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Modal Analysis of Percussion Instrum

Using Vibrational Holography

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- **Abstract**
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■ Many musical instruments, including numerous percussion instru
sound produced from vibrating plates. With vibrational holograph
gain very accurate information about the patterns of vibration (m
Time-average vibrational holography uses two beams of a cohere
on a high-resolution holography film. One beam with a fixed path
reference beam, while the object beam is incident on the vibrating
vibrating plate changes the path length of the object beam to crea
about the antinodes of vibration. We studied systems of increasin
starting with a simple case of a uniform circular plate. We then co
metal plates, such as specialty cymbals, and hand bells. In real-tim
holography, it is necessary to produce a plate hologram of the no
We develop the plate and place it back in the holder, superimposi
the real object. If we vibrate the object, there will be interference
image of the hologram, and the vibrating object. We use a beam c
slow down the interference shifts throughout the period of the vib
technique to study modal patterns in a cymbal.

- **Introduction**

All sound can be thought of as a superposition of pure sine waves. We can represent this o
displacement versus time on a graph which, because of simple harmonic motion, will resu
When an object vibrates at multiple frequencies, its representative wave becomes no longe
complex wave, not easily describable with simple trigonometry. We know that this wave c
broken up into several sine waves with distinct frequencies using Fourier transformation. I
an object vibrates at those distinct frequencies, known as resonance or modes, we can pred
will create a sound and what sound it will create.

Ernst Chladni was the first to study these modes in 1787. He used flat plates, now known
study the vibrational behavior of flat surfaces. The classic plate is of a uniform thickness t
understanding of the natural modes of vibration. It is difficult to analyze modes of vibratio
eye, the plate does not appear to move. To study this, we need to drive the plate at natural
wave frequencies. Chladni did this by bowing the plates, and we are able to do it with a fu
a small-massed driving magnet and electromagnetic drive coil. Certain modes, dependent
the metal, the shape, and the thickness, will resonate with specific frequencies. A circular
vibrate with nodal diameters and nodal circles. An elementary way to study this is to use s
and drive the plate at its resonant frequencies. The shavings will move to the lines on the p
vibrate, called nodal lines. The result is a plate with sand lines at the nodes of vibration of
gives us a good idea of the manner of vibration. This technique relies on the assumption th
massless and moves with little force at all. It is also limited by the fact that the surface mu

2

horizontal and relatively flat. Unfortunately this is rarely the case.

A much more sensitive and versatile way to understand the production of sound is by usin holography. The technique of vibrational holography applies what we know of optics and to study the vibrational modes of vibrating surfaces. Vibrational holography, or holograph uses laser superposition to either constructively or destructively interfere a reference and o where the previously mentioned nodal or antinodal areas of the plate vibrate.

As with any study using wave superposition, it is necessary to have a single coherent wave interference. Please refer to Figure 1. One He-Ne laser is used and split into two beams wi reflective mirror. One beam, the object beam, travels along a path to the object. The objec given frequency by a contact magnet. The laser is reflected off of the object to an emulsifi the beam splitter, a second laser beam, known as the reference beam, follows a different p emulsion of the holographic plates, there will be an interference pattern on the plate that is different path lengths.

If our object is not vibrating, there will be interference at many different levels due to the our subject. The interference from the path differences creates a complex diffraction gratin we develop our plate, the areas of constructive interference become spots on the film. Whe view the image, we will diffract the reference beam to form a virtual image of our object f complex diffraction pattern.

When an object is vibrating in a sinusoidal manner, it spends most of its time at the point o displacement. The interference pattern created when the vibrating object is at the positive maximum displacement will emphasize certain areas in the grid of the points of interferen create our visualization of the vibration. While the plate is in between the positive and neg result on our plate is *averaged out* in the time average and, therefore, does not embellish o object vibrates, the path length will be changed from positive amplitude to negative ampli emulsified glass plate, the two wave fronts interfere depending on how much that particula vibrating surface moved. Because the laser wavelength is on the order of 632.8 nm (6.328 surface must only move about one half of a wavelength to change from constructive interf *dark fringe*. Because of the small wavelength produced by the laser, spatial resolution is e amplitudes. These fringes get closer together when the path difference increases and a bull created where the antinodes are located, surrounded by a nodal circle. We can study these functions to help us describe the vibration of the object.

