

Introduction to Holography

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Holography is treated in a manner suitable for inclusion in an undergraduate optics course. The student is assumed to understand the elementary treatments of interference and diffraction.

HOLOGRAPHY, or wavefront reconstruction,¹ is a process by which the amplitude and phase variation across a wavefront may be recorded and subsequently reproduced. The reproduced (or reconstructed) wavefront will be identical in form to the original wavefront and, upon entering the eye or any other optical instrument, will produce the same effects as would have been produced by the original wavefront.

Let us first consider as an example the most easily understood method of holography. From the left, in Fig. 1, a collimated beam of monochromatic light falls upon a small angle prism P and an object O . The collimated beam is coherent across its width, having been produced either by a pinhole at the focal point of a lens or by a laser. The object is a lantern slide (or photographic transparency) which by diffraction, scattering, or thickness variation disturbs the wavefront passing through it. The prism P deviates the light by a few degrees but does not disturb its collimation. To the right at H , the light deviated by the prism and the light passing directly through the object have their maximum overlap. A fine-grained photographic plate is placed at H , exposed, and then developed. This developed photographic plate is called a hologram. Unless the object was very simple, there will be no recognizable similarity between the hologram and its object.

In Fig. 2 we show how this hologram may be reconstructed. The same beam of collimated, monochromatic light is incident upon the hologram. The hologram, acting somewhat like a diffraction grating, allows much of the light to pass through into the zero order but diffracts

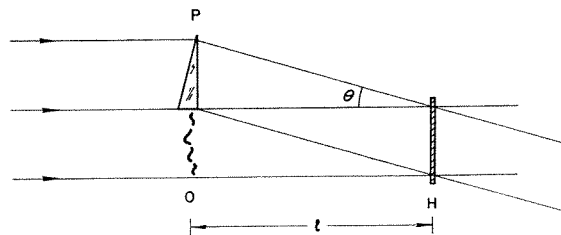


FIG. 1. A simple experimental arrangement for producing a hologram. Collimated monochromatic light from the left falls on the object O and the prism P which deviates the beam by the angle θ . The two beams overlap and produce interference at the photographic plate H .

some of it into the first order on each side. Near the hologram these three beams (or orders) overlap but separate as they propagate to the right. The upper order is a reproduction of the wavefront which fell upon the hologram from the object; an eye placed in this beam will, upon looking through the hologram, see an image or reproduction of the object. With the particular arrangement used here, the lower order forms a wavefront similar to that in the upper order except that the phase variations are of opposite sign. This wavefront therefore converges upon

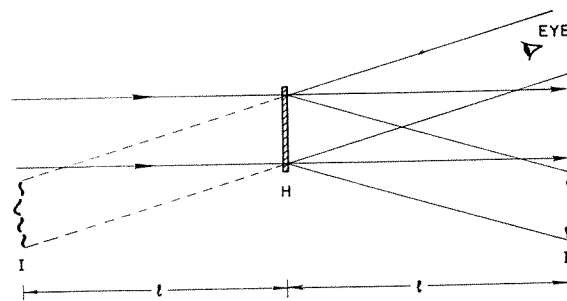


FIG. 2. Reconstruction using the hologram H produced in Fig. 1. The photographic plate after development is placed in a collimated beam. A virtual image of the original object appears at I . A real image is formed at I' and is the mirror image of I in the plane of H .

¹D. Gabor, Proc. Roy. Soc. (London) A197, 454 (1949).

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I' and forms there a real image having the orientation and location of the mirror image of I reflected in the plane of the hologram. These few paragraphs have been a presentation of experimental observations; their explanation is the subject of the following discussion.

First of all, let us remember that if monochromatic, coherent light enters the eye or any other optical instrument, the only information furnished to the eye is the amplitude and phase of the light as a function of position in the entrance aperture. If this information is given, then the light is completely specified and its behavior as it passes on into the instrument is determined. Furthermore, diffraction calculations show us that if the phase and amplitude are known for every point on a surface through which the light passes then its phase and amplitude distribution over any subsequent surface are determined and may be calculated. If we are required to simulate or reconstruct some wavefront, it will be sufficient, therefore, to reproduce the phase and amplitude distribution over any surface through which the original light wave passed. The reconstructed wavefront will thereafter propagate through free space and through optical instruments just like the original wavefront.

