A bit about reverse correlation stimulus design and analysis

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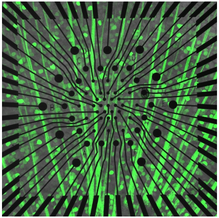
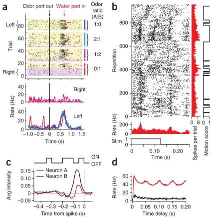
1/27/2016, UCSD

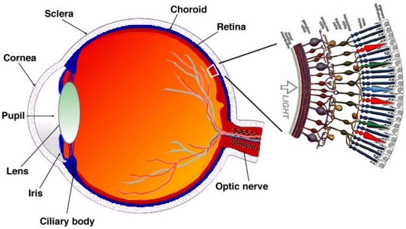



a
Odor port out
Water port in
Odor ratio (A:B)
Left
Right
Trial
1:0
2:1
1:2
0:1
Rate (Hz)
Time (s)
b
Repetition
Rate (Hz)
Stim
Time (s)
Spikes per trial
Motion score
c
ON
OFF
Neuron A
Neuron B
Avg intensity
Time from spike (s)
d
Rate (Hz)
Time delay (s)

Outline:

- Reverse correlation experiments and linear non-linear models
- Why use white noise?
- Why not use white noise?
- Analysis of responses to non-white noise stimuli
  - synthetic model
  - rat thalamus

Reverse correlation experiments:



The model we will construct:

$$\text{response}(t) \quad = \quad f[\text{stimulus}(t' < t), \text{response}(t' < t)]$$

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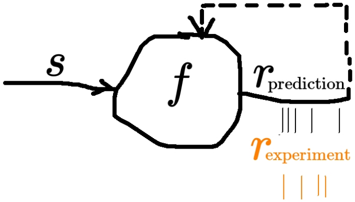

If by knowing f we can predict responses to stimuli that weren't used to fit f we have hope to understand the computation the cell is performing.

Important model assumptions: no explicit time dependence (no adaptation), fixed stimulus statistics

Two steps of constructing model
1. dimensionality reduction

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We want to focus on stimulus statistics, so small number = 1.

The first component is the Spike Triggered Average:

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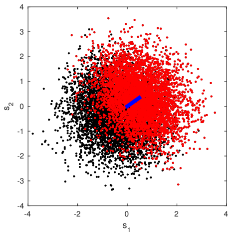
$$\begin{aligned} \varphi_{\text{STA}} &= \left(\frac{1}{N_{\text{spike}}} \sum_{t = t_{\text{spike}}} s(t) \right) - \langle s \rangle \\ r(t) &= f[\varphi_{\text{STA}} \cdot s] \end{aligned}$$



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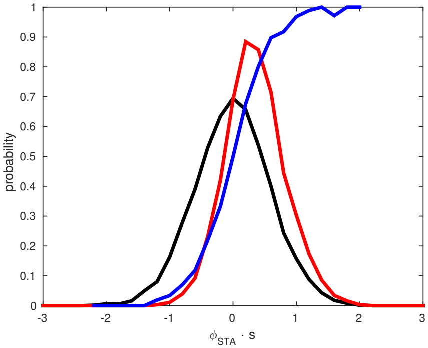

Why use white noise stimuli?

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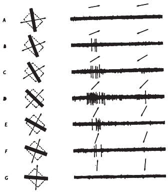
Avoid *putting the answer in*



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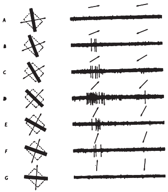

Makes analysis easy

$$\begin{aligned} r(t) &= f[\varphi_{STA} \cdot s] \\ p(\text{spike}|s) &= \frac{p(s|\text{spike})\,p(\text{spike})}{p(s)} \end{aligned}$$

Why not use white noise stimuli?

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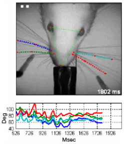
active sensing
(self generated stimulus)



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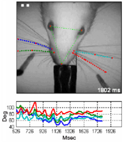
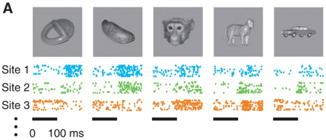


some neurons will never spike



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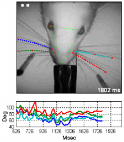

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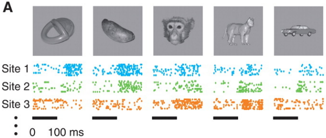
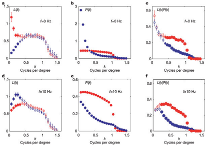
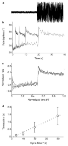
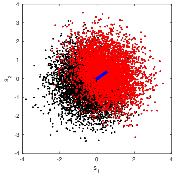


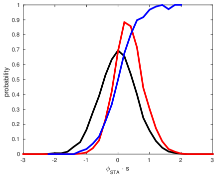
ask questions about
adaptation, optimal processing ...





What is the big deal?





