Problem 7.14 (BW).

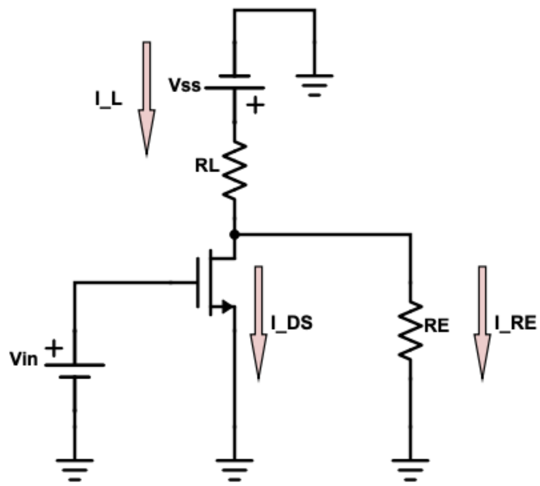

Figure 1: Figure 7.85 from your text. Current directions are drawn in for KCL analysis. All currents flow from high to low potential.

We are told that the MOSFET in Figure 1 operates in Saturation Mode. This means that the MOSFET drives some current, I_{DS}, across it, dropping a voltage, V_{DS}, on its way between the Source and the Drain. V_{out} is concerned with the drop across the MOSFET and R_E. R_E is in parallel with the MOSFET, so $V_{RE} = V_{DS}$. Thus, $V_{out} = I_E * R_E$. We can write down the following relations from the get go:

$$V_{out} = I_E * R_E \tag{1}$$
$$V_{SS} - V_{out} = I_L * R_L \tag{2}$$
$$I_L = I_{DS} + I_{RE} \tag{3}$$

Since the MOSFET is in saturation mode, we know I_{DS} is equal to $I_{DS} = \frac{K}{2}(V_{In} - V_T)^2$ where V_{In} is equivalent to V_{GS} and V_T is the threshold voltage for the MOSFET to turn on (assuming n-channel enhancement mode MOSFET characteristics). V_T is equivalently known as $V_{GS,off}$ in JFETs.
Note that K is a parameter equivalent to $\frac{\mu_n C_{ox} W}{L}$, where μ_n is electron carrier mobility, C_{ox} is the capacitance of the oxide layer, W is the channel width (lateral), and L is the channel length (along the current vector). Now that all three currents are known, we can solve the KCL relation:

$$\frac{V_{SS} - V_{out}}{R_L} = \frac{V_{out}}{R_E} + \frac{K}{2}(V_{In} - V_T)^2 \tag{4}$$
$$\frac{V_{out}}{R_E} + \frac{V_{out}}{R_L} = \frac{V_{SS}}{R_L} - \frac{K}{2}(V_{In} - V_T)^2 \tag{5}$$
$$V_{out} = \frac{R_E R_L}{R_E + R_L} * \frac{2V_{SS} - R_L K (V_{In} - V_T)^2}{2R_L} \tag{6}$$
$$\boxed{V_{out} = R_E \frac{2V_{SS} - R_L K (V_{In} - V_T)^2}{2R_E + R_L}} \tag{7}$$

To ensure that our MOSFET remains in Saturation Mode, we also have to impose an inequality relation on V_{In}: $V_T < V_{In} < V_{triodecutoff} \approx V_{out} + V_T$.

Problem 7.15 (BW).

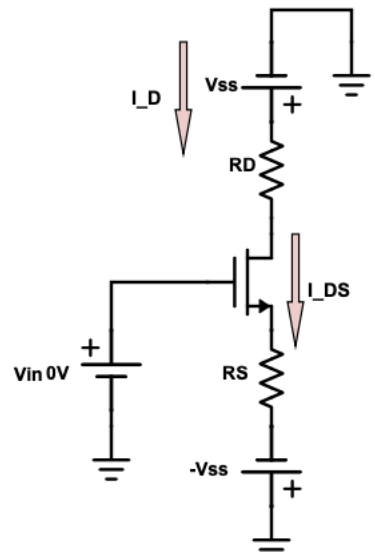

Figure 2: Figure 7.86 from your text. Current direction is drawn in, flowing from high to low potential.

