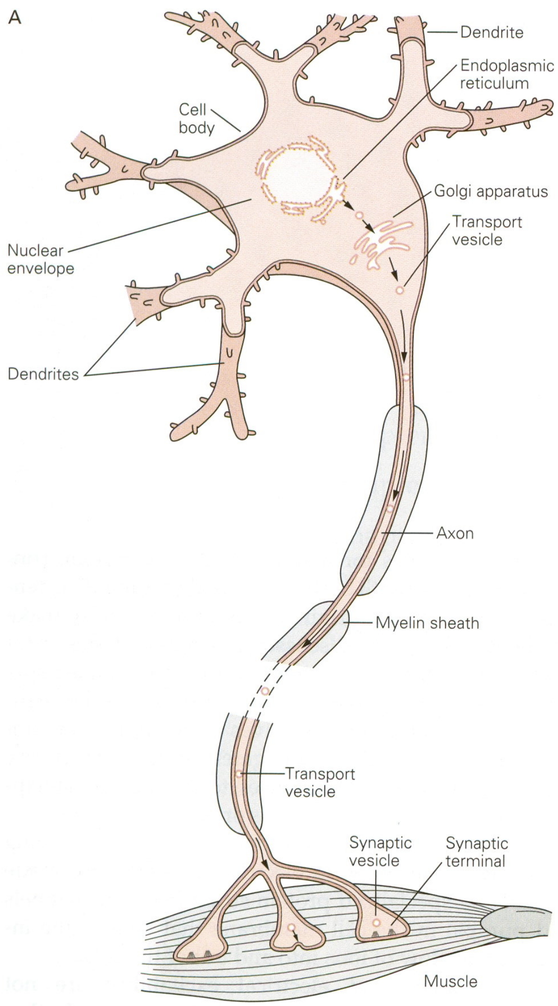
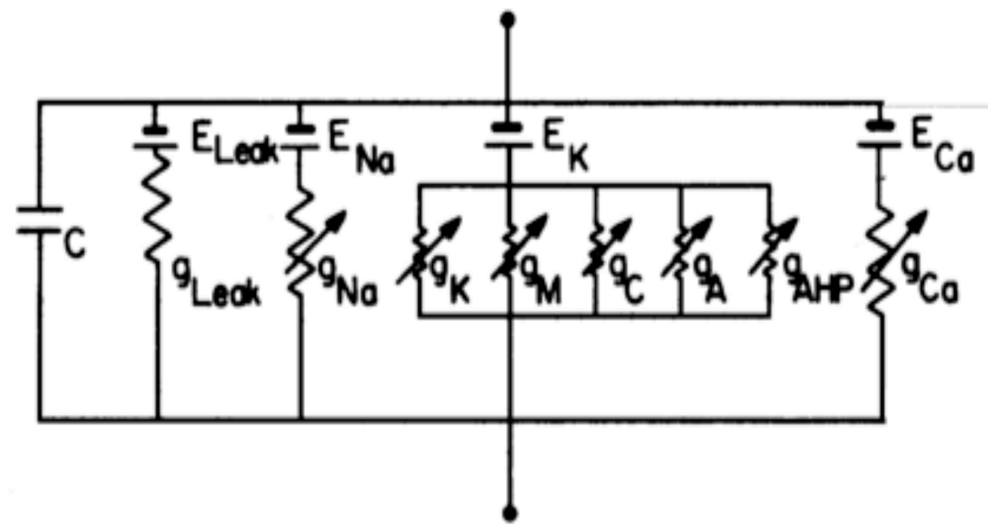
Illustrated Notes on
Basic Cellular Biophysics

Modified from Michael J Berry II
MCN 2010 Lecture Notes

Equivalent Circuit Model of a Neuron

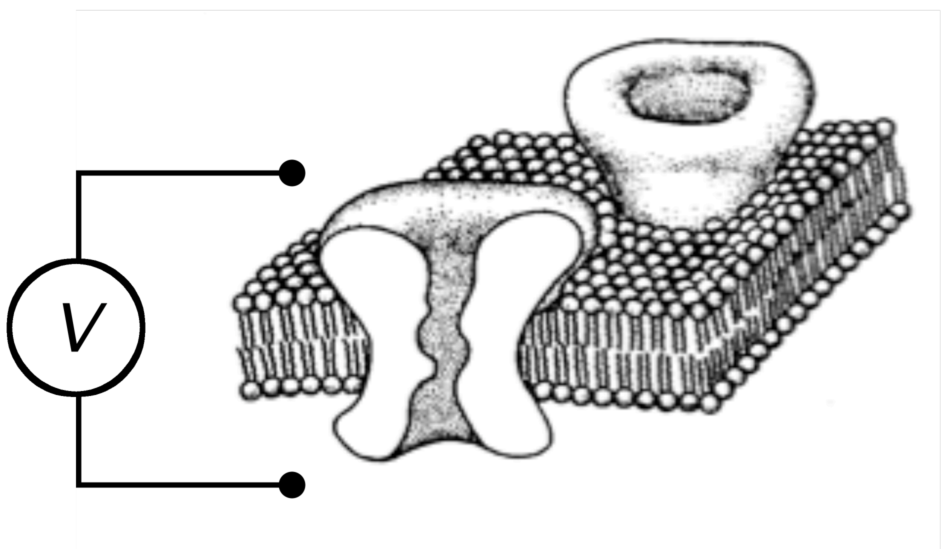


→



Resting Potential: Electrochemical Equilibrium

$$V = \frac{K_B T}{e} \ln \frac{[X]_{out}}{[X]_{in}}$$



$V = -65\,mV$

Common Ionic Concentrations

TABLE 2.1
Extracellular and Intracellular Ion Concentrations

Ion	*Concentration (mM)* Intracellular	Extracellular
Squid neuron		
Potassium (K^+)	400	20
Sodium (Na^+)	50	440
Chloride (Cl^-)	40–150	560
Calcium (Ca^{2+})	0.0001	10
Mammalian neuron		
Potassium (K^+)	140	5
Sodium (Na^+)	5–15	145
Chloride (Cl^-)	4–30	110
Calcium (Ca^{2+})	0.0001	1–2

$$E_K = -75\ mV$$

$$E_{Na} = +55\ mV$$

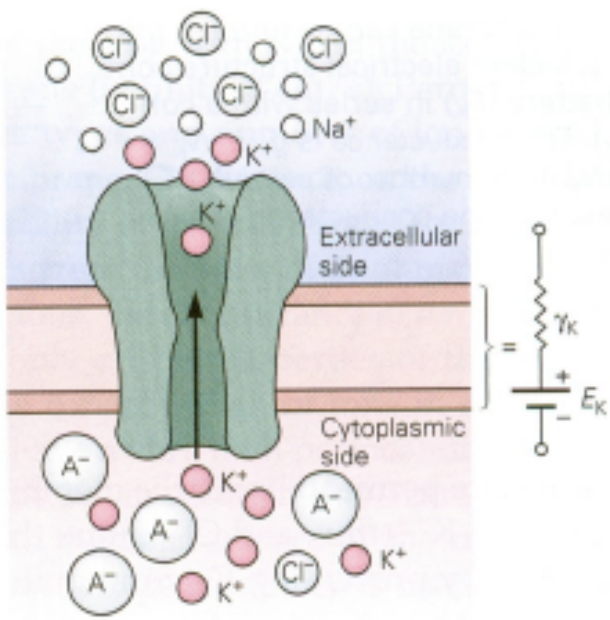
$$E_{Cl} = -70\ mV$$

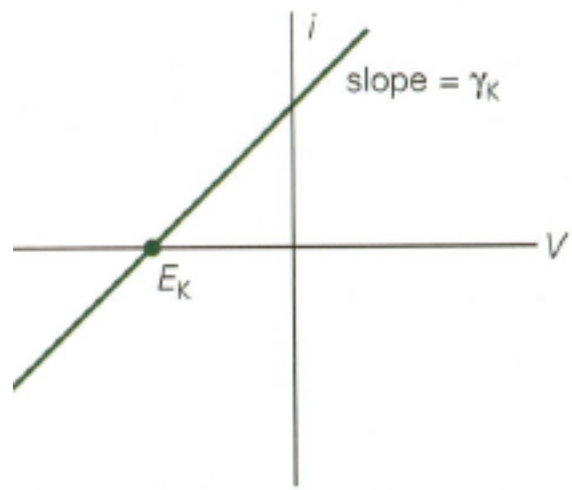
$$E_{Ca} \approx +150\ mV$$

Out of Equilibrium

- For $V > V_X$ current flows into cell: $I_X > 0$
- For $V < V_X$ current flows out of cell: $I_X < 0$

- *I-V* curve (current-voltage):