- **Theory**

As an introduction, we can look at the behavior of a single exposure hologram, reconstruc field of the following:

We can show the ***E*** field incident on a holographic plate is simply the function of a vector fields (the reference beam and the object beam).

$$E_{ref} = re^{i\omega t}e^{i\frac{2\pi}{\lambda}x\,Sin(\alpha)}$$

$$E_{subj} = se^{i\omega t}e^{i\frac{2\pi}{\lambda}x\,Sin(\beta)}$$

As in most cases, what is recorded is the irradiance of the total field. We can show:

3

$$I = (E_{subj} + E_{ref})(E^{*}_{subj} + E^{*}_{ref})$$

$$I = r^{2} + s^{2} + E_{subj}E^{*}_{ref} + E_{ref}E^{*}_{subj}$$

$$i(\frac{2\pi}{}xSin(-i(\frac{2\pi}{}xSin(\beta)) \qquad -i(\frac{2\pi}{}xS$$

This irradiance is recorded on the film as a light and dark pattern (a diffraction pattern if y reilluminate the film with the reference beam, the transmitted electric field will have an el attenuated (albeit nonuniformly) by the pattern on the film.

~~Now we~~ can show through a series of calculations, that the resultant electric field created w reilluminate the holography plate with just the reference beam:

$$E_{resultant} = (r^{2} + s^{2})E_{ref} + r^{2}se^{i(\omega t + \frac{2\pi}{\lambda}xSin(\alpha))} + r^{2}se^{i(2\frac{2\pi}{\lambda}xSin(\beta))}$$

The first term of this is actually the reference beam, modulated in amplitude but not phase incident beam passes through the film unchanged. The second term describes the electric f come from the object. The third term actually is the **E** field due to a *real image* of the subj important to our data.

The phase of this second term is due to the term $xSin(\alpha)$. This phase is for only a singly e we look at a moving source, we must take into consideration the optical path length differe can be explained with respect to the unit vector $\boldsymbol{n_1}$, representing the direction of illuminati $\boldsymbol{n_2}$ the direction of recording (or the direction to the film), and $\boldsymbol{n_m}$, the direction of vibratio

$$\Delta = An_{m} \cdot (n_{1} + n_{2})$$

A is the displacement of vibration at any particular point on our object. If we change this a by multiplying it by $\sin(\omega t)$, we get an $\boldsymbol{E}$ field as follows:

$$E \qquad = \quad E\ e^{i(\frac{2\pi}{\lambda}An_{i}}$$

where E_0 is a function varying with time as:

$$E_{0} = r^{2}se$$

If we take the time average of this function, we get the integral:

$$\langle E_{image} \rangle = E_{0}\frac{1}{T}\int^{T} e^{i(\frac{2\pi}{\lambda}An_{m}}$$

According to Brown, Grant, and Stroke, this image has an intensity that goes like a first or

$$I = |E_0|^2 \left| J_0\left(\frac{2\pi}{\lambda} A m_m \cdot (n_1 + n_2)\right) \right|^2$$

This equation is the intensity of a single point on our object, but can be used to describe al object with some creative thinking. If we think of every point on our object as a single sou sum over all points to reconstruct the object.

This technique of time-average holography has been used at length studying the productio instruments such as classical and acoustic guitar, percussion instruments such as snare dru steel drums.

Vibrational interferometry research at Stetson University was initiated in 1996 with Frank research under Dr. Kevin Rigg's mentoring on *Modal Analysis of Musical Instruments Usi Holography*. Stetson now has an active acoustical holography lab that is ready for in depth summer of 1996the lab was used to analyze the modes of steel drums and guitar bodies. T for research on a subject that has not previously been investigated anywhere.

