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Receptive Field

$$Z(t) = g\left[I_0 + \int d^2r \int_{-\infty}^{t} dt'\, R(\vec{r}, t-t')\, S(\vec{r}, t)\right]$$

prob of spiking
or
Instantaneous spike rate

Receptive Field

Stimulus

when the stimulus driven part is small compared to the background Z_0

$$Z(t) = \underbrace{g(I_0)}_{\equiv Z_0} + \underbrace{\left.\frac{dg}{dI}\right|_{I=I_0}}_{\equiv g'} \underbrace{\int d^2r \int_{-\infty}^{t} dt'\, R(\vec{r}, t-t')\, S(\vec{r}, t)}_{\text{Modulation in firing rate by convolution of stimulus with receptive field}}$$

Probability of no spikes in the interval [0, t] and one spike in (t, t+dt] is

$$e^{-\int_0^t dt' Z(t')} \cdot Z(t)$$

for a Poisson process

* For $R(\vec{r}, t) \equiv \sum_n \lambda_n \underbrace{F_n(\vec{r})\, G_n(t)}_{\text{modes of receptive field}}$

* $$Z(t) = Z_0 + g' \sum_n \lambda_n \int d^2r\, F(\vec{r}) \int_{-\infty}^{t} dt'\, G_n(t-t')\, S(\vec{r}, t')$$

For $S(\vec{r}, t') = X(\vec{r})\, \delta(t)$ (In practice, rapidly changing stimulus

$$Z(t) = Z_0 + g' \sum_n \lambda_n G_n(t) \underbrace{\int d^2r\, F(\vec{r})\, X(\vec{r})}_{\text{Rate depends on spatial pattern}}$$

Let us reverse process and ask if we can reconstruct a stimulus from the spike train

For simplicity, let us ignore space. The previous description of receptive field gives

Averaged and smoothed spike train $\rightarrow$ $$Z(t) = Z_0 + g' \int_{-\infty}^{t} dt'\, R(t-t')\, S(t')$$

$\downarrow$ Fourier Transform

$$\tilde{Z}(\omega) = Z_0\, \delta(0) + g'\, \tilde{R}(\omega)\, \tilde{S}(\omega)$$

$$\tilde{S}(\omega) = \frac{\tilde{Z}(\omega) - Z_0\, \delta(0)}{g'\, \tilde{R}(\omega)}$$ not very informative

Optimal Reconstruction

How well can we reconstruct stimulus from a given spike train

Let $\Lambda_\alpha(t) \equiv \sum_{\text{spikes}} \delta(t - t_s)$ spike train for α-trial

predicted $\rightarrow$ $$S_\alpha(t) = \int_{-\infty}^{t} dt'\, T(t-t')\, \Lambda_\alpha(t)$$

$\uparrow$ Transfer Function $\left(\tilde{T}(\omega) \propto 1/R(\omega)\, ? \right)$

Compare $S_\alpha^{\text{actual}}(t)$ vs. $S_\alpha^{\text{predict}}(t)$

Integrated Error $$\equiv \sum_\alpha \int dt \left[S_\alpha^{\text{actual}}(t) - S_\alpha^{\text{predict}}(t) \right]^2$$

$\downarrow$ Fourier Transform (different ω's are uncorrelated)

$$\text{Error} = \sum_\alpha \left[\tilde{S}_\alpha^{pred}(\omega) - \tilde{S}_\alpha^{actual}(\omega)\right]^2$$

$$'' = \sum_\alpha \left|\tilde{T}(\omega)\tilde{\Lambda}_\alpha(\omega) - \tilde{S}_\alpha^{actual}(\omega)\right|^2$$

$$'' = \sum_\alpha \Big[\tilde{T}(\omega)\tilde{T}^*(\omega)\tilde{\Lambda}_\alpha(\omega)\tilde{\Lambda}_\alpha^*(\omega) + \tilde{S}_\alpha^{act}(\omega)\tilde{S}_\alpha^{act*}(\omega) - \tilde{T}(\omega)\tilde{\Lambda}_\alpha(\omega)\tilde{S}_\alpha^{act*}(\omega) - \tilde{T}^*(\omega)\tilde{\Lambda}_\alpha^*(\omega)\tilde{S}_\alpha(\omega)\Big]$$

$$\frac{\partial E}{\partial T^*(\omega)} = \sum_\alpha \left[\tilde{T}(\omega)\tilde{\Lambda}_\alpha(\omega)\tilde{\Lambda}_\alpha^*(\omega) - \tilde{\Lambda}_\alpha^*(\omega)\tilde{S}_\alpha^{act}(\omega)\right]$$