Let the hologram of Fig. 2 be replaced by a horizontally lined grating for which the transmitted amplitude varies according to the expression

$$A_T = a + b \cos q y, \quad (1)$$

where A_T is the transmitted amplitude, y is the vertical coordinate in the plane of the grating, $2\pi/q$ is the grating space (or period of the grating), and a and b are constants which determine the average transmission and the contrast of the grating. We require that $b < a$. This grating will put into the zero order light of amplitude a and into the first order on each side amplitude of $\frac{1}{2}b$, since the amplitude in any order of a grating is proportional to the coefficient of the corresponding harmonic term in the Fourier analysis of the amplitude transmission function of the grating. Also, it is clear that if $b = 0$ no light goes into the side orders since the grating no longer exists, but has become a uniform (i.e., unlined) screen. The important point is that the

amplitude diffracted into each of the two first orders is proportional to b .

A second point to remember is that a vertical displacement of this grating will alter the phase of the light in the two side orders. A downward displacement of the grating retards the phase of the upper beam and advances the phase of the lower beam. A displacement of one grating space causes a phase change of 2π , the equivalent of one wavelength in path, so phase is a sensitive function of the vertical position of the grating.

Return now to Fig. 1 and consider an object which contains no detail but uniformly attenuates the wave passing through it. The light falling on the photographic plate H consists of two beams, the reference beam which passes through the prism and falls upon the plate at angle θ and the beam through the object which falls normally upon the plate. If a_r represents the amplitude of the reference beam and a_o the amplitude of the object beam, then interference fringes will be formed on the plate described by the equation

$$I(y) = a_r^2 + a_o^2 + 2a_r a_o \cos [y(2\pi/\lambda) \sin \theta - \phi]. \quad (2)$$

In this equation $I(y)$ is the intensity at the plate as a function of the vertical coordinate y , and ϕ is the phase of the object beam relative to the phase of the reference beam at $y = 0$. The maximum intensity is at those values of y for which

$$y_{\max} [(2\pi/\lambda) \sin \theta] - \phi = 2n\pi, \quad (3)$$

and the maximum intensity is

$$I_{\max} = (a_r + a_o)^2. \quad (4)$$

Likewise, the minima occur where

$$y_{\min} [(2\pi/\lambda) \sin \theta] - \phi = (2n + 1)\pi \quad (5)$$

and have the value

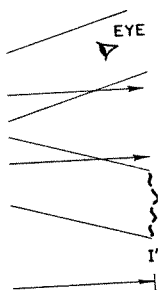
$$I_{\min} = (a_r - a_o)^2. \quad (6)$$

From Eqs. (4) and (6), we find the contrast or visibility of the fringes V to be

$$V \equiv (I_{\max} - I_{\min}) / (I_{\max} + I_{\min}) = 2a_r a_o / (a_r^2 + a_o^2). \quad (7)$$

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This equation shows that if the reference beam is constant, then the amplitude of the object beam may be unambiguously deduced from an observation of the fringe visibility. Further, we see that in case $a_o \ll a_r$, the visibility is approximately a linear function of a_o .

Equations (3) and (5) show us that the locations of the maxima and minima are related to ϕ . For example, consider the maximum corresponding to $n = 0$ in Eq. (3), in this case

$$y_{\max} = \frac{\phi}{[(2\pi/\lambda) \sin \theta]}, \quad (8)$$

which says that the y coordinate of the maximum is a linear function of the phase of the object beam. If we initially specify that the object beam is less intense than the reference beam, then an examination of the fringe pattern can give us complete and unambiguous information about the object beam. Furthermore, this information can be recorded photographically.

Even though the photographic process, per se, responds to intensity and is insensitive to the phase of incident light, it can be used to record the intensity pattern described by Eq. (2). As we have seen, this pattern contains both phase and amplitude information. The photographic record resembles the grating described by Eq. (1); we must explore this similarity with some care.

Let us now extend our thinking to the case in which the object is more complicated than a simple partially transparent plate with uniform phase delay. It may be a lantern slide of a complex line drawing or of an attractive scene. In either case the amplitude and phase of the wave transmitted through the object will be a complicated function of position. We need not be directly concerned with specifying these quantities at the object plane, it is sufficient to realize that by diffraction they determine the amplitude distribution and phase of the object wave falling upon the plate at H in Fig. 1. That this is true is clear from the fact that an eye placed in the position H can see the object.

