Problem 7.18 (KG).

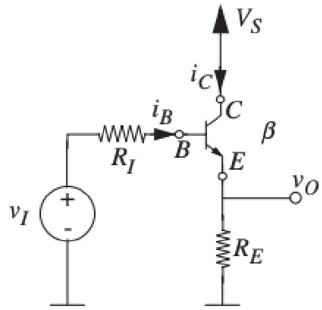

Figure 1: Common-collector amplifier circuit (also called source follower) from Agarwal and Lang.

We are told to assume this BJT operates in the active region and to use the piece-wise linear model in Exercise 7.8.

(a) **Draw the active-region equivalent circuit of the BJT source follower by replacing the BJT to its piecewise-linear model.**

In the active region, we have

$$I_C = \begin{cases} \beta I_B & \text{for } I_B > 0 \text{ and } V_{CE} > V_{BE} - 0.4V \\ 0 & \text{otherwise} \end{cases}$$

along with $I_E = (\beta + 1)I_B$ and $V_{BE} = 0.6V$ where the collector diode is off and the emitter diode is on. The condition that $V_{CE} > V_{BE} - 0.4V$ ensures that the base-to-collector diode stays off, which means we can treat it as an open circuit. Furthermore, since the emitter diode is on, it will appear as a short circuit as shown below.

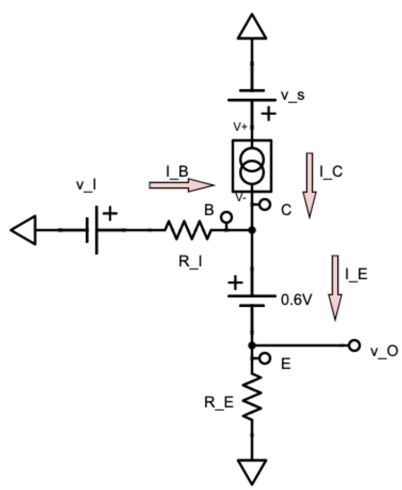

Figure 2: Simplified active-region equivalent circuit where the emitter diode is on and the collector diode is off.

For reference, the piece-wise linear model for a BJT in the active region is shown below.

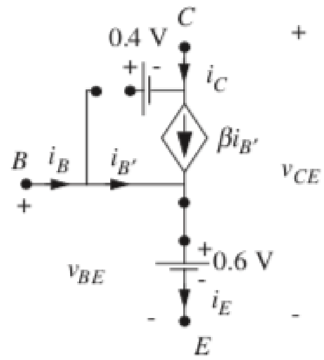

Figure 3: Piece-wise linear model of a BJT in the active region from Agarwal and Lang

(b) **Assuming active region operation, determine V_o in terms of V_I, R_I, R_E, and β.**

Observe $V_o = I_E R_E$ where $I_E = I_C + I_B = (\beta + 1)I_B$. Then, using KVL and Ohm's Law at the base-emitter, we see that

$$I_B R_I = V_I - (V_{BE} + V_o) \implies I_B = \frac{V_I - 0.6 - V_o}{R_I}$$

This means that

$$V_o = I_E R_E = (\beta + 1)\left(V_I - 0.6 - V_o\right)\frac{R_E}{R_I}$$

After rearranging, we get $\boxed{V_o = \dfrac{V_I - 0.6}{\frac{R_I}{R_E(\beta+1)} + 1}}$.

(c) **What is the value of V_o when $\beta R_E \gg R_I$?**

We take $\lim\limits_{\beta R_E \gg R_I} \frac{R_I}{(\beta+1)R_E} = 0$ where $\beta + 1 \approx \beta$ for large β. It follows that $\boxed{V_o \approx V_I - 0.6}$.

(d) **Compute the value of V_o given that $V_I = 3V$, $R_I = 10k\Omega$, $R_E = 100k\Omega$, $\beta = 100$, and $V_S = 100V$.**

Using the answer from part (b) we plug in the values and get $\boxed{V_o \approx 2.4V}$. Notice that this is consistent with our answer in part (c).

