Physics 178/278
Assignment 1

January 12, 2025

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Physics seniors and graduate students solve the problem 2,3,4. Everyone else solves problem 1,2,3.

1. (10 pts)

   Use Kirchhoff's law to find the current I_1, I_2, I_3 in terms of resistence and the EMF of the batteries.

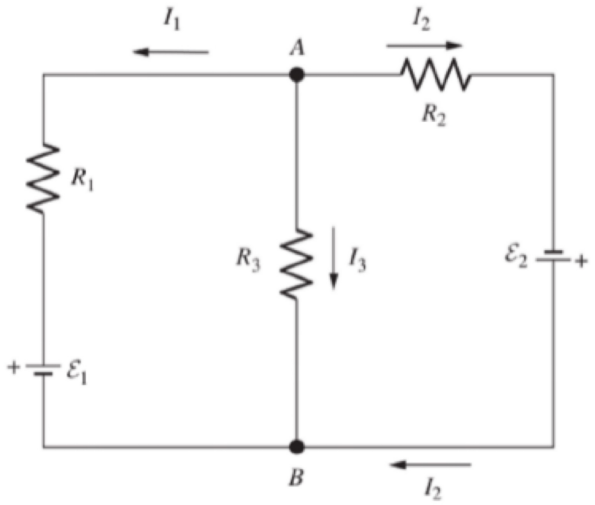

Figure 1: Circuit for problem 1.

2. (10 pts)

   For the following RC circuit, we have the a constant current source I_0, and the switch is closed when $t = 0$. In this problem, you are asked to calculate the voltage of the capacitor as a function of time and other parameters in the circuit.

   (a) RC in parallel. Fig 2 left

   (b) RC in series. Fig 2 Right

3. The Integrate-and-Fire (IF) model is a simplified representation of a spiking neuron, commonly used in computational neuroscience. The model describes how the membrane potential $V(t)$ of a neuron evolves over time in response to an input current $I(t)$.

Figure 2: Circuit for problem 2. Left, RC in parallel. Right, RC in series

Consider a neuron described by the following simple integrate-and-fire model in the figure where the neuron firing can be modeled as a simple RC circuit with current source I and an additional switch s. The switch is turned of when the voltage across the capacitor is below $V_{thresdhold}$ and turned on when the voltage across the capacitor is equal or above $V_{thresdhold}$. In this problem, you are going to model the voltage across the capacitor $V(t)$ as the membrane potential.

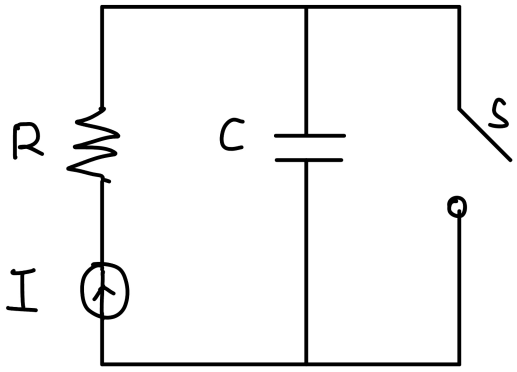

Figure 3: Circuit for problem 3. A simple model of the neuron firing with a RC circuit with an additional switch s.

(a) (2pts) Use the Kirchhoff's current conservation law to construct a differential equation to describe the dynamics of the membrane voltage or voltage of the capacitor (You can ignore the switch at this part)

(b) (3pts) Solve the differential equation for a constant current source $I = I_0$ (You can ignore the switch at this part)

(c) (5pts) Now we add the switch into consideration, plot the time series of the voltage across the capacitor $V(t)$ as a function of time. Do some research on the actual value for the current, resistance and the capacitance and calculate the firing rate of this neuron across different values of current ($1mA$, $5mA$, $10mA$).

4. Similar to problem 2 Find the voltage across the capacitor when the current is time dependent:

$$I(t) = \begin{cases} 0 & \text{if } t \leq 0, \\ I_0 \sin(w_0 t) & \text{if } t > 0. \end{cases}$$

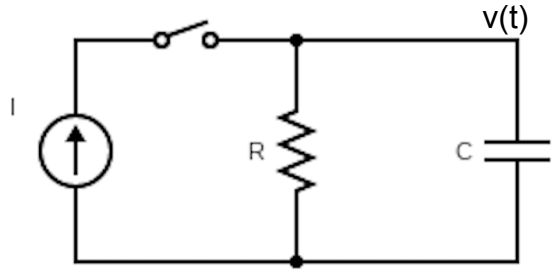

Figure 4: Circuit for problem 4.