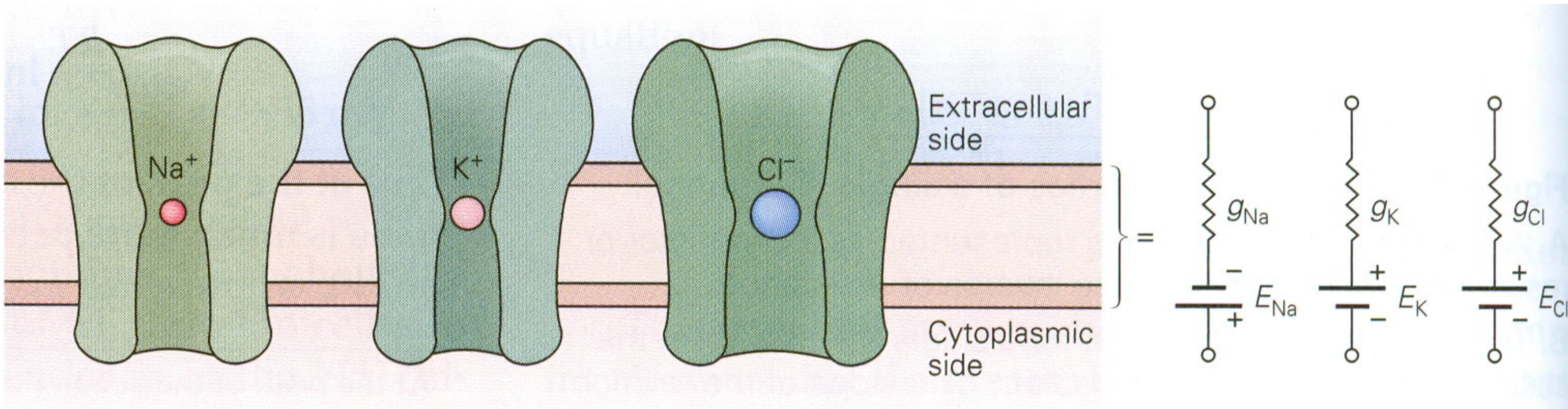




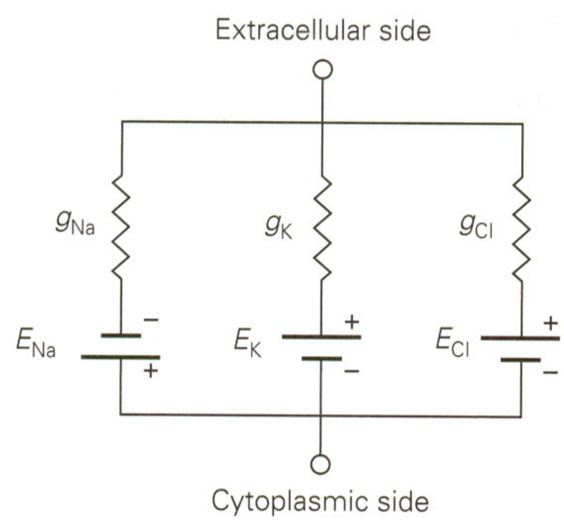
$$I_K = \gamma_K \left(V - V_K \right)$$

Multiple Kinds of Ions

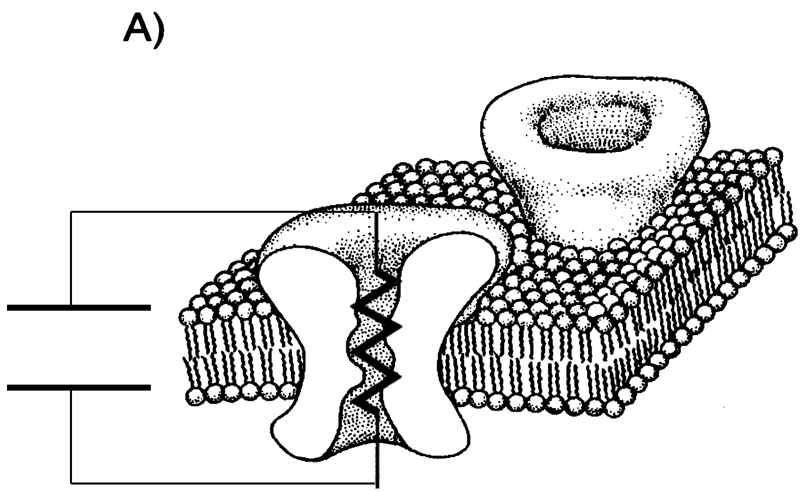
- Several *I-V* curves in parallel:

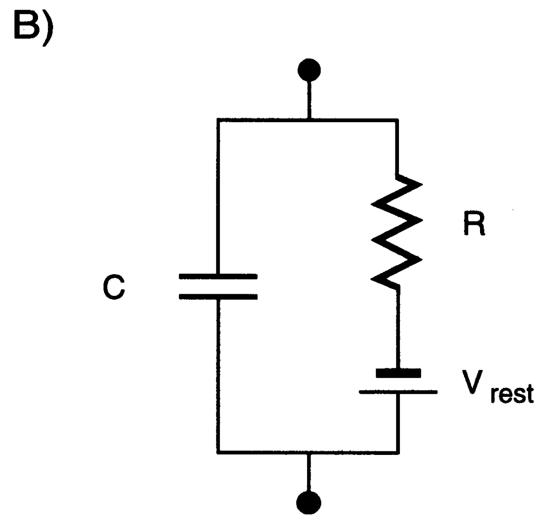


- New equivalent circuit:



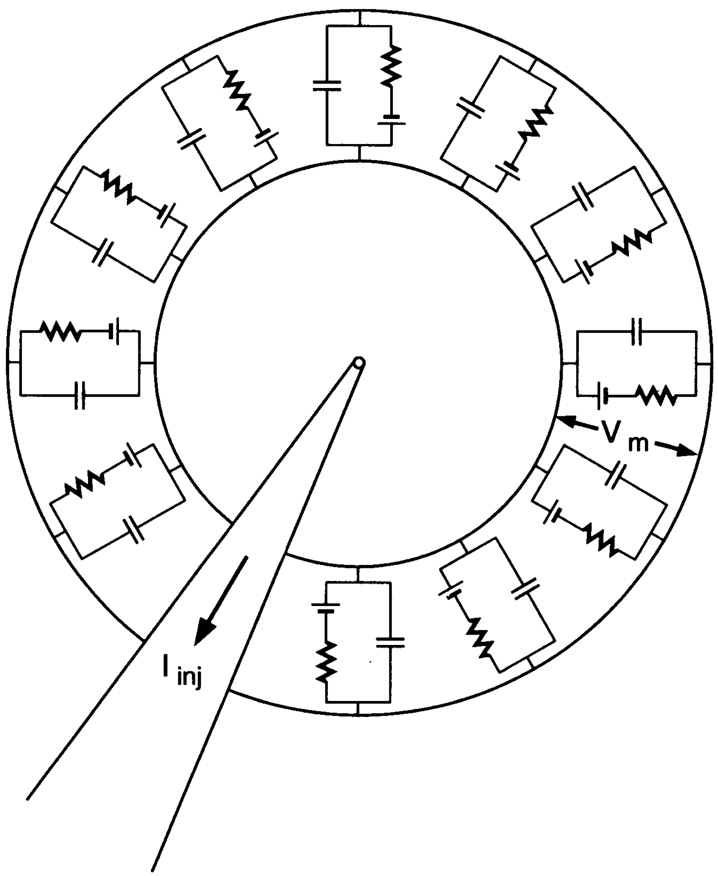
Passive Membrane Properties

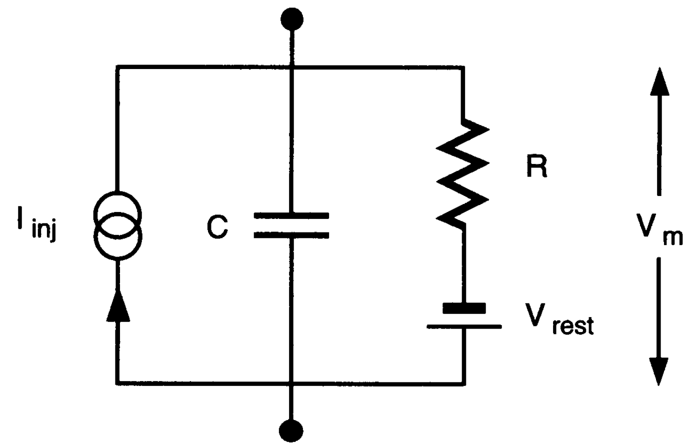




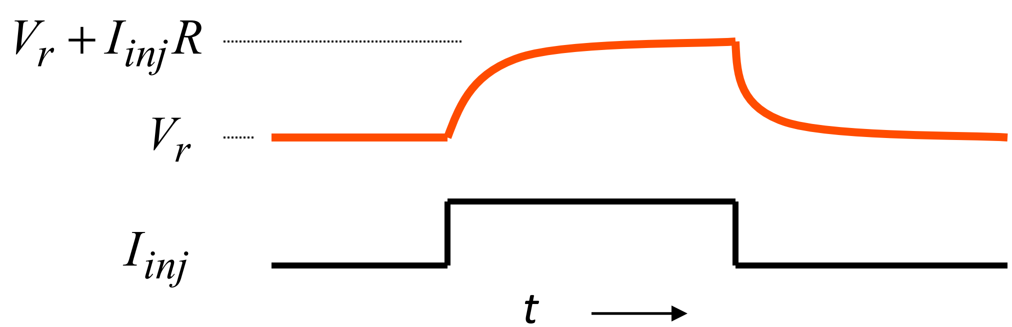
$$C \approx 1\mu F / cm^2$$

Compact Cell Model (isopotential voltage)

