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Physics 171/271
Fall 2009

Receptive Field

$$\underset{\substack{\uparrow \\ \text{Prob of spiking} \\ \underline{\text{or}} \\ \text{Instantaneous spike rate}}}{z(t)} = g\left[I_0 + \int d^2r \int_{-\infty}^{t} dt' \underset{\substack{\uparrow \\ \text{Receptive Field}}}{R(\vec{r}, t-t')} \underset{\substack{\uparrow \\ \text{Stimulus}}}{S(\vec{r}, t)} \right]$$

when the stimulus driven part is small compared to the background z_0

$$z(t) = \underbrace{g(I_0)}_{\equiv z_0} + \underbrace{\left.\frac{dg}{dI}\right|_{I=I_0}}_{\equiv g'} \underbrace{\int d^2r \int_{-\infty}^{t} dt' R(\vec{r}, t-t') S(\vec{r}, t)}_{\text{Modulation in firing rate by convolution of stimulus with receptive field}}$$

$$\boxed{\text{Prob of no spikes from } (0,t) \text{ and one spike at time } t \text{ is } e^{-\int_0^t dt' z(t')} \cdot z(t) \text{ for a Poisson process}}$$

$*$ For $R(\vec{r}, t) \equiv \sum_n \lambda_n \underbrace{F_n(\vec{r}) G_n(t)}_{\text{modes of receptive field}}$

$*$ $$z(t) = z_0 + g' \sum_n \lambda_n \int d^2r\, F(\vec{r}) \int_{-\infty}^{t} dt' G_n(t-t') S(\vec{r}, t')$$

For $S(\vec{r}, t') = X(\vec{r})\, \delta(t)$ (In practice rapidly changing stimulus)

$$z(t) = z_0 + g' \sum_n \lambda_n \underbrace{\int d^2r\, F(\vec{r}) X(\vec{r})}_{\text{Rate depends on spatial pattern}}$$

Let us reverse process and ask if we can reconstruct a stimulus from the spike train

For simplicity, let us ignore space. The previous description of receptive field gives

Averaged and smoothed spike train

$$Z(t) = Z_0 + g'\int_{-\infty}^{t} dt'\, R(t-t')\, S(t')$$

$\big\downarrow$ Fourier Transform

$$\tilde{Z}(\omega) = Z_0\,\delta(0) + g'\,\tilde{R}(\omega)\,\tilde{S}(\omega)$$

$$\tilde{S}(\omega) = \frac{\tilde{Z}(\omega) - Z_0\,\delta(0)}{g'\,\tilde{R}(\omega)}$$ not very informative

<u>Optimal Reconstruction</u>

How well can we reconstruct stimulus from a given spike train

Let $\Lambda_\alpha(t) \equiv \sum_{\text{spikes}} \delta(t-t_s)$ spike train for α-trial

predicted

$$S_\alpha(t) = \int_{-\infty}^{t} dt'\, T(t-t')\,\Lambda_\alpha(t)$$

Transfer Function $\left(\tilde{T}(\omega) \propto 1/R(\omega)\,?\right)$

Compare $S_\alpha^{actual}(t)$ vs. $S_\alpha^{predict}(t)$

Integrated Error $\equiv \sum_\alpha \int dt \left[S_\alpha^{actual}(t) - S_\alpha^{predict}(t)\right]^2$

$\big\downarrow$ Fourier Transform (different ω's are uncorrelated

$$\text{Error} = \sum_\alpha \left[\tilde{S}_\alpha^{pred}(\omega) - \tilde{S}_\alpha^{actual}(\omega) \right]^2$$

$$" = \sum_\alpha \left| \tilde{T}(\omega) \tilde{\Lambda}_\alpha(\omega) - \tilde{S}_\alpha^{actual}(\omega) \right|^2$$

$$" = \sum_\alpha \Big[\tilde{T}(\omega) \tilde{T}^*(\omega) \tilde{\Lambda}_\alpha(\omega) \tilde{\Lambda}_\alpha^*(\omega) + \tilde{S}_\alpha^{act}(\omega) \tilde{S}_\alpha^{act*}(\omega) - \tilde{T}(\omega) \tilde{\Lambda}_\alpha(\omega) \tilde{S}_\alpha^{act*}(\omega) - \tilde{T}^*(\omega) \tilde{\Lambda}_\alpha^*(\omega) \tilde{S}_\alpha(\omega) \Big]$$

$$\frac{\partial E}{\partial T^*(\omega)} = \sum_\alpha \left[\tilde{T}(\omega) \tilde{\Lambda}_\alpha(\omega) \tilde{\Lambda}_\alpha^*(\omega) - \tilde{\Lambda}_\alpha^*(\omega) \tilde{S}_\alpha^{act}(\omega) \right]$$

holds for each value of $T^*(\omega)$, so really differentiation w.r.t. a scalar

Let $\frac{\partial E}{\partial T^*(\omega)} = 0 \quad \therefore \tilde{T}(\omega) |\tilde{\Lambda}_\alpha(\omega)|^2 = \tilde{\Lambda}_\alpha^*(\omega) \tilde{S}_\alpha^{act}(\omega)$

$$\therefore T(\omega) = \frac{\sum_\alpha \tilde{\Lambda}_\alpha^*(\omega) \tilde{S}_\alpha^{act}}{\sum_\alpha |\tilde{\Lambda}_\alpha(\omega)|^2}$$

← Cross Power

← Power in Spike Train

And $\tilde{S}_\alpha^{pred}(\omega) = \tilde{T}(\omega) \tilde{\Lambda}_\alpha(\omega)$

↑ Best Filter

Notes on Singular Value Decomposition

$$R(r,t) \equiv \sum_n \lambda_n F_n(r)\, G_n(t)$$

with $\int d^2r\, F_n(r) F_m(r) = \delta_{mn}$ and $\int dt\, G_n(t)\, G_m(t) = \delta_{mn}$ } Orthonormal Functions

Consider contraction to symmetric correlation matrix

$$C(t,t') \equiv \int dr\, R(r,t)\, R(r,t')$$

$$" = \sum_n \sum_m \lambda_n \lambda_m \underbrace{\int dr\, F_n(r) F_m(r)}_{\delta_{nm}}\, G_n(t)\, G_m(t')$$

$$" = \sum_n \lambda_n^2\, G_n(t)\, G_n(t')$$

Then

$$\int dt'\, C(t,t')\, G_m(t') = \sum_n \lambda_n^2 G_n(t) \underbrace{\int dt'\, G_n(t') G_m(t')}_{\delta_{nm}}$$

$$\int dt' \;\; " \;\; G_m(t') = \lambda_n^2\, G_m(t)$$ ↖ Eigenvalue problem

and the $F_n(t)$ are found by

$$\int dt\, R(r,t)\, G_m(t) = \sum_n F_n(r) \underbrace{\int dt\, G_n(t)\, G_m(t)}_{\delta_{nm}}$$

$$" = F_m(t)$$

Bottom line: Space-time modes from measured RF