(e) **Determine the range of values of V_I for which the BJT operates in its active region for the parameter values given in (d). What is the corresponding range of V_o?**

We can establish a lower bound for V_I in active region since we know that $V_I > 0.6V$ in order to operate. Using the condition that corresponds with $I_B > 0$, we use the fact that

$$V_{CE} = V_S - V_o > 0.6 - 0.4 = 0.2 \quad \implies \quad 10 - 0.2 = 9.8 > V_o$$

Using our answer from part (b), we find the upper bound of V_I from the upper bound of V_o:

$$9.8V > \frac{V_I - 0.6}{\frac{R_I}{R_E(\beta+1)} + 1} \implies 10.4 > V_I$$

Hence, $\boxed{0.6V < V_I < 10.4V}$ and $\boxed{0V < V_o < 9.8V}$.

Problem 7.19 (SH).

a) This configuration is known as a Darlington Pair, designed to have a much higher current gain than that of an individual transistor. In order to represent this with the piece-wise linear model, we simply replace each transistor with the equivalent circuit. However, while the input transistor at node B' looks virtually the exact same as that in the book, keep in mind the the base current of the output transistor is in fact the emitter current of the input transistor.

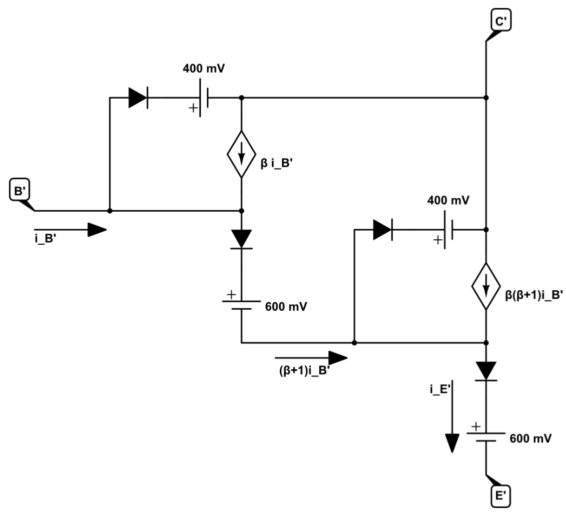

Figure 4: Piece-wise linear model of Darlington transistor configuration.

b) As we can see from the simplified diagram, the emitter current $i_{E'}$ is the sum of the input BJT's emitter current with the output current's collecter current:

$$i_{E'} = \beta(\beta+1)i_{B'} + (\beta+1)i_{B'}$$

To find the current gain, $i_{E'}/i_{B'}$:

$$\beta' = \frac{i_E'}{i_B'} = (\beta+1)^2$$

In the limit that $\beta \gg 1$, we can do the following expansion:

$$\beta' = \beta^2\left(1+\frac{1}{\beta}\right)^2 \approx \beta^2\left(1+\frac{2}{\beta}\right)$$

Thus, the current gain of the Darlington Pair is:

$$\boxed{\beta' \approx \beta(\beta+2)}$$

However, for even larger β, we can take

$$\beta' \approx \beta^2$$

For $\beta = 100$, we have a current gain of roughly 10,200 or about 10k, which is much greater than our initial β, as we wanted!

c) As the two transistors are both active when $i_{B'} > 0$, we simply follow the path from the B' node down to the E' node and apply KVL. This ends up being simply the sum of the two transistor's V_{BE}'s, therefore giving:

$$\boxed{V_{B'E'} = V_{BE_1} + V_{BE_2} = 1.2 \text{ V}}$$

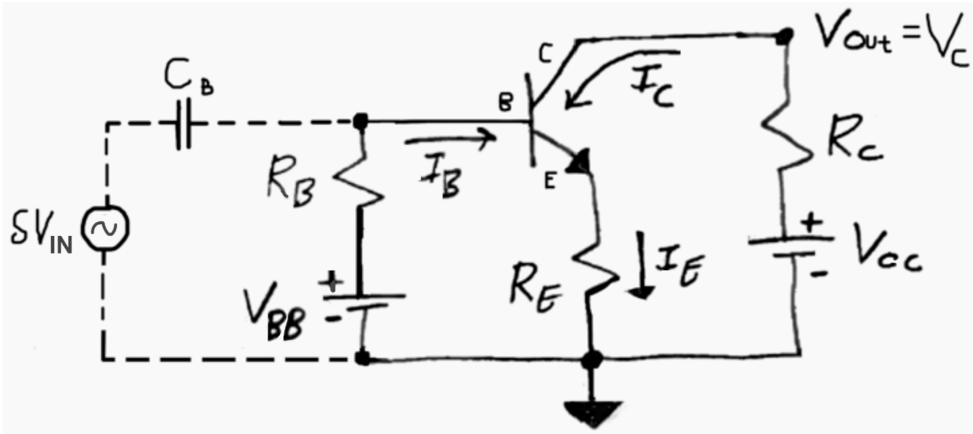

Figure 5: We are presented with this Common Emitter Amplifier circuit from the Handout. We see that $V_{CC} = +15V$. V_{BB}, R_B, R_E, and R_C are yet to be determined per the quiescent operating points: $V_{CE,Q} = V_{CC} - V_{CE,Sat}$ and $I_{C,Q} = 10mA$.