holds for each value of $T^*(\omega)$, so really differentiation w.r.t. a scalar

Let $\frac{\partial E}{\partial T^*(\omega)} = 0 \quad \therefore \tilde{T}(\omega)\left|\tilde{\Lambda}_\alpha(\omega)\right|^2 = \tilde{\Lambda}_\alpha^*(\omega)\tilde{S}_\alpha^{exp}(\omega)$

$$\therefore T(\omega) = \frac{\sum_\alpha \tilde{\Lambda}_\alpha^*(\omega)\tilde{S}_\alpha^{exp}}{\sum_\alpha \left|\tilde{\Lambda}_\alpha(\omega)\right|^2}$$

← Cross Power

← Power in Spike Train

And $\tilde{S}_\alpha^{pred}(\omega) = \tilde{T}(\omega)\tilde{\Lambda}_\alpha(\omega)$

↑ Best Filter

The transfwr function is just the corss-corrletion, divided by the autocorrelation to correct for trends of correlation aming the neuronal response $\Lambda(\omega)$.

We shall see next how the filter has a simple form - and reduces to the expression for the reverse correlstion or spike-triggered average, when the correlation between spikes is zero. This holds for low spike rates; at high rates the refractory period leads to correlations in the rat.e

How do we interpret the result for the optimal filter? Look at the case of $\Lambda(\omega)$ for a spike train

First - the numerator is just the cross power, or the Fourier Transform of the cross-correlation, i.e.

$$\sum_\sigma \tilde{\Lambda}^*_\sigma(\omega)\tilde{S}_\sigma(\omega) = \sum_\sigma \int_{-\infty}^{\infty} d\tau e^{-i\omega\tau} \int dt \Lambda_\sigma(t) S_\sigma(t-\tau)$$

But $\sum_\sigma \Lambda(t) \rightarrow \sum_{\text{spike times}}^{N} \delta(t-t_s)$

$$\therefore \sum_\sigma \tilde{\Lambda}^*_\sigma(\omega)\tilde{S}_\sigma(\omega) = \int_{-\infty}^{\infty} d\tau e^{-i\omega\tau} \sum_{\text{spike times}}^{N} \int dt \delta(t-t_s) S(t-\tau)$$

$$'' = \int_{-\infty}^{\infty} d\tau e^{-i\omega\tau} \underbrace{\sum_{\text{spike times}}^{N} S(t_s-\tau)}_{\text{Spike Trigger Stimulus Avg.}}$$

Second - the denominator is just the power, or the Fourier transform of the auto-correlation

$$\sum_\sigma |\tilde{\Lambda}_\sigma(\omega)|^2 = \int_{-\infty}^{+\infty} d\tau e^{i\omega\tau} \underbrace{\sum_{\text{spike times}}^{N} \underbrace{\int dt \delta(t-t_s)\delta(t-t_s+\tau)}_{\delta(t_s-t_s+\tau)=\delta(\tau)}}_{N\delta(\tau)}$$