The MOSFET operates in Saturation Mode, so it drives a current, I_{DS} and sinks V_{DS} across the channel. Note that the current enters the voltage sources at both the Source and the Drain through the negative terminal first. We can write a KVL loop equation and simplify as follows:

$$-V_{SS} + I_{DS} * R_D + V_{DS} + I_{DS} * R_S - V_{SS} = 0 \tag{8}$$

$$V_{out} = I_{DS} * R_S - V_{SS} \tag{9}$$

$$V_{out} = 2V_{SS} - V_{DS} - I_{DS} * R_D \tag{10}$$

I_{DS} can be generally written down as in the previous problem and expressed in terms of V_{In} and V_{out}

$$I_{DS} = \frac{K}{2}\left(V_{GS} - V_T\right)^2 \tag{11}$$

$$= \frac{K}{2}\left(V_{in} - V_{out} - V_T\right)^2 \tag{12}$$

$$= \frac{K}{2}\left(0 - I_{DS} * R_S + V_{SS} - V_T\right)^2 \tag{13}$$

$$I_{DS} = \frac{K}{2}\left(I_{DS}^2 R_S^2 - 2I_{DS}R_S V_{SS} + V_{SS}^2 + 2I_{DS}R_S V_T - 2V_{SS}V_T + V_T^2\right) \tag{14}$$

Solve (13) for I_{DS}

$$I_{DS} = \frac{1 + KR_S V_{SS} - KR_S V_T + \sqrt{1 + 2KR_S V_{SS} - 2KR_S V_T}}{2KR_S^2} \tag{15}$$

$$I_{DS} = \frac{1 + KR_S V_{SS} - KR_S V_T - \sqrt{1 + 2KR_S V_{SS} - 2KR_S V_T}}{2KR_S^2} \tag{16}$$

Simplifying further yields:

$$I_{DS} = \frac{V_{SS} - V_T}{R_S} + \frac{1}{KR_S^2} - \sqrt{\frac{2\left(V_{SS} - V_T\right)}{KR_S^3} + \frac{1}{K^2 R_S^4}} \tag{17}$$

Obviously the quadratic in (14) will yield 2 roots. I_{DS} is required to be positive in our circuit. Since increasing R_S is expected to decrease the value of I_{DS}, we choose (16) as our value of I_{DS}.

$$V_{out} = I_{DS} * R_S - V_{SS} \tag{18}$$

$$= \boxed{-V_T + \frac{1}{KR_S} - \sqrt{\frac{2\left(V_{SS} - V_T\right)}{KR_S} + \frac{1}{K^2 R_S^2}}} \tag{19}$$

Problem 7.16 (BW).

This problem starts exactly as 7.15 did. However, the V_{out} is now placed such that we need to include V_{DS} from the MOSFET as well. There is no reason why I_{DS} should change. Writing the same KVL loop equation as before:

$$-V_{SS} + I_{DS} * R_D + V_{DS} + I_{DS} * R_S - V_{SS} = 0 \tag{20}$$

$$V_{out} = V_{DS} + I_{DS} * R_S - V_{SS} \tag{21}$$

$$-2V_{SS} + I_{DS} * R_D + V_{out} + V_{SS} = 0 \tag{22}$$

$$V_{out} = V_{SS} - I_{DS} * R_D \tag{23}$$

$$\boxed{V_{out} = V_{SS} - R_D \left(\frac{V_{SS} - V_T}{R_S} + \frac{1}{KR_S^2} - \sqrt{\frac{2\left(V_{SS} - V_T\right)}{KR_S^3} + \frac{1}{K^2 R_S^4}} \right)} \tag{24}$$

Problem 7.17 (BW).

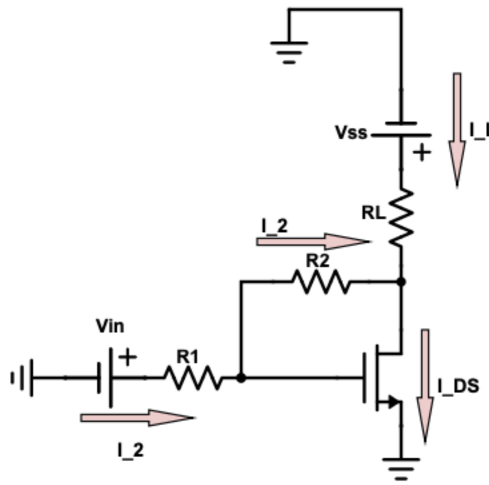

Figure 3: Figure 7.88 from your text. Current directions are drawn in for KCL analysis, flowing from high to low potential. Note that no current flows into the gate of the MOSFET as the input impedance at the gate is so high.