In this case (1997), we will be examining the complex plates found in Turkish and Chines These specimens, specifics to be named later, have the more complex variable thickness a and center bells. These will produce more complex modes of vibration, changing and bend of the cymbal. The Turkish cymbal, for example, tends to be of a thicker brass, which mea amplitude of the vibrations will be less. In comparison, the Chinese cymbals have more of the edge of the cymbal the plate folds up at an angle.

There is also a technique using these interference patterns called *real-time interferometry.* exposure of the object that is not vibrating and develop the plate, we can replace it in the e gravity plate holder using ball bearings and set-pins. This plate holder restricts all six degr that we can superpose the hologram and the object. When we set the Chladni plate in vibra pattern appears on the object and, therefore, is called *live-fringe*. We will be using the lab nondestructive testing also. We will use video and photography to record this.

We will be using photography and emulsified glass plates as a record medium for our find Chladni plates, and define each of the resonating frequencies into the accepted form of (m diameters, and n= nodal circles. There are bound to be some perturbations from the classic to the complex nature of the system. Though this is too complex to solve directly, we can physics and solve it numerically.

- *G_3 Hand Bell-Amplitude Analysis*

We can learn about the amplitude analysis ability of vibrational holography. We can look the hand bell at two different amplitudes (Fig 1 –2):

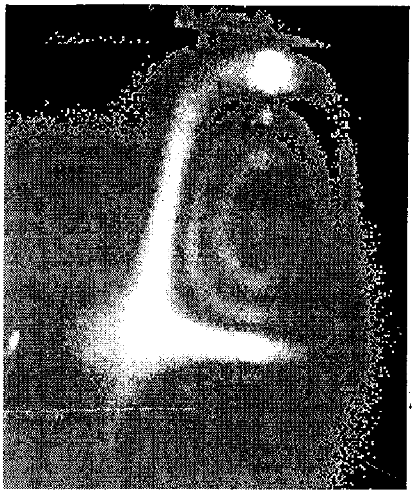

Figure 1-(3,1) mode being driven at -60.2 dB (relative)

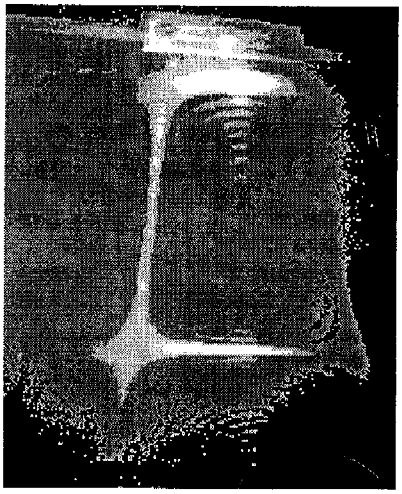

Figure 2-(3,1) mode driven at -50.0 dB (relative)

If you notice, the nodal lines and circles are in the same places, letting us know it is the sa
be confirmed by looking at the holography log and noting that they were both done when
driven at 1342.3 Hz. The driving amplitudes are measured with a near-field microphone an
decibels. The dB scale is a logarithmic scale. We can describe the *Sound Intensity Level* (o
defined as:

$$SIL = 10Log(\frac{I}{I_0})$$

where *I* is intensity, which is proportional to displacement squared.