Problem Handout 1 (BW).

We are told that the small-signal current gain, β, of the transistor between the Base and the Emitter is 100. We are also told that we desire a DC voltage gain, $|\frac{V_{out}}{V_{in}}| = 30$.

Let's first write some relations that relate I_B, I_C, and I_E to represent gain.

$$I_C = I_B + I_E \qquad \text{By KCL} \tag{1a}$$
$$-V_{BB} + I_B R_B + V_{BE} + I_E R_E = 0 \qquad \text{By KVL} \tag{1b}$$
$$-V_{CC} + I_C R_C + V_{CE} + I_E R_E = 0 \qquad \text{By KVL} \tag{1c}$$
$$I_C = \beta I_B \qquad \text{By Definition of } \beta \tag{1d}$$

From (1a) and (1d): $I_C = \frac{I_C}{\beta} + I_E \approx I_E$ since $\beta >> 1$.

From (1b):

$$-V_{BB} + \frac{R_B I_C}{\beta} + V_{BE} + I_E R_E = 0 \tag{2}$$
$$I_E(\frac{R_B}{\beta} + R_E) \approx I_C(\frac{R_B}{\beta} + R_E) = V_{BB} - V_{BE} \tag{3}$$
$$I_C \approx \frac{V_{BB} - V_{BE}}{\frac{R_B}{\beta} + R_E} \tag{4}$$
$$I_C \approx \frac{V_{BB} - V_{BE}}{R_E} \qquad \text{Since } \beta >> 1 \tag{5}$$

From (1c):

$$-V_{CC} + I_C R_C + V_{CE} + I_E R_E = 0 \tag{6}$$
$$I_C(R_C + R_E) \approx V_{CC} - V_{CE} \tag{7}$$
$$I_C \approx \frac{V_{CC} - V_{CE}}{(R_C + R_E)} \tag{8}$$
$$V_{CE} = V_{CC} - I_C(R_C + R_E) \qquad \text{From (6)} \tag{9}$$

From Figure 1, we can see that

$$\begin{aligned} V_{out} &= V_{CE} + I_E R_E && (10)\\ &= V_{CC} - I_C(R_C + R_E) + I_E R_E && (11)\\ &= V_{CC} - I_C R_C && (12) \end{aligned}$$

Recall that we are in the business of finding a ratio of output to input voltage. Input voltage in this case is V_{BB}. Plug (5) into (12) to arrive at the following:

$$V_{out} = V_{CC} - R_C \left(\frac{V_{BB} - V_{BE}}{R_E} \right) \tag{13}$$

$$V_{out} = \left(V_{CC} + \frac{R_C}{R_E} V_{BE} \right) - \frac{R_C}{R_E} V_{BB} \tag{14}$$

$$V_{out} = (Constant) - \frac{R_C}{R_E} V_{BB} \tag{15}$$

Since this circuit is an AC Amplifier (It does different things to DC Signals!), we can drop the constant and say that the Gain is $-\frac{R_C}{R_E}$. We can drop the negative sign when analyzing resistor values. We can now write the following expressions:

$$R_C = 30 R_E \qquad \text{Desiring a Gain of 30} \tag{16}$$

$$R_C + R_E = \frac{V_{CC} - V_{CE}}{I_C} \qquad \text{This additional constraint is from (8)} \tag{17}$$