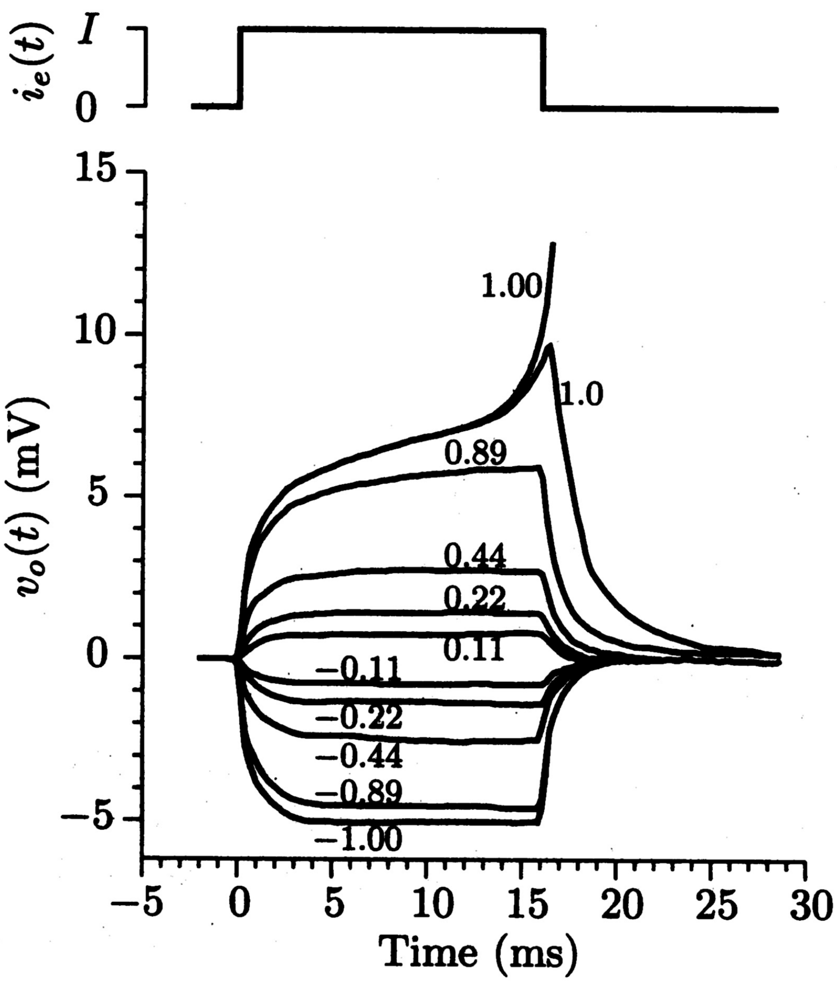


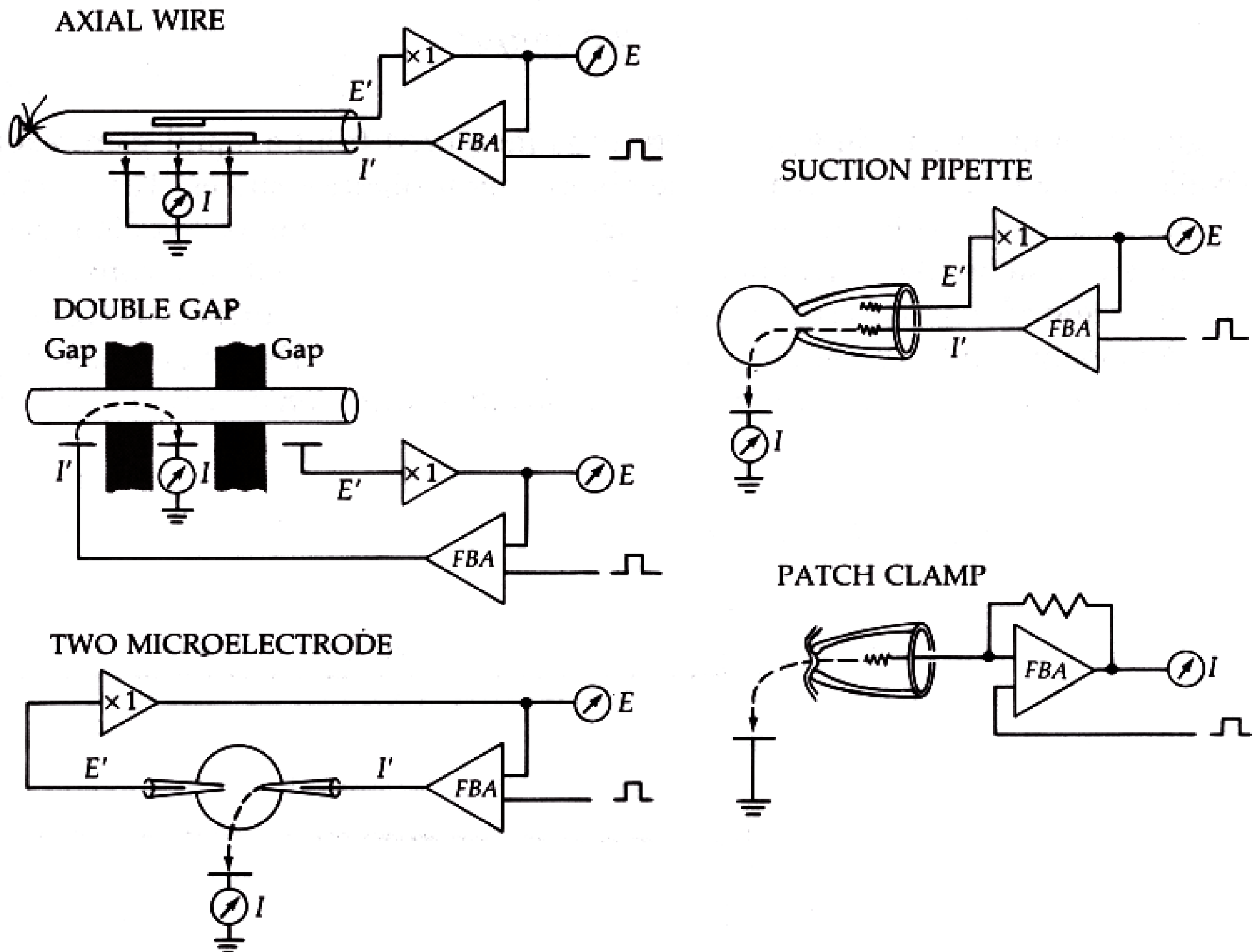


$$C\frac{dV_m}{dt} = I_{inj} + \frac{(V_m - V_r)}{R}$$



Real Data from Lobster Axon

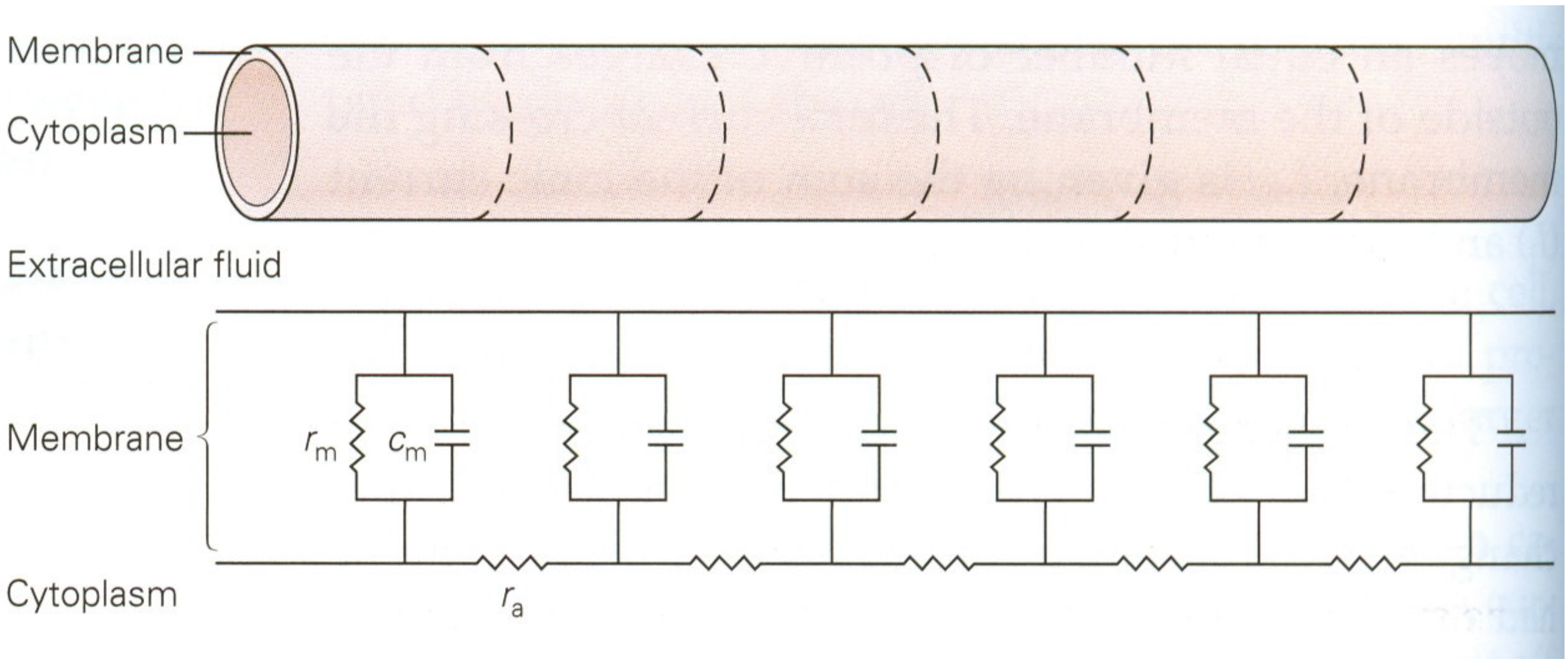


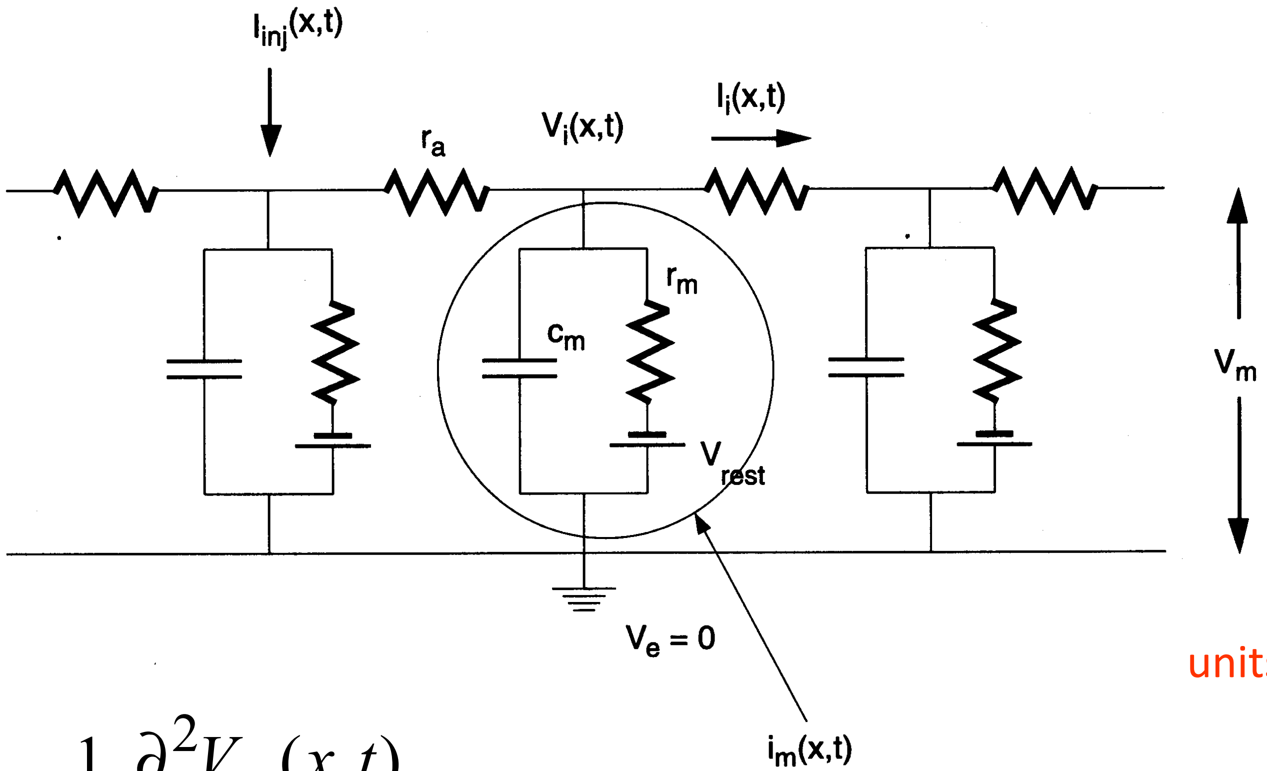


chapt_7_voltage_clamp.eps

Passive Properties: Long Cable

- Describe cable as a series of RC circuits
- Allow the thickness of each segment $\Delta x \to$ zero





units:

r_m: ohm-cm

r_a: ohm/cm

i_m: A/cm