We are again told that the MOSFET operates in Saturation Mode. Because we now know what I_{DS} is supposed to be, we can write KCL relations that relate V_{In} to V_{out}:

$$I_L + I_2 = I_{DS} = \frac{K}{2}\left(V_{GS} - V_T\right)^2 \tag{25}$$

$$I_L = \frac{V_{SS} - V_{out}}{R_L} \tag{26}$$

$$I_2 = \frac{V_{In} - V_G}{R_1} = \frac{V_G - V_{out}}{R_2} \tag{27}$$

$$\tag{28}$$

Where V_G is the voltage immediately to the right of R_1 and to the left of R_2. Since V_S (the voltage at the source) is grounded, $V_{GS} = V_G$. Equating the two different expressions for I_2 yields the expression for V_G:

$$\frac{V_{In} - V_G}{R_1} = \frac{V_G - V_{out}}{R_2} \tag{29}$$

$$V_G * \left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{V_{In}}{R_1} + \frac{V_{out}}{R_2} \tag{30}$$

$$V_G = \frac{R_1 R_2}{R_1 + R_2} * \frac{R_2 V_{In} + R_1 V_{out}}{R_1 R_2} \tag{31}$$

$$V_G = \frac{R_2 V_{In} + R_1 V_{out}}{R_1 + R_2} = V_{GS} \tag{32}$$

$$\tag{33}$$

Plugging V_G back into (26) puts I_2 in terms of V_{In} and V_{out}.

$$I_2 = \frac{\frac{R_2 V_{In} + R_1 V_{out}}{R_1 + R_2} - V_{out}}{R_2} \tag{34}$$

$$= \frac{V_{In} - V_{out}}{R_1 + R_2} \tag{35}$$

Combining all of the current relations yields:

$$I_{DS} = \frac{K}{2}\left(V_G - V_T\right)^2 \tag{36}$$

$$= \frac{K}{2}\left(\frac{R_2 V_{In} + R_1 V_{out}}{R_1 + R_2} - V_T\right)^2 \tag{37}$$

$$= \frac{K}{2}\left(\frac{R_2^2 V_{In}^2}{(R_1 + R_2)^2} + \frac{2R_1 R_2 V_{In} V_{out}}{(R_1 + R_2)^2} + \frac{R_1^2 V_{out}^2}{(R_1 + R_2)^2} - \frac{2R_2 V_{In} V_T}{R_! + R_2} - \frac{2R_1 V_{out} V_T}{R_1 + R_2} + V_T^2\right) \tag{38}$$

$$= \frac{V_{SS} - V_{out}}{R_L} + \frac{V_{In} - V_{out}}{R_1 + R_2} \tag{39}$$

At this point, we simply solve a quadratic equation for V_{out}. This may seem very daunting but you should be able to identify easily separable terms with different orders of V_{out}^n. Note that you will arrive at two roots once again. The voltage drop is expected to be positive, so pick the one that is more likely to produce a positive value. This kind of analysis is common in experimental physics. The result is calculated using Mathematica and is stated below:

$$\boxed{\begin{aligned} V_{out} = -\frac{1}{KR_1^2}\left(R_1 + R_2\right)^2 \Bigg(& -\frac{1}{R_1 + R_2} + \frac{1}{R_L} + \frac{KR_1 R_2 V_{In}}{\left(R_1 + R_2\right)^2} - \frac{KR_1 V_T}{R_1 + R_2} \\ & - \sqrt{\left(\frac{1}{R_1 + R_2} - \frac{1}{R_L} - \frac{KR_1 R_2 V_{In}}{\left(R_1 + R_2\right)^2} + \frac{KR_1 V_T}{R_1 + R_2}\right)^2 + \frac{2KR_1^2\left(-\frac{V_{In}}{R_1 + R_2} - \frac{KR_2^2 V_{In}^2}{2(R_1 + R_2)^2} + \frac{V_{SS}}{R_L} + \frac{KR_2 V_{In} V_T}{R_1 + R_2} - \frac{K}{2}V_T^2\right)}{\left(R_1 + R_2\right)^2}} \Bigg) \end{aligned}}$$