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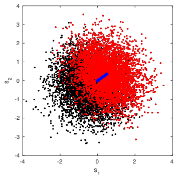
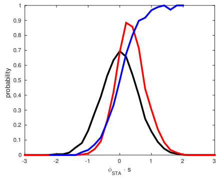
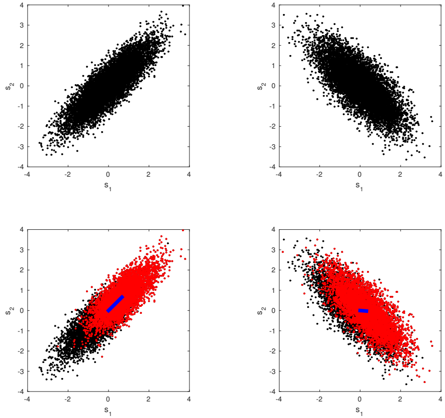


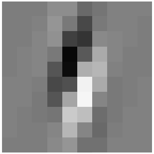


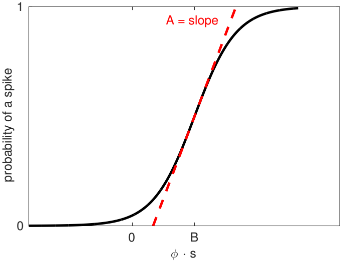


A simple exercise, model

$$p(\text{spike}|s) = \frac{1}{1 + \exp\left[-A(\phi \cdot s - B)\right]}$$

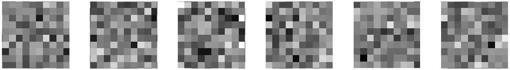
$\phi =$



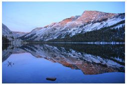


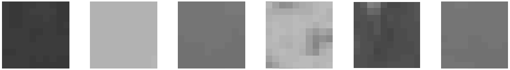
A simple exercise, stimulus

white noise

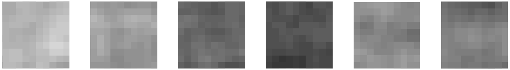


10×10 patches from



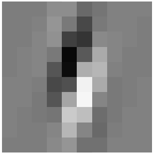


strongly correlated Gaussians

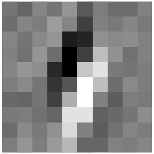


Spike Triggered Average

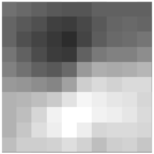
ϕ



STA computed from responses to white noise



STA computed from responses to correlated noise



Decorrelation

Analyze responses with decorrelated stimulus $\tilde{s}$

$$\begin{aligned} C &= \mathrm{cov}(S) = \frac{1}{T}\sum_{t=1}^{T} s(t)s(t)^\top \\ \tilde{s}(t) &= C^{-\frac{1}{2}} s(t) \end{aligned}$$

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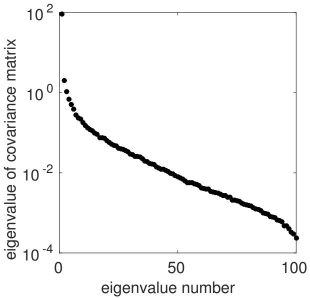
&= \mathbb{I}
\end{aligned}
$$

For finite data, need regularization



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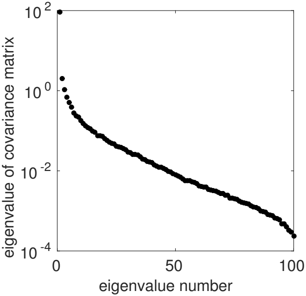
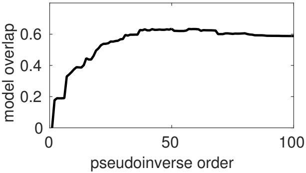
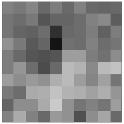
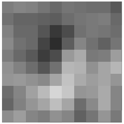




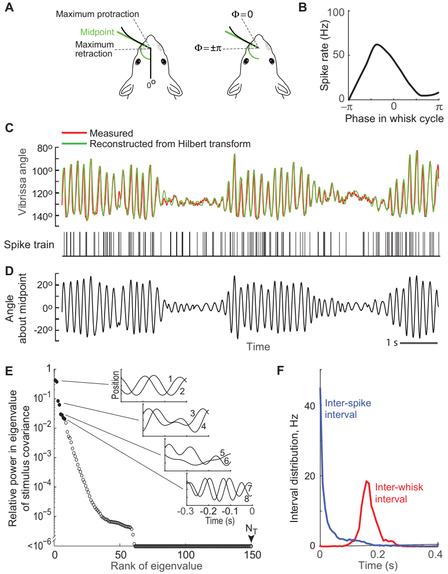


computed from full inverse



computed from pseudoinverse

Rat thalamus data



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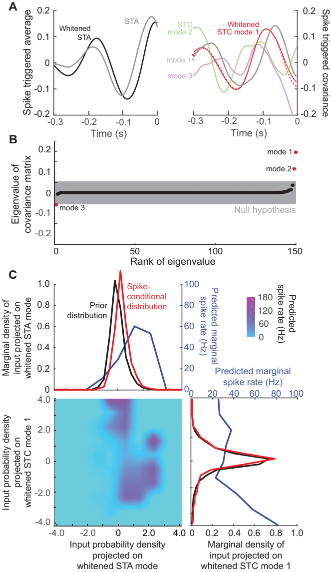

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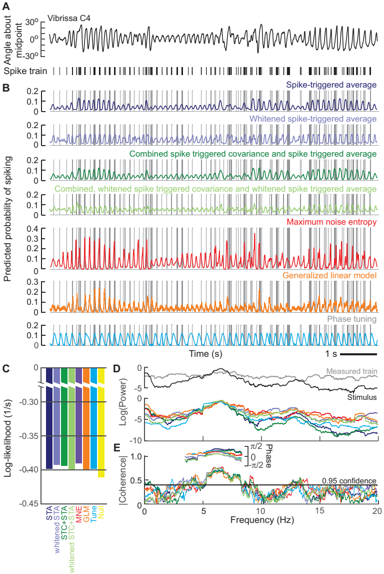

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