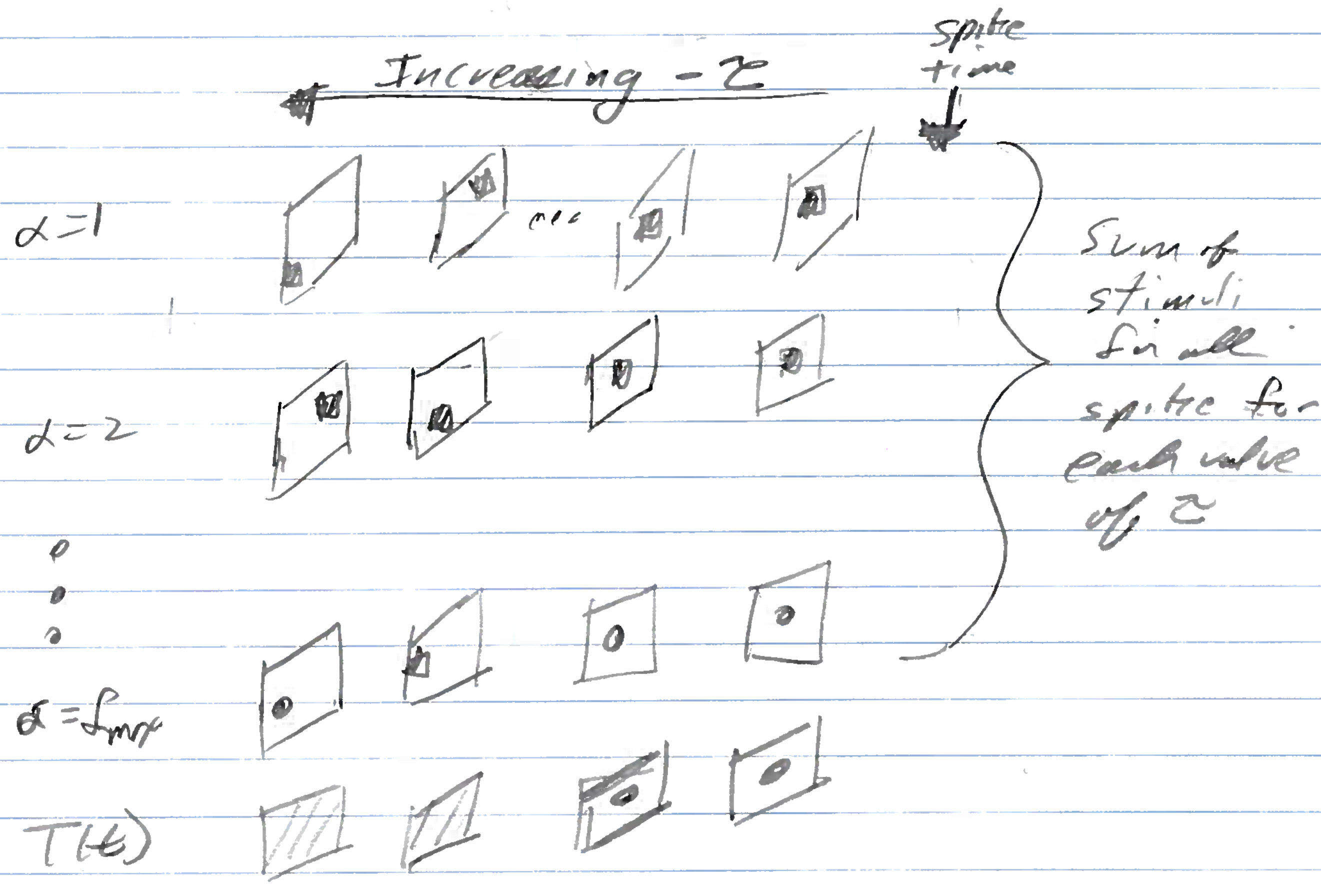
$$'' = N$$

∴ We can write that for the case of uncorrelated spikes

$$T(\tau) = \int_{\infty}^{-\infty} d\tau\, e^{-i\omega\tau}\, \tilde{T}(\omega) = \frac{1}{N}\int_{-\infty}^{\infty} d\tau\, e^{-i\omega\tau} \sum_{\alpha} \tilde{R}_{\alpha}(\omega)\tilde{S}_{\alpha}(\omega)$$

$$\text{"} = \underbrace{\frac{1}{N}\sum_{\substack{\text{spike}\\\text{time}}} S(t_s - \tau)}_{\text{Reverse Correlation Formula.}}$$

This formula is very easy to apply. Wait for a spike, sum up stimulus at time τ is the past, for all



Notes on Singular Value Decomposition

$$R(r,t) \equiv \sum_n \lambda_n F_n(r)\, G_n(t)$$

with $\int d^2r\, F_n(r)\, F_m(r) = \delta_{mn}$ } Orthonormal Functions

and $\int dt\, G_n(t)\, G_m(t) = \delta_{mn}$

Consider contraction to symmetric correlation matrix

$$C(t,t') \equiv \int dr\, R(r,t)\, R(r,t')$$

$$'' \quad = \sum_n \sum_m \lambda_n \lambda_m \underbrace{\int dr\, F_n(r)\, F_m(r)}_{\delta_{nm}}\, G_n(t)\, G_m(t')$$

$$'' \quad = \sum_n \lambda_n^2\, G_n(t)\, G_n(t')$$

Then $\int dt'\, C(t,t')\, G_m(t') = \sum_n \lambda_n^2\, G_n(t) \underbrace{\int dt'\, G_n(t')\, G_m(t')}_{\delta_{nm}}$

$$\int dt' \; '' \; '' \quad = \lambda_n^2\, G_m(t)$$ ↖ Eigenvalue Problem

and the $F_n(t)$ are found by

$$\int dt\, R(r,t)\, G_m(t) = \sum_n F_n(r) \underbrace{\int dt\, G_n(t)\, G_m(t)}_{\delta_{nm}}$$

$$'' \quad = F_m(t)$$

Bottom line; Space-time modes from measured RF