If we describe the amplitude of the object wave at the plane H by $a_o(x, y)$ and its phase

by $\phi(x, y)$ then Eq. (2) becomes

$$I(x, y) = a_r^2 + [a_o(x, y)]^2 + 2a_r a_o(x, y) \cos \{y[(2\pi/\lambda) \sin \theta] - \phi(x, y)\}. \quad (9)$$

Here x and y represent horizontal and vertical coordinates in the plane H . In each small area of the plane H , the fringe contrast serves as a record of the amplitude of the object wave in that area, and the vertical position of fringes is a record of the phase of the object wave in that area. The exposed and developed plate, which becomes the hologram, in this way carries the information about $a_o(x, y)$ and $\phi(x, y)$.

When the wavefront is reconstructed, as in Fig. 2, each small area acts as a grating to determine the amplitude and phase of the light diffracted into the upper and lower beams from that area of the plate. The upper beam contains the correct phase information, the lower beam has all phases of reverse sign. The entire upper wavefront is a combination or mosaic of waves produced by the individual small areas and thus has the same amplitude and phase distribution as the original object wave. An eye placed in the upper beam and looking back through (not at) the hologram will see a virtual image of the object.

In the lower beam the amplitude distribution is the same but the signs of all the phases are reversed. In this way divergent wavefronts are converted to convergent wavefronts and a real image is produced at I' .

These arguments, although qualitative in nature, are clearly valid as long as the amplitude and phase of the object wave [i.e., $a_o(x, y)$ and $\phi(x, y)$] are slowly varying functions of position when compared with the $\cos \{y[2\pi/\lambda \sin \theta]\}$ term of Eq. (2). Under these conditions the elemental areas described in the previous paragraphs will contain several fringes. It is perhaps an act of faith to extend these arguments to the situation in which $a_o(x, y)$ and $\phi(x, y)$ contain spatial frequencies comparable to $(\sin \theta/\lambda)$.

We have also assumed that the photographic process is such that an interference pattern of the form described by Eq. (2) will give rise, upon development, to a grating of the simple form described in Eq. (1). This is certainly

an oversimplification but only giving a part of the picture. The details shall not be discussed.

We have seen in Fig. 2 that the reconstructed wavefront is a photographic plate of the object. Therefore we have seen that we have been a simple grating viewed through it is much more complicated by holding the illuminated plate. Unless the only part of the small portion of the entrance is very unsharp view of a light, i.e., a grating. Under the conditions may be seen that the grating may be seen to the left of the object and the reconstructed image consists of a virtual image through a grating. The scattered light is a very complex pattern of light by the reconstructed image wave. The detail of the entire reconstructed image is the hologram. The primary beam of the object beam

an oversimplification, but fortunately introduces no error into predicted location of the images but only gives some loss of intensity by diverting a part of the light into images of higher order. The details of these photographic effects shall not occupy our attention.

We have made the point that the upper beam in Fig. 2 results from a reconstruction of the wavefront which initially fell upon the photographic plate H of Fig. 1 after passing through the object. An eye placed in this upper beam therefore sees the virtual image just as it would have seen the object if plate H of Fig. 1 had been a simple glass window and the object viewed through it. This is not necessarily good, it is much like trying to view a transparency by holding it a foot or two from the eye but illuminated by a point source 10 or 12 ft away. Unless the transparency is highly scattering, the only part which will appear illuminated is the small portion directly between the source and the entrance pupil of the eye; the resultant view is very unsatisfactory. A much more satisfactory view of a transparency is to view it in diffuse light, i.e., against the background of an illuminated sheet of white paper or ground glass. Under these conditions the entire transparency may be seen at a glance. This same advantage may be obtained in holography by putting a lightly ground glass, called a "scatter plate," to the left of the object in Fig. 1. The scatter plate and the transparency now become the total object and the object wave at any point on H consists of light which has been scattered there through all parts of the transparency. Because of the scatter plate, the object wave at H will be a very complicated function of position and will no longer represent the Fresnel diffraction pattern of the transparency; instead it is produced by the scatter plate *and* the transparency. In the reconstruction from this hologram the virtual image will reproduce the transparency and the scatter plate, but since the scatter plate has no detail of interest in it, it serves only to make the entire transparency visible through all parts of the hologram.