c_m: F/cm

$$\frac{1}{r_a}\frac{\partial^2 V_m(x,t)}{\partial x^2} = i_m(x,t)$$

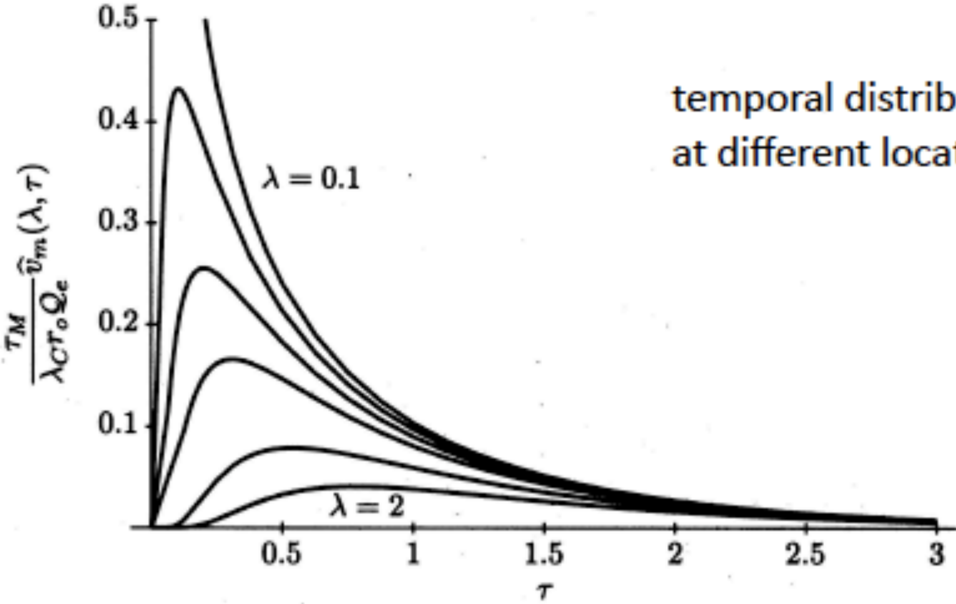
$$i_m(x,t) = \frac{V_m(x,t) - V_{rest}}{r_m} + c_m\frac{\partial V_m}{\partial t} - I_{inj}(x,t)$$

$$\tau_m = r_m c_m$$

$$\lambda = \sqrt{\frac{r_m}{r_a}}$$

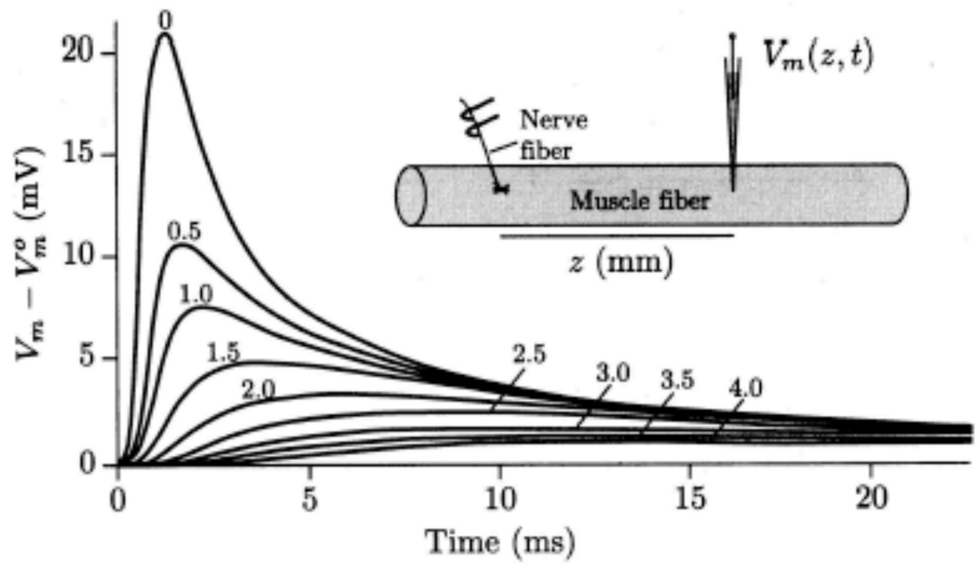
$$\lambda^2\frac{\partial^2 V_m(x,t)}{\partial x^2} = \tau_m\frac{\partial V_m(x,t)}{\partial t} + (V_m(x,t) - V_{rest}) - r_m I_{inj}(x,t)$$

