Contour Integration for Simple poles

$$\oint ds\, f(s) = 2\pi i \sum \text{residues}$$

When $f(s) = \dfrac{\text{regular function}}{\text{polynomial function with simple zero's}}$

$$'' \equiv \frac{q(s)}{p(s)}$$

The residue at each pole is given by the expression $\longrightarrow \left. \dfrac{q(s)}{\frac{\partial p(s)}{\partial s}} \right|_{s=s_{pole}}$

Example with $p(s)^{*} = (s-a)(s-b)$; $q(s) = e^{st}$

$$\oint ds \frac{e^{st}}{(s-a)(s-b)} = 2\pi i \left\{ \left. \frac{e^{st}}{(s-b)} \right|_{s=a} + \left. \frac{e^{st}}{(s-a)} \right|_{s=b} \right\}$$

$$'' = 2\pi i \left\{ \frac{e^{at}}{a-b} + \frac{e^{bt}}{b-a} \right\}$$

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*As indicated in notes (p.) for $p(s) = \sum_n (s-s_n)$

$$\frac{\partial p(s)}{\partial s} = \sum_n \sum_{k \neq n} (s-s_k)$$