The presence of the scatter plate in the object beam obviously reduces the intensity of the object beam reaching the plate, H , so that it is no

longer comparable with the reference beam. This reduces the contrast of the fringes formed at H and results in a generally unsatisfactory hologram. The obvious solution to this problem is to reduce the reference beam intensity until the two are again comparable. The optimum intensity ratio at H of reference to object beam is a matter of some debate but a ratio of 3 to 1 gives good results and even 10 to 1 is not objectionable. The reference beam should be more intense than the object beam.

The presence of the scatter plate so distorts the object wavefront that the fringes as we have described them are no longer recognizable as such. The hologram plate appearing rather as a random collection of minute light and dark blotches. In spite of appearances the blotches cannot be random since in the simple arrangement of Fig. 2 an image of the original object will be reproduced. It is safe to say, therefore, that sufficient information is stored on the hologram to reproduce the image of the object, but that this information is stored or encoded in such an unfamiliar form that direct examination of the hologram plate is essentially useless.

There is an alternate approach to the subject of holography² which is often useful. If we can master them both, we are in the fortunate position of being able to use whichever one offers the most immediate solution to the particular problem at hand. This second point of view states that a hologram is a collection, i.e., a superposition, of zone plates; and under proper illumination each zone plate will produce one point in the image. Thus the hologram will point by point reproduce the image of the object. In order to understand and appreciate this statement fully, let us consider the following simple experiment.

A beam of monochromatic collimated light from the left (Fig. 3) falls upon a photographic plate H . Some of the light falls directly upon H as a plane wave but some is intercepted by the negative lens L , which converts the plane wave into a spherical convex wave which also falls upon the plate H . This spherical wave has its center of curvature at the focal point of the

² G. L. Rogers, Nature 166, 237 (1950).

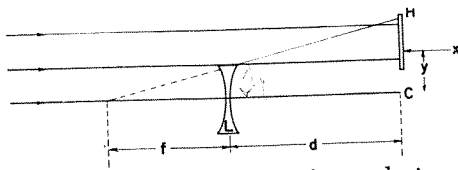


FIG. 3. Experimental arrangement for producing a zone-plate hologram. *H* represents the photographic plate, *L* a simple negative lens of focal length $(-f)$, *C* the point at which light through the center of the lens strikes the plane of *H*; *y* is the distance from some arbitrary reference mark *x* to *C*.

divergent lens so that its radius of curvature at the plane of *H* is $(f + d)$. Since these two waves are coherent, interference fringes will be produced wherever they overlap. One wave is plane and the other spherical, just as in the "Newtons rings" experiment, so the pattern will be similar and given by the expression

$$I(y) = a_p^2 + a_s^2 + 2a_p a_s \cos(\pi y^2 / \lambda r). \quad (10)$$

In this application *I* is the intensity on the plane *H* as a function of *y* which is distance along the plate measured from *C*, the point at which the plane and spherical waves are tangent; a_p and a_s represent the amplitude of the plane and spherical waves, respectively; λ is the wavelength of the light; and *r* is the radius of curvature of the spherical wavefront, in this case $r = (f + d)$. The fringes are of course circular with center at *C* but exist only in that portion of the figure where both the plane and spherical wavefronts reach the plane *H*. If *H* is a photographic plate, these fringes may be recorded.

From the equation above the intensity will be a maximum, resulting in maximum photographic darkening, whenever

$$\pi y^2 / \lambda r = 2n\pi \quad (11)$$

or

$$y = (2n\lambda r)^{1/2}.$$

This is the equation for the radii (*y*) of the dark rings of a zone plate of focal length *r*. The conventional zone plate differs from this only in having abrupt changes between the light and dark zones instead of the gradual transition described by our equation. After development the plate *H* may be considered as an off-axis piece of a zone plate.