Putting it All Together



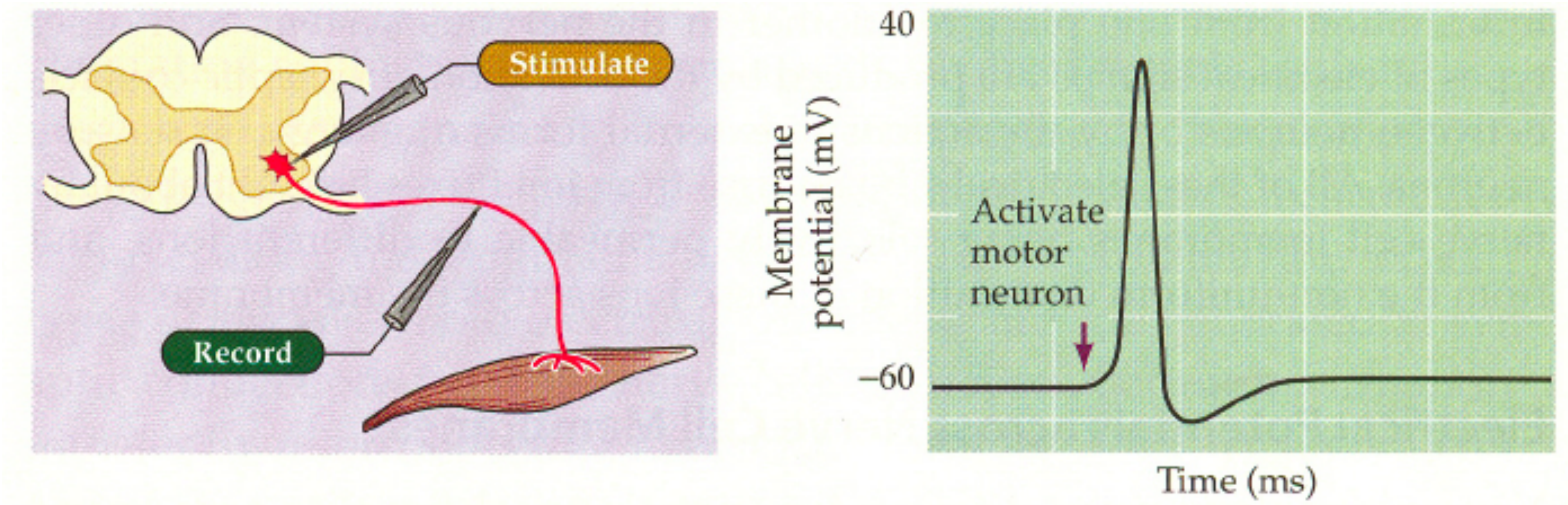
temporal distribution at different locations