If we look at comparing two levels, we can show:

$$SIL_1 = 10Log(\frac{I_1}{I_0})$$

$$SIL_2 = 10Log(\frac{I_2}{I_0})$$

$$SIL_2 - SIL_1 = 10Log\left(\frac{I_2}{I_0}\right) - 10Log\left(\frac{I_1}{I_0}\right) = 10\left(Log(\frac{\frac{I_2}{I_0}}{\frac{I_1}{I_0}})\right) = 10Log\left(\frac{I_2}{I_1}\right.$$

$$SIL_2 - SIL_1 = 10Log\left(\left(\frac{d_2}{d_1}\right)^2\right) = 20Log\left(\frac{d_2}{d_1}\right)$$

If we look at our specific case, we can calculate the ratio of d_2/d_1 is 3.16. If we assume tha
function of interference, we should be able to count up the number of fringes and take the
Fig.15/Fig.14. When we do this, we get 3, which compares very well. The difference is du
assumption of linear functionality, and the fact that when we count the fringes, this is a qu
which would not give us a fraction as a ratio. Actually the fringes correspond to zeros of B
Our calculation is rough approximation is hopeful for a future paper that studies more accu
dependency of vibrational holograms.

- **Conclusion**

There are many applications for this technique in industry. The method of vibrational holo be used to study stress in turbines among other things. In our particular laboratory, we spe dedicated to percussion instruments, particularly of the cymbal family. These metal subjec stated, broken up into two families: those that are clamped at the edges, and those free to v The cymbals were the ones that were free to vibrate at the edges. These were successfully be modeled by a uniform circular plate. Even a hand bell could conform to this model. Th thickness, and contour of the actual cymbal were relatively negligible, until we studied the Mini-Chinese® Cymbal, and the Paiste Alpha Splash® Cymbal are when the perturbations non-uniformity of the cymbal were too great to conform to the uniform plate model. Thou should have been modeled by the square plate clamped at the edges, the modes of this wer This was no doubt due to the fact that the notes on a steel drum are not square, but if we m a regular shape, it would be that of a trapezoid. The thickness variations due to the hamme also play an important role in the normal modes of vibration.

There were several problems with the execution of this technique that need to be dealt wit first is the topic of coherence length. Unfortunately, the He-Ne laser used in this experime length that was only 20-30 cm. The attention paid to the coherence length and its relation difference however should have been greater. This lack of attention leads to a contrast in t was less than that achievable. In the future, there should be more a greater amount of time the contrast in the holograms but matching the path lengths. This lack of contrast lead to th the steel drum results from this paper.

The second of our problems had to do with the issue of photographing our holograms. Init film close to our subject so that we could collect as much light reflected off the vibrating s holography film as possible. We were still able to evaluate each subject's modal attributes the parallax phenomena available in the hologram. Unfortunately however, the camera doe to the parallax of the hologram, and therefore in some of the subjects, our publishable data photographs) turned out to be simply unpublishable. We need to take this into consideratio

In the vein of the photographs, one of the most difficult things in post-research processing quality photographs of the holograms. Though the best results quickly came from high spe i.e. exposure times of $3.125*10^{-4}$ s), the nature of this film is there are large crystals of silv film to collect as much light as possible. The high resolution achieved be the holographic drastically reduced in this process. Theoretically, we should be able to use *much* slower fil photography is also done in a light vacant room, similarly to the exposure of the holograph be too difficult to spend a little more time at perfecting this portion of the research to attai publishable data.

Some of the records or subject modes were damaged in developing the actual holography to remember that the emulsion is not fixed to the plastic (or glass) backing. In developing must remember this fact and not damage the emulsification of the holography film.

As a whole, we were able to successfully perform modal analysis on our subjects. We circ would have prevented us from using this highly sensitive method of vibrational study. Tho preliminary "rough" fit of amplitude analysis, a future endeavor is logically to study the ra amplitude analysis on our subjects. The work involved is simply to further research of the equation involving Bessel functions

$$I = |E_0|^2 \left| J_0\left(\frac{2\pi}{\lambda} Am_{m} \cdot (n_1 + n_2)\right) \right|$$

The amplitude information falls out of this equation. The data is taken and there is no labo necessary for this, though better contrast holograms and better photographs would be reco

7

Future projects from this are numerous. In the summer of 1998, research is planned for ins of TV holography, a method of real time Interferometry *directly* incident on a camera lens wave-recombiner. TV Holography is a much more efficient and will prove to be very usef