If we take this zone plate and illuminate it as shown in Fig. 4 with collimated light of the

same wavelength as used in its production, then most of the light will continue on through undeviated but the zone plate will have (among others) both negative and positive focal length $r = (f + d)$. Consequently, a portion of the light emerging from the zone plate will form a divergent wave as though it had come from the point *V*. This is a virtual image and is located, relative to the photographic plate *H*, in exactly the same position as was the focal point of the lens *L* during exposure. Corresponding to the positive focal length of the zone plate, some of the incident light will converge to form a real image at the point *R*. *R* is the mirror image of *V* in the plane *H*. A quick calculation will reveal that if *f* and *d* are each 50 cm, *y* is 3 cm, and λ is 600 nm, then the fringe spacing should be about 20 μ . The eye cannot resolve these fringes, but a microscopic examination of the plate will reveal them. Obviously, the photographic plates used must be capable of resolving these fringes.

Let us suppose that with the available light intensity an exposure time *T* will give a satisfactory density in the developed plate *H*. Neglecting reciprocity problems this could be replaced by several, let us say 10 to be specific, partial exposures each of time *T*/10. After each partial exposure we make a small change in either the position of the lens *L* or the plate *H* in Fig. 3. The motions should be small because we wish a large area of the plate *H* to record fringes during each and every partial exposure. In this area of the plate there will after development be a superposition or overlay of ten zone plates. In reconstruction each zone plate will produce a virtual image at the same position relative to *H* as was the focal point of the lens *L* during its partial exposure. By properly selecting the position of *H* (or of *L*) during each of the partial exposures the 10 bright spots in the image *V* may be arranged in any desired pattern, even

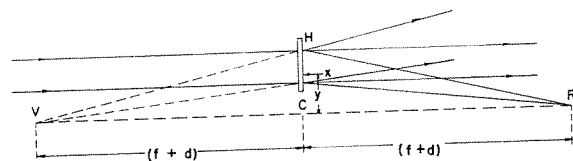


FIG. 4. Reconstruction using the zone-plate hologram produced in Fig. 3. A virtual-point image is produced at *V* and a real-point image at *R*.

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three dimensional, by changing d . As before, a real image will be formed at R which is the mirror image of V in the plane of H .

The experiments just described clearly lend support to the statement that a hologram is a superposition of zone plates and that under proper illumination each zone plate will produce one point in the image. In this way the hologram will point by point reproduce an image of the object. This is a valuable point of view which is particularly useful in helping us understand where the images are located; there is one slight defect or simplification in this point of view.

In these experiments the zone plates, corresponding to the various points in the object, were presented to the plate sequentially. If one were to make an ordinary hologram of an object of 10 bright spots on a dark background, the entire exposure would be made simultaneously. In each case the 10 spherical waves would interfere with the plane reference wave to produce 10 zone-plate patterns. In the simultaneous case there will also be fringes produced by the interference of the 10 spherical waves with each other; in the sequential case these fringes cannot be produced since only one spherical wavefront is present at a time.

The two cases of simultaneous and sequential exposure may be compared mathematically. In the simultaneous case $I(x, y)$, the intensity as a function of position on the hologram plane, is

$$I(x, y) = E_R^2 + \sum_n E_n^2 + 2 \sum_n E_R E_n \cos(ky\theta - k\Phi_n) + \sum_n \sum_{m \neq n} E_m E_n \cos(k\Phi_m - k\Phi_n). \quad (12)$$

This equation applies specifically to the experimental situation shown in Fig. 5, and the notation is as follows. E_R is the amplitude of the reference wave and E_n the amplitude, at the photographic plate, of the spherical wave originating from point n . The reference wave falls upon the photographic plate at the angle θ ; k is $2\pi/\lambda$. The quantities

$$\Phi_n \equiv \frac{(x - x_n)^2 + (y - y_n)^2}{2D} + \Lambda_n \quad (13)$$

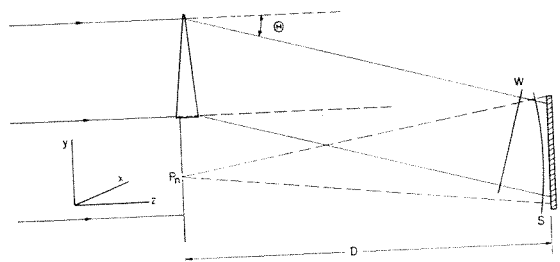


FIG. 5. The production of zone-plate holograms in sequence. P_n , pinhole in position n ; S , spherical wavefront centered at P_n ; W , plane wavefront from prism; H , photographic plate.

are defined by this equation, where (x, y) are coordinates on the photographic plate (which is perpendicular) to the z direction and (x_n, y_n) are the coordinates of the n th bright point in the object plane. D is the separation (in the z direction) between the object plane and the plane of the photographic plate. Λ_n represents some arbitrary (but constant) phase shift in the process of scattering or diffraction at point n . It is expressed in units of equivalent path length.