theory

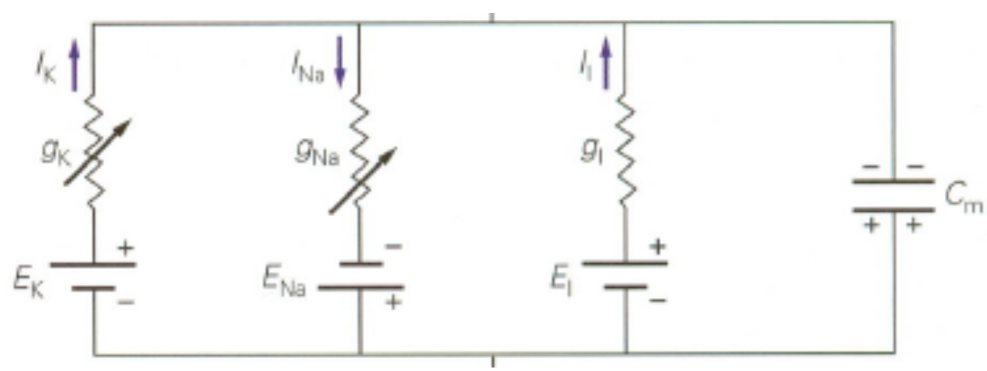


experiment

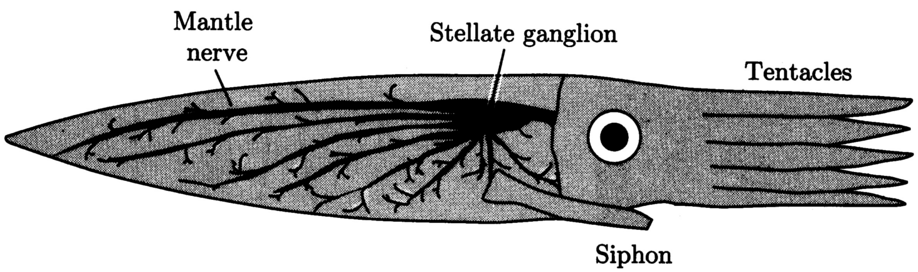
The Action Potential

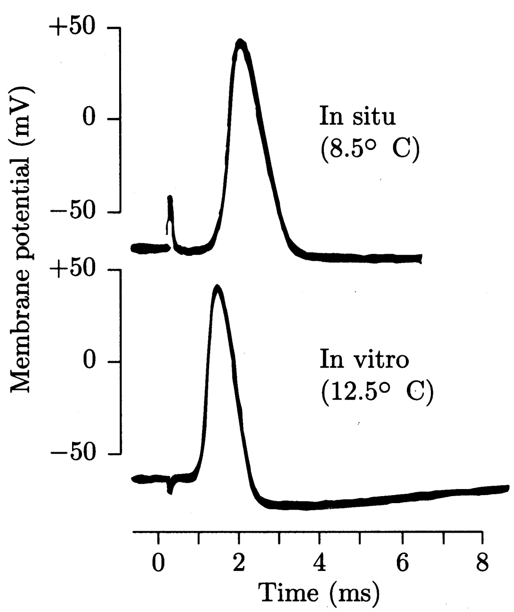


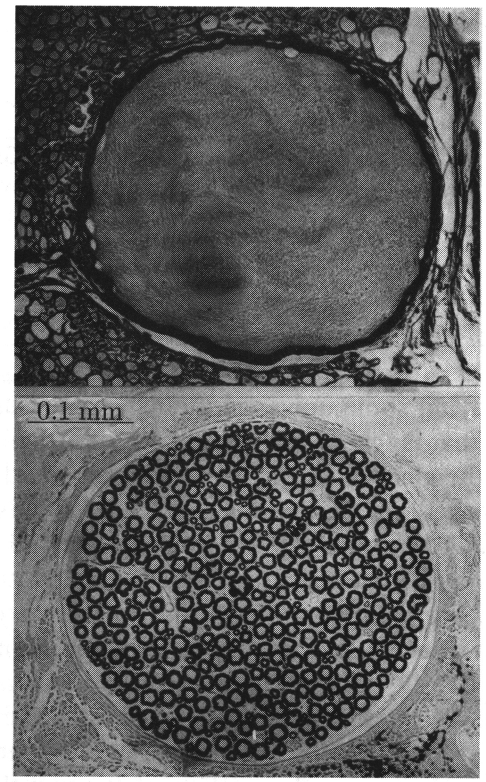
- Equivalent circuit model



METHODS: Squid Giant Axon

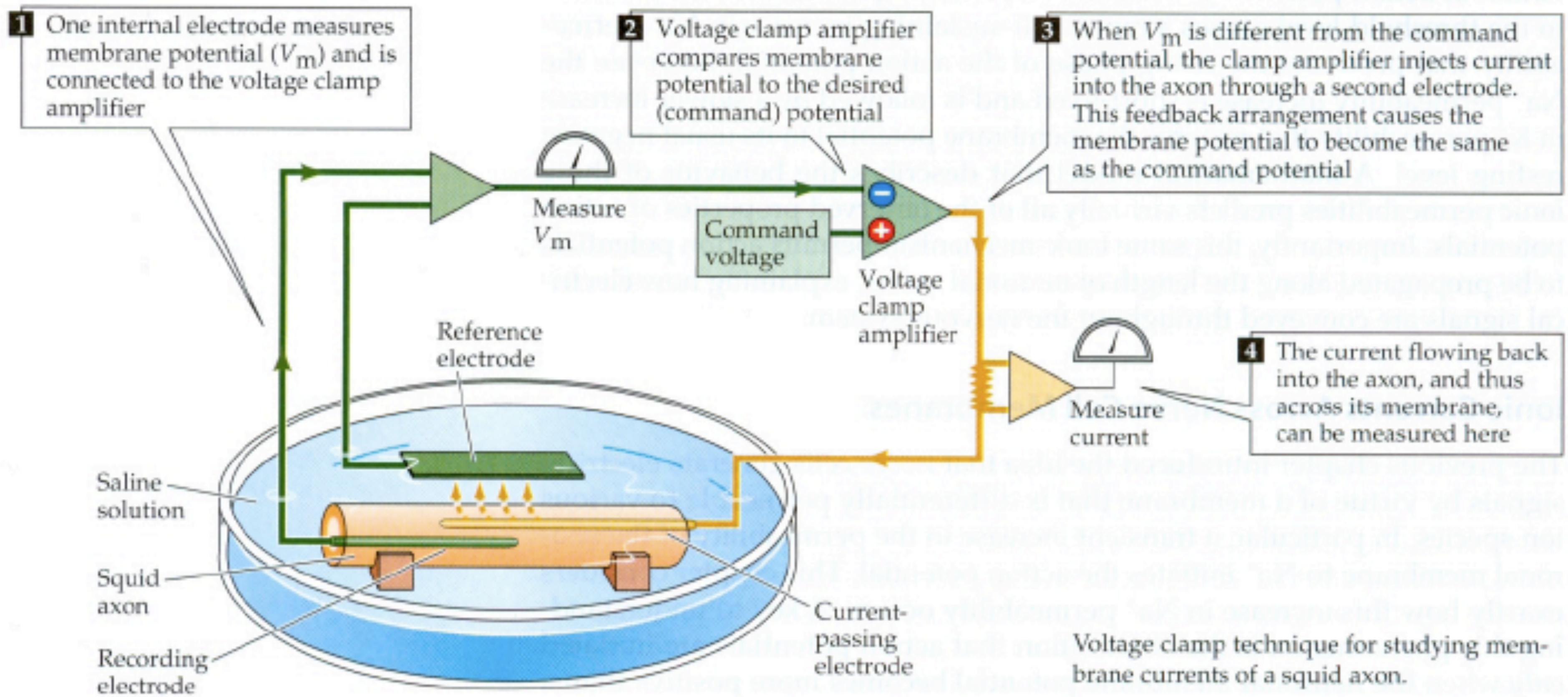






METHODS: Voltage Clamp

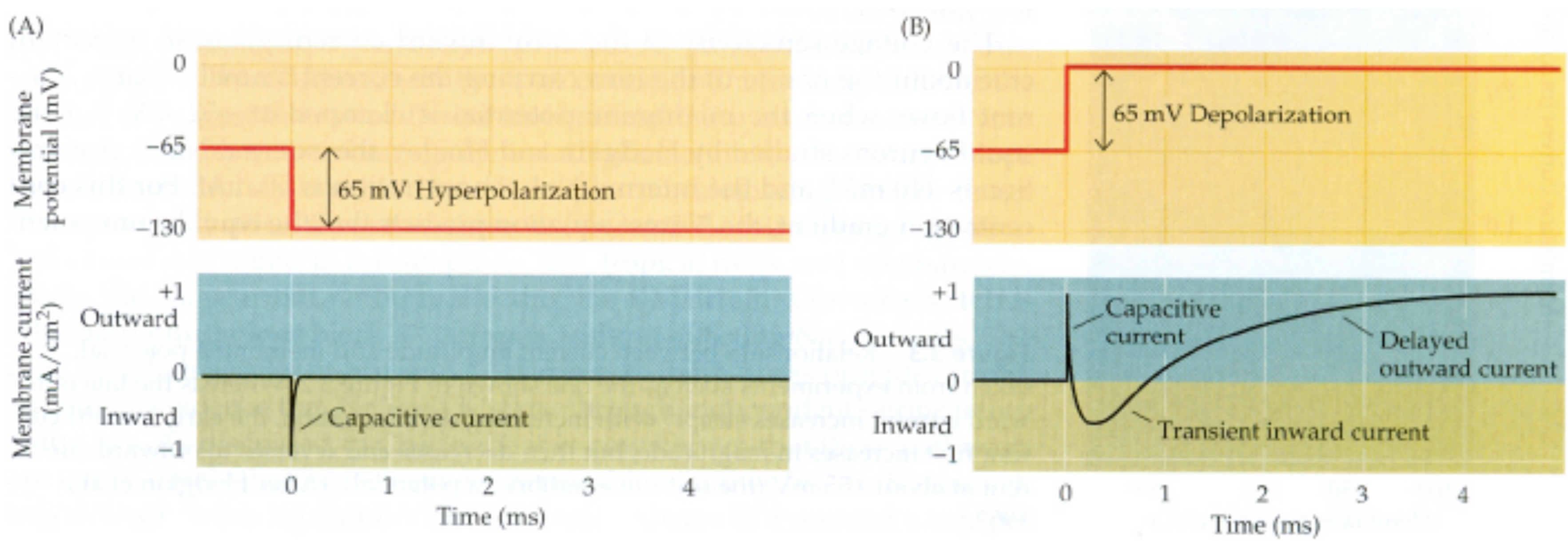
- Currents are voltage-gated, so must control voltage



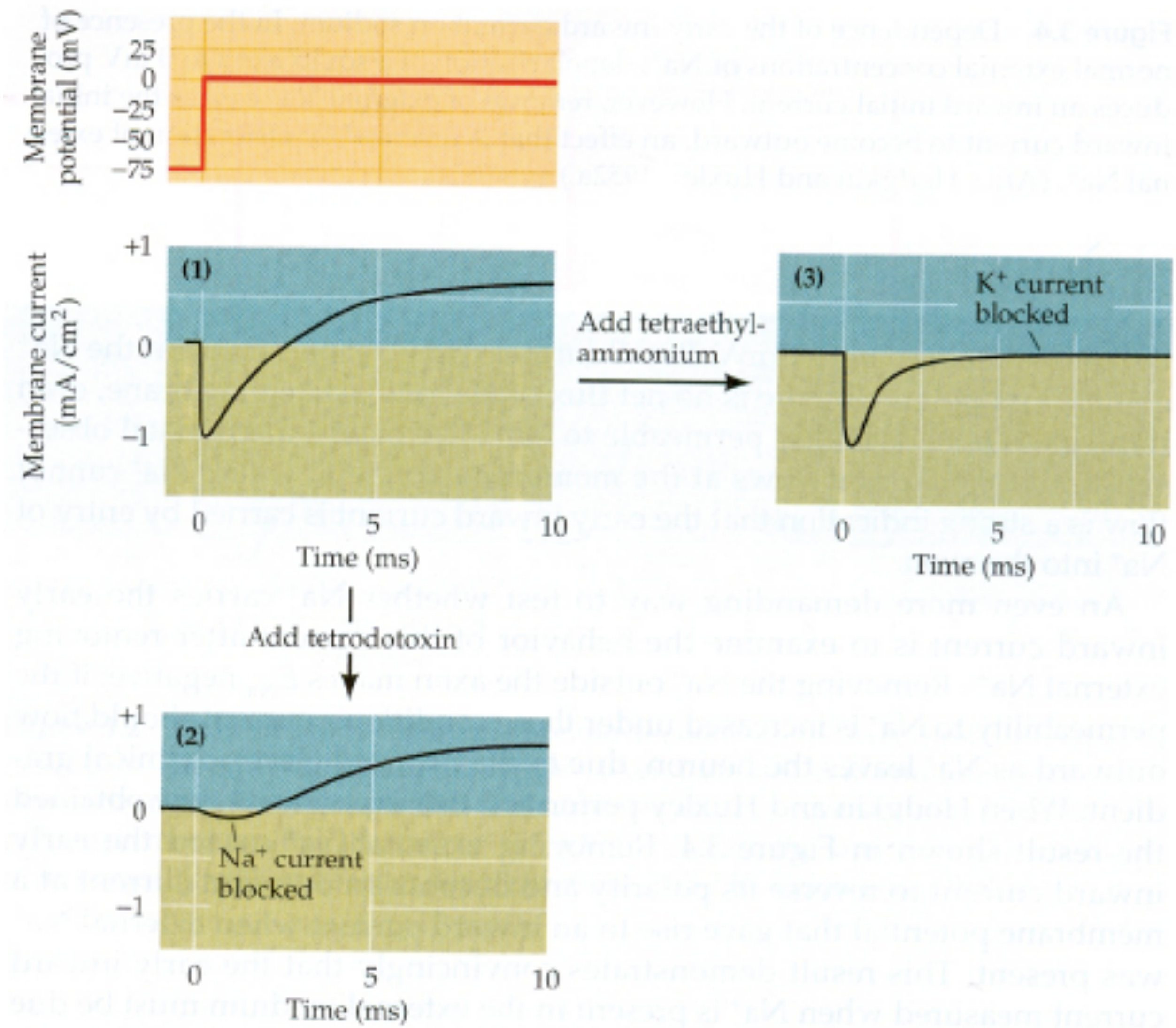
Voltage clamp technique for studying membrane currents of a squid axon.

METHODS: Voltage Clamp II

- Capacitive current when applying voltage command
- Inward and outward currents when depolarizing