At this point, we shall consider the operating/quiescent points to determine the constraint for $R_C + R_E$. Substituting quiescent voltages and currents as well as relevant values into (17) yields:

$$\begin{aligned} R_C + R_E &= 30 R_E + R_E && (18)\\ &= \frac{V_{CC} - V_{CE,Q}}{I_{C,Q}} && (19)\\ &= \frac{V_{CC} - (V_{CC} - V_{CE,Sat})}{I_{C,Q}} && (20)\\ &= \frac{V_{CE,Sat}}{I_{C,Q}} && (21)\\ &= \frac{0.2V}{10mA} && (22)\\ &= 20\Omega && (23) \end{aligned}$$

$$\boxed{R_E = \frac{20\Omega}{31} = 0.645\Omega} \tag{24}$$

$$\boxed{R_C = 30 * 0.645\Omega = 19.355\Omega} \tag{25}$$

In order to ignore β, we need to establish a relationship between R_B, R_E, and β.

$$I_C \ \frac{1}{R_E} \qquad \text{From (5)} \tag{26}$$

$$I_B \ \frac{1}{R_B} \qquad \text{By similar logic} \tag{27}$$

$$I_C = \beta I_B \tag{28}$$

$$\frac{1}{R_E} = \frac{\beta}{R_B} \tag{29}$$

$$R_B = \beta R_E \tag{30}$$

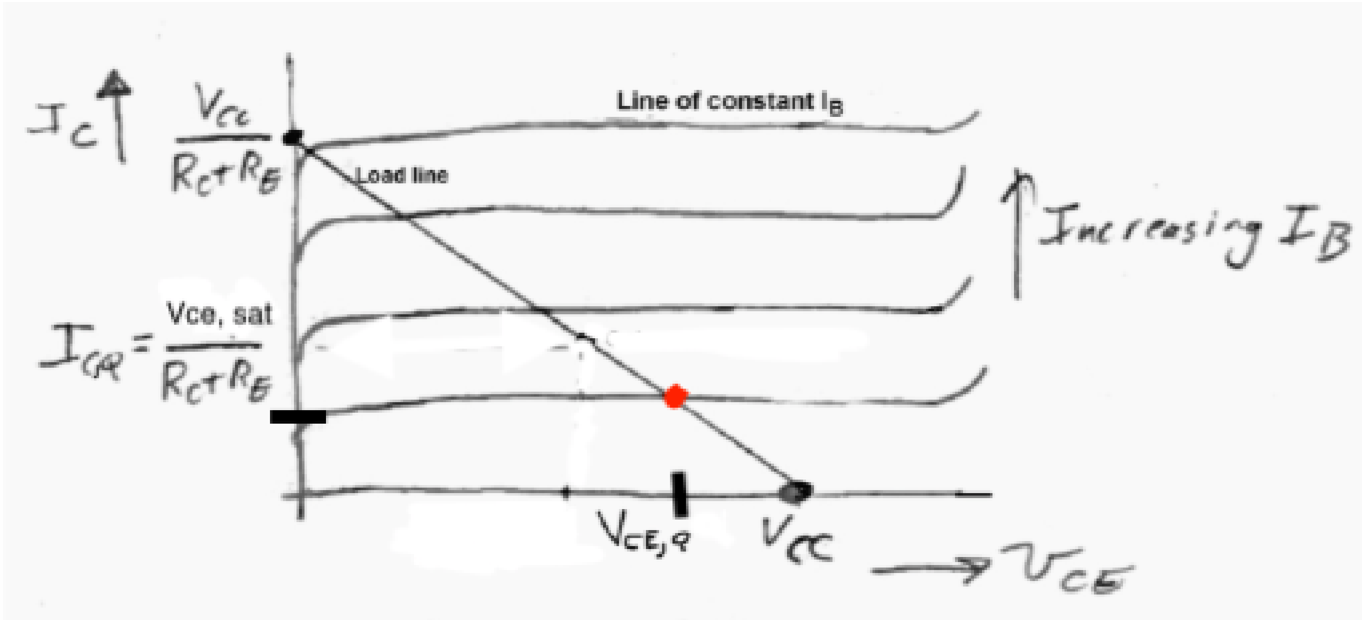

Figure 6: Rough sketch of where the load line intersects the constitutive relation between I_{CE} and V_{CE} of this BJT

Now, we choose a R_B value such that $\frac{R_B}{R_E} << 100$. For simplicity, let's choose $\boxed{\mathrm{R}_B = 10 * R_E = 6.45\Omega}$.

Problem Handout 2 (BW).

