ &= \frac{V_{carrier}}{\frac{2I_{DSS}R_D}{V_{GS}^2(\text{off})}\left(\frac{V_{mod}}{2} - V_{GS}(\text{off})\right) + 1} \\ &= \frac{V_c^1 \cos(\omega_c t)}{\frac{I_{DSS}R_D}{V_{GS}^2(\text{off})}\left(V_m^0 + V_m^1 \cos(\omega_m t) - 2V_{GS}(\text{off})\right) + 1} \qquad \checkmark \end{aligned}$$

Observe from the hint we are given that

$$V_{out} \approx A\cos(\omega_c t) + B\frac{V_m^1}{V_{GS}(\text{off})}\cos\omega_c t \cos\omega_m t = A\cos(\omega_c t)\left(1 + \frac{B}{A}\frac{V_m^1}{V_{GS}(\text{off})}\cos\omega_m t\right)$$

From our expression of V_{out} we see that we need to perform a Taylor expansion for a small parameter ϵ such that $(1+\epsilon)^n \approx 1 + n\epsilon$ where $n = -1$. This means that

$$\epsilon \propto \frac{B'}{A'} \frac{V_m^1}{V_{GS}(\text{off})} \cos(\omega_m t)$$

where I have relabeled with A' and B' and I will show that these are related to A and B by V_c^1. Now let's rearrange the terms in the denominator of V_{out} to isolate this ϵ term

$$\begin{aligned}
V_{out} &= V_{carrier}\left(\frac{I_{DSS}R_D}{V_{GS}^2(\text{off})}\left[V_m^0 + V_m^1\cos(\omega_m t) - 2V_{GS}(\text{off})\right] + 1\right)^{-1} \\
&= V_{carrier}\left(1 - 2\frac{I_{DSS}R_D}{V_{GS}(\text{off})} + \frac{I_{DSS}R_D}{V_{GS}^2(\text{off})}\left[V_m^0 + V_m^1\cos(\omega_m t)\right]\right)^{-1} \\
&\approx V_{carrier}\left(1 - 2\frac{I_{DSS}R_D}{V_{GS}(\text{off})} + \frac{I_{DSS}R_D}{V_{GS}^2(\text{off})}\left[V_m^1\cos(\omega_m t)\right]\right)^{-1} \qquad (V_m^0 \text{ is just a DC bias shift. Let it be zero.}) \\
&= V_{carrier}\left(A' + B'\frac{V_m^1}{V_{GS}(\text{off})}\cos(\omega_m t)\right)^{-1} \\
&= \frac{V_{carrier}}{A'}\left(1 + \frac{B'}{A'}\frac{V_m^1}{V_{GS}(\text{off})}\cos(\omega_m t)\right)^{-1} \checkmark
\end{aligned}$$

Here, we see that $A' = 1 - 2\frac{I_{DSS}R_D}{V_{GS}(\text{off})} = 51$ and $B' = \frac{I_{DSS}R_D}{V_{GS}(\text{off})} = 25 \implies \frac{B'}{A'} \approx 0.5$ from using $I_{DSS} = 10mA$, $V_{GS}(\text{off}) = -4V$, and $R_D = 10k\Omega$. Recall that in the actual lab we used $V_m^1 \approx 0.5V$. Then,

$$\frac{B'}{A'}\frac{V_m^1}{V_{GS}(\text{off})} \approx -0.06 \implies \frac{B'}{A'}\frac{V_m^1}{V_{GS}(\text{off})}\cos(\omega_m t) \ll 1$$

where we know that cosine varies between 0 and 1 and it's clear that a small number times a small number is a smaller number (small means less than 1). This shows us what our final solution indeed matches our hint and you can infer A and B from inspection.

$$\begin{aligned}
V_{out} &\approx \frac{V_c^1}{A'}\cos(\omega_c t) - \frac{V_c^1}{A'}\frac{B'}{A'}\frac{V_m^1}{V_{GS}(\text{off})}\cos(\omega_c t)\cos(\omega_m t) \\
&\approx A\cos(\omega_c t) + B\frac{V_m^1}{V_{GS}(\text{off})}\cos(\omega_c t)\cos(\omega_m t) \quad \checkmark
\end{aligned}$$