Separation of Currents



Two Sources of Voltage Dependence

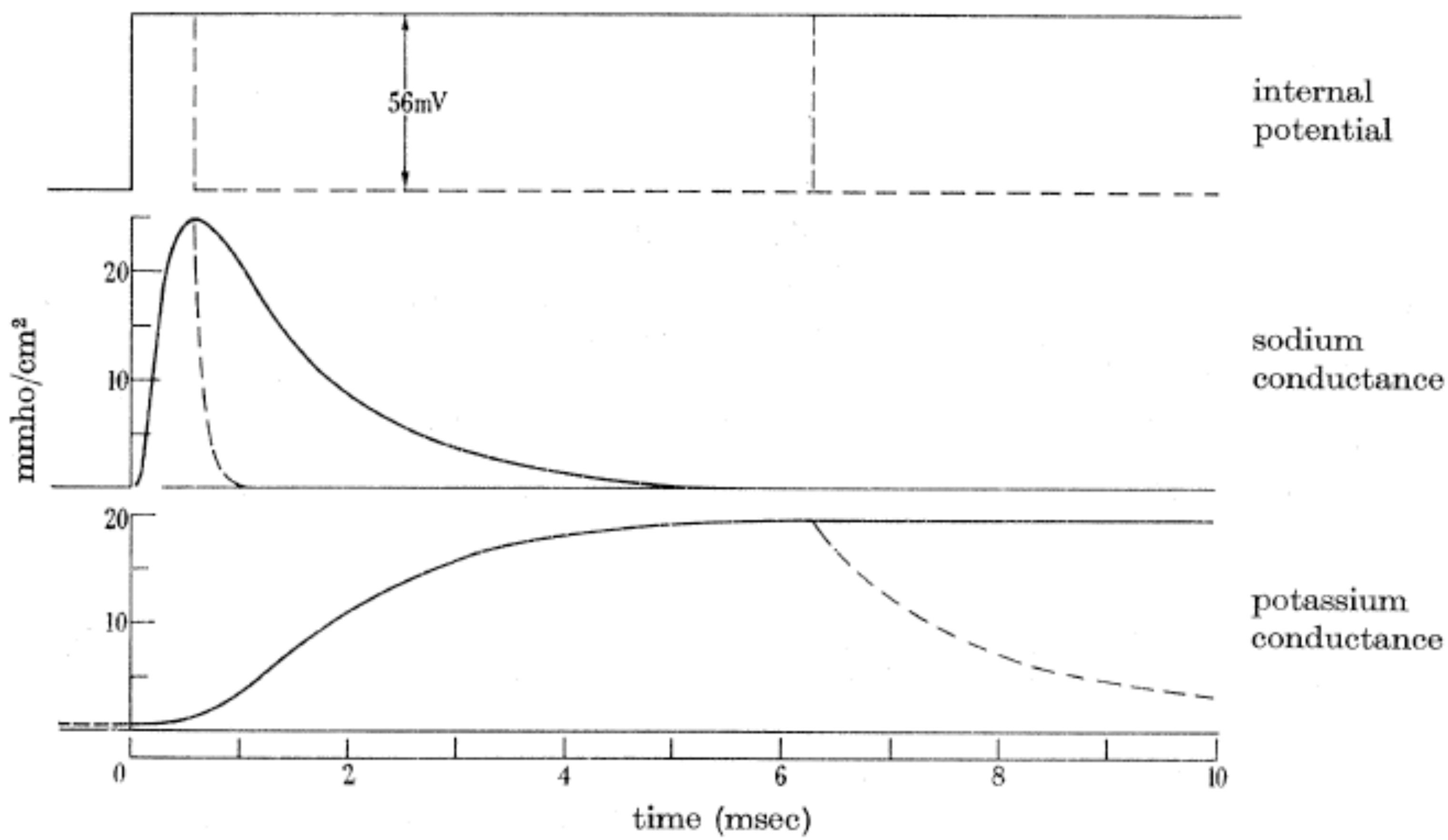
$$I_{Na}(V,t) = g_{Na}(V,t) \cdot (V - V_{Na})$$

conductance   driving force

$$g_{Na}(V,t) = \frac{I_{Na}(V,t)}{V - V_{Na}} \qquad g_K(V,t) = \frac{I_K(V,t)}{V - V_K}$$

Measuring Conductances

- I_{Na} activates, then inactivates
- I_K has shallow S-shaped activation, but exponential inactivation

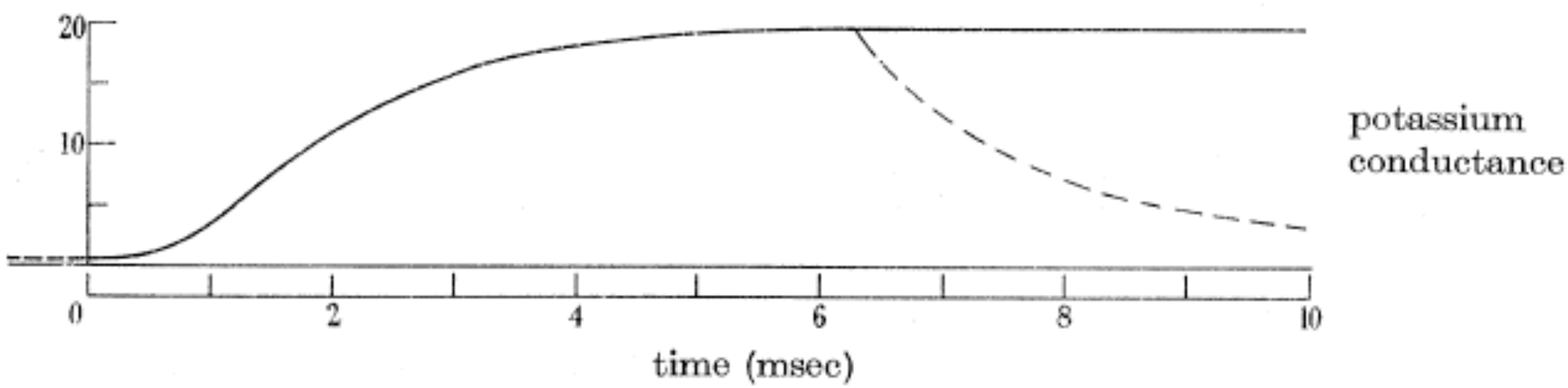


Temporal Dynamics

- *Ansatz:*

$$g_K\left(V,t\right) = \bar{g}_K \, n\left(V,t\right)^4$$

decay: $n\left(t\right) \sim \exp\left(-t/\tau\right)$



Activation Variable: *n(V,t)*

- For a fixed voltage, *V*:

$$\tau_n \frac{dn(t)}{dt} = n_\infty - n(t)$$

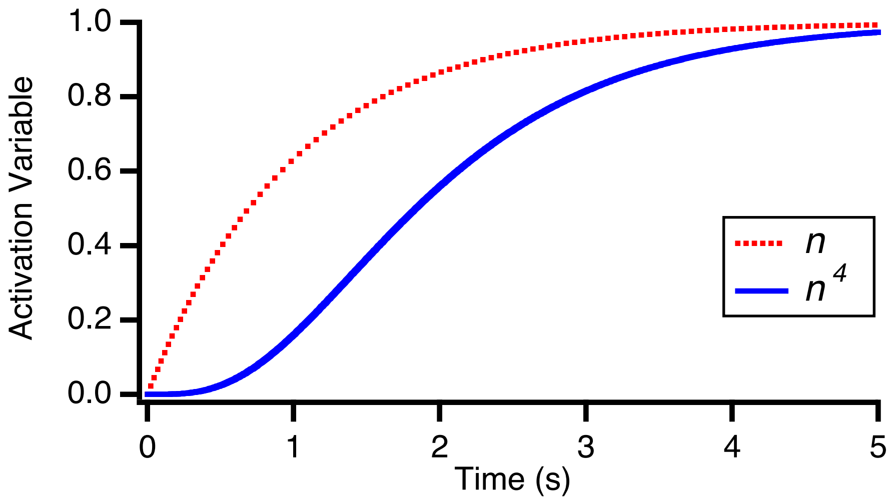
- Change of variables:

$$\eta(t) \equiv n(t) - n_\infty \qquad \tau_n \frac{d\eta(t)}{dt} = -\eta(t)$$

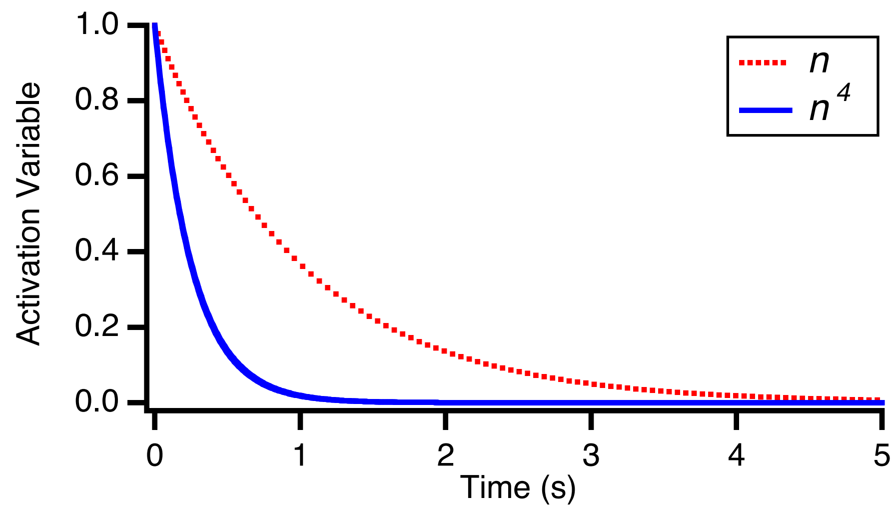
$$T \equiv \frac{t}{\tau_n} \qquad \frac{d\eta(T)}{dT} = -\eta(T)$$