Most of the work has already been laid out in the previous problem. From (17) and (19), we can infer that:

$$\boxed{I_{C,Q} = \frac{V_{CC} - V_{CE,Q}}{R_C + R_E} = \frac{V_{CE,Sat}}{R_C + R_E} = 10mA} \tag{31}$$

Problem Handout 3 (BW).

Recall that, in the context of BJTs and bias resistors (the resistors that surround the transistor and allow for feedback pathways), the y-intercept of a load line is the maximum current achievable; this generally implies to treat the transistor as a short with no voltage drop. The x-intercept of a load line is the maximum voltage achievable; this is the supply voltage.

Since we have specified the Q point (set point) first, we know that the load line will cross one of the bottom-most (low I_B curve) curves as shown in Figure 2. Whereever this load line crosses the constitutive curve represents a match between allowed values of current and voltage of this transistor and allowed values of this circuit.

We know that $V_{CE,Q}$ will be around where it is in the figure because the problem defined $V_{CE,Q} = V_{CC} - V_{CE,Sat} = 15V - 0.2V$. Since the operating voltage is so high, it makes sense that the operating current will be lower.

Problem Handout 4 (BW).

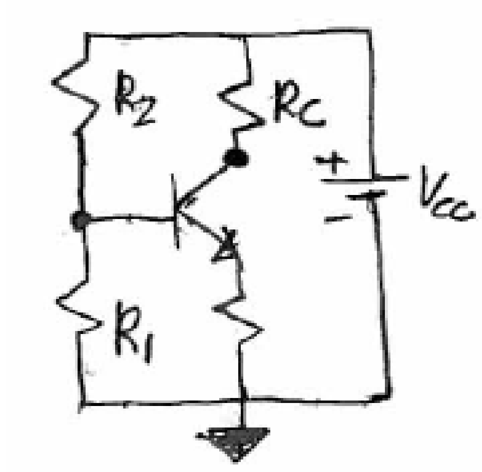

Figure 7: Schematic for what the single supply circuit will look like. Notice that we have turned R_B into two resistors, R_1 and R_2. These two resistors are in parallel when performing Thevenin equivalence analysis

First, we need to know what V_{BB} is. From (5), we see that:

$$V_{BB} \approx I_C R_E + V_{BE} \tag{32}$$
$$= I_{C,Q} R_E + V_{BE} \tag{33}$$
$$= 10mA * 0.645\Omega + 0.7V \tag{34}$$

Plugging in values from the previous part as well as V_{BE} = Forward Bias necessary for BE diode to work.

$$\tag{35}$$
$$= 0.706V \tag{36}$$

This is valid because the BE diode will remain forward biased and the BC diode will remain reverse biased as $V_{CC} = 15V > 0.706V$.

To convert Figure 1 into a single supply circuit, engineer a voltage divider with two resistors R_1 and R_2 to turn the 15V V_{CC} supply into "V_{BB}" at the correct node.

We wish to solve the following equations for R_1 and R_2:

$$R_B = \frac{R_1 R_2}{R_1 + R_2}$$

The Thevenin resistance calls for the shorting of the voltage source to ground, making $R_1||R_2$

$$V_{BB} = \frac{R_1}{R_2 + R_1} V_{CC}$$

V_{BB} is set up to be the voltage division of V$_{CC}$

Solving the above two relations for R_1 and R_2 yields $\boxed{\mathrm{R}_1 = R_B \frac{V_{CC}}{V_{CC}-V_{BB}}}$ and $\boxed{\mathrm{R}_2 = R_B \frac{V_{CC}}{V_{BB}}}$. Plug in numbers to get values of the resistors.
V_{CC} is always at a higher potential, so you do not risk much current leaking between the branches through the voltage supply.

Problem Handout 5 (BW).

Recall the left part of the circuit shown in Figure 1. The capacitor, C_B, in series with R_B to ground (V_{BB} is a DC source and is treated as a short to AC signals) forms a high pass filter.

Recall that cut-on frequency (analogous to "cut-off" frequency but high pass filters are concerned with when the signal starts to pass rather than when it starts to die off) is:

$$\begin{aligned}
f_{cut-on} &= \frac{1}{2\pi R_B C_B} && \text{by definition} && (40)\\
C_B &= \frac{1}{2\pi R_B f_{cut-on}} && && (41)\\
&= \frac{1}{2\pi * 6.45\Omega * 100Hz} && && (42)\\
&= \boxed{247\mu\mathrm{F}} && && (43)
\end{aligned}$$