Physics 178/278
Assignment 3

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Please upload your homework as a pdf version of the jupyter notebook with code and the running output of the code.

We have a ring of N neurons indexed by $i = 0, 1, \ldots, N-1$. Each neuron i corresponds to an angle

$$\phi_i = \frac{2\pi i}{N},$$

and we denote the firing rate of neuron i at time t by $r_i(t)$. We will simulate the discrete-time dynamics:

$$r_i(t + \Delta t) \;=\; r_i(t) \;+\; \Delta t\Big[- r_i(t) \;+\; \sum_{j=0}^{N-1} W_{i,j}\, r_j(t) \;+\; I_i^{ext}(t)\Big].$$

- The term $-r_i(t)$ models a linear decay/leak of activity.
- $W_{i,j}$ is the synaptic connectivity from neuron j to neuron i.
- $I_i^{ext}(t)$ is an external input to neuron i.

1 Problem description

(1) Define Connectivity

Use a ring-based kernel of the form

$$W(\Delta\phi) = W_0 \;+\; W_1 \cos\big(\Delta\phi\big),$$

where $\Delta\phi$ is the angular difference between neurons. Discretely, set

$$W_{i,j} \;=\; W_0 \;+\; W_1 \cos\Big(\phi_i - \phi_j\Big),$$

(2)External Input

Apply a localized Gaussian input to break the continuous symmetry:

$$I_i^{ext}(t) \;=\; \begin{cases} I_0 \exp\Big(-\frac{(\phi_i - \phi_0)^2}{2\sigma^2}\Big), & \text{for } 0 \le t < T_{\text{in}}, \\ 0, & \text{for } t \ge T_{\text{in}}. \end{cases}$$

This means each neuron i receives a bump of input centered at some angle ϕ_0 for a limited time $0 \leq t < T_{\text{in}}$, after which the input is removed.

2 Task

Make sure to use the following parameters

1. $N = 1000$
2. $W_0 = -1$, $W_1 = 3$
3. $\Delta t = 0.01$
4. $T_{max} = 1000$
5. $T_{input} = 500$
6. $I_0 = 2$
7. $\sigma = 0.5$
8. $\phi_0 = 0$

Question

1. Initialize the rates $r_i(0)$ to small random values (e.g. uniform in $[-0.01, 0.01]$).
2. Construct the weight matrix $W_{i,j}$ based on W_0 and W_1.
3. Simulate Include the Gaussian input. the above update equation for $t = 0$ to T_{max} using a small time step Δt (e.g. 0.01). $I_i^{ext}(t)$ only for $0 \leq t < T_{\text{in}}$, then set $I_i^{ext}(t) = 0$ afterward.
4. Plot/Analyze:
   - Plot $r_i(t)$ at several time slices.
   - Create a heatmap (angle vs. time) to visualize bump formation. (The value at each site in heatmap is the value of the neuron at a particular angle and a particular time)
5. After the external input is turned off, does the bump persist? Or does the activity vanish or diffuse?
6. Try different values of W_1 from 1 to 5 (for example, 1, 1.1, ..., 5). How does changing W_1 affect the stability and width of the bump?

7. (Graduate students only) Explore different combinations of the W_0, W_1 to see if you can get the phase diagram in Figure 20 of the lecture note Chapter 4: Phase diagram for different ranges of synaptic weights.

Figure 20: Phase diagram for different ranges of synaptic weights