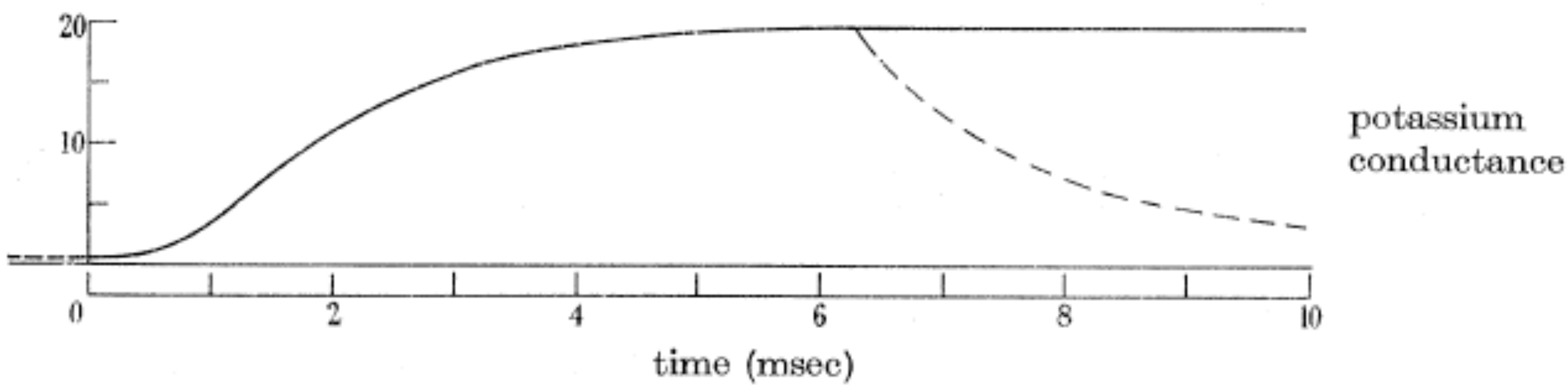
Temporal Dynamics

- *Rise:*



- *Decay:*





Equations for the Active Currents

- Conductances:

$$g_K = \bar{g}_K \, n^4$$

$$g_{Na} = \bar{g}_{Na} \, m^3 \, h$$

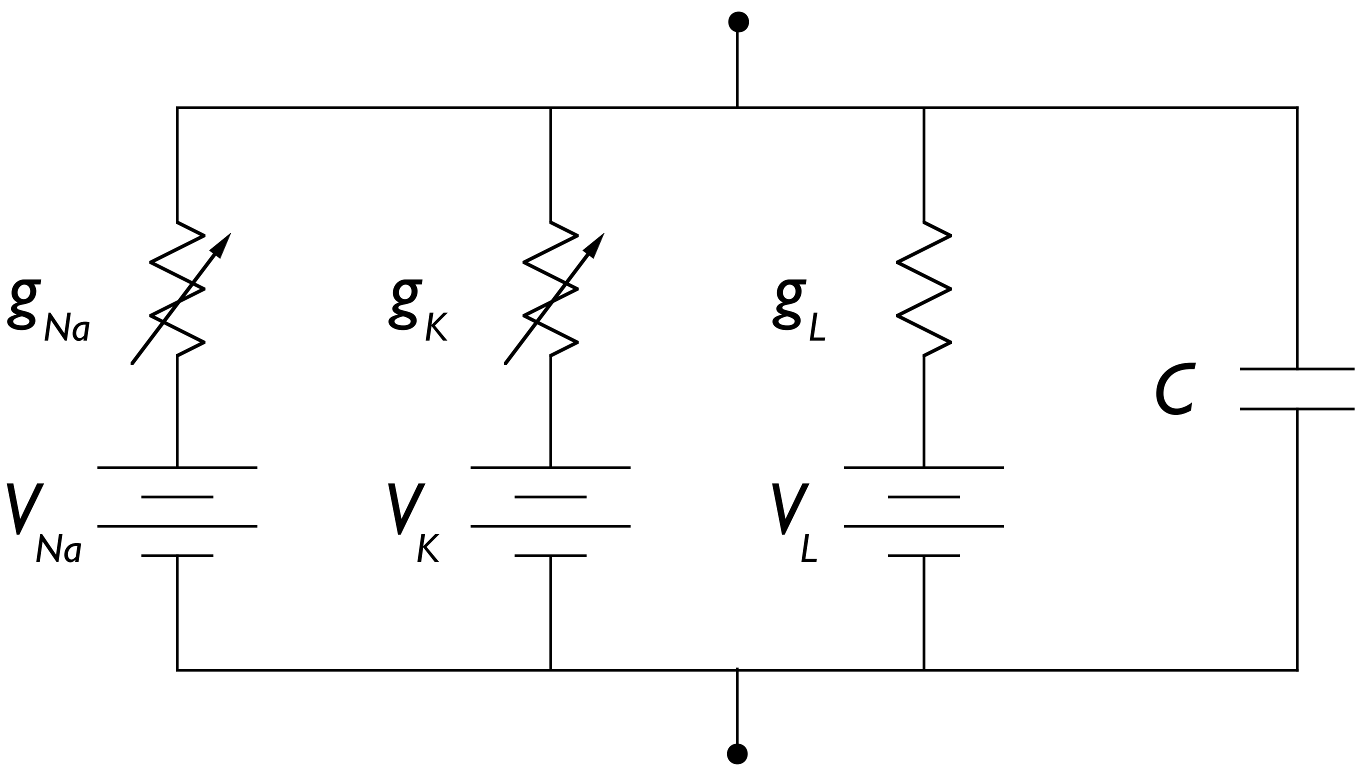
- Currents:

$$I_K = g_K \left(V - V_K\right)$$

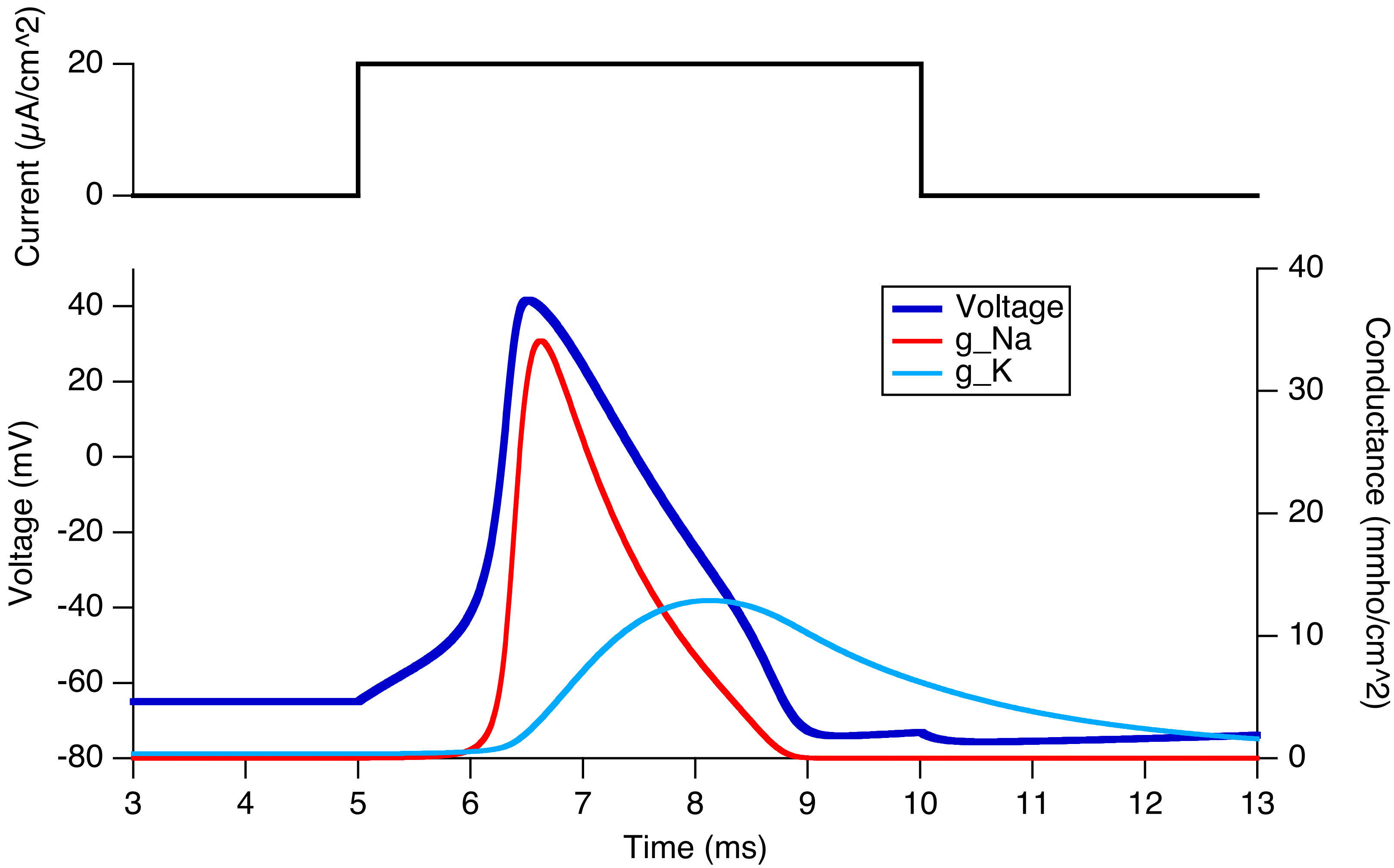
$$I_{Na} = g_{Na} \left(V - V_{Na}\right)$$

Putting All the Channels Together

$$C\frac{dV}{dt} = -\bar{g}_L\left(V - V_L\right) - \bar{g}_K n^4\left(V - V_K\right) - \bar{g}_{Na} m^3 h\left(V - V_{Na}\right) + I_{ext}$$



Results: Single Spike



Results: Activation Variable