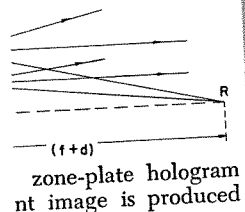
In the sequential case the time integrated intensity, or total exposure is

$$I(x, y) = \sum_n E_{Rn}^2 + \sum_n E_n^2 + 2 \sum_n E_{Rn} E_n \cos(ky\theta - k\Phi_n). \quad (14)$$

In this equation E_{Rn} is the amplitude of the reference wave during the n th exposure. The time integration has been omitted for simplicity. The rest of the notation remains unchanged.

Equations (12) and (14) are similar. The first terms differ in form but are constants independent of x and y . The significant difference is the last term of Eq. (12) which has no counterpart in the sequential situation described by Eq. (14). This last term of the simultaneous case represents the interference among the spherical waves. If we assume the n -point scatterers are close together in the dark field of the object plane, then the spherical waves reaching the plate will intersect at small angles giving rise to interference fringes of low spatial frequency. This is indicated in the mathematics if $|\Phi_m - \Phi_n|$ is small as compared with $y\theta$. Physically this means that the maximum angular spread of the n bright spots,

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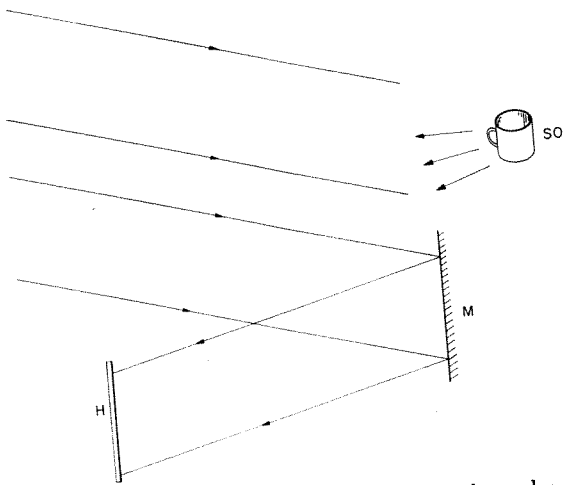


FIG. 6. Formation of hologram of opaque object by reflection and scattering. *H*, photographic plate; *M*, plane mirror; *SO*, scattering object.

when viewed from the recording plate, is small compared to the angle θ .

These low spatial frequencies as they appear on the developed hologram correspond to a coarse grating and such a grating diffracts the light by only a small angle. In this case the angle is so small that these terms contribute no light to the virtual and real images *V* and *R* but only serve to give the zero-order beam some structure.

We see therefore that although the zone-plate approach is slightly less than perfect, its defects are not of practical consequence, whereas its use is very often of value in helping us predict the location of the two images.

If we have understood the discussions so far, then we understand the essentials of holography. There are many variations in the methods, some of them minor and some very ingenious, but generally understandable as variations on the basic methods already described. We mention some of them briefly.

The simplest and most obvious extension³ is shown in Fig. 6 and involves using light diffusely reflected from the object and a reference beam directed onto the recording plate by reflection from a mirror. Here, as with the scatter plate, the reference beam must be attenuated in some

way so that at the recording plate the illumination by the reference beam and the object beam are in a ratio of about 3 to 1. As in most holographic work, the exposure should be such that the developed hologram has an average photographic density of $\frac{1}{4}$ to $\frac{1}{2}$. The general tendency of the beginner in this field is to overexpose. There is one problem which this situation brings sharply into focus. Holography is essentially a recording of interference fringes and their subsequent use to reconstruct a wavefront. If, during exposure, there is sufficient motion of any of the elements to "wash out" the fringes, then the resulting photographic plate will be useless. For the earlier cases this motion tolerance was not too severe, particularly if the angle θ were small. In the reflection case motion of the object or the reference beam mirror as little as half a wavelength can prove disastrous.

Next in complexity we consider the situations shown in Fig. 7. The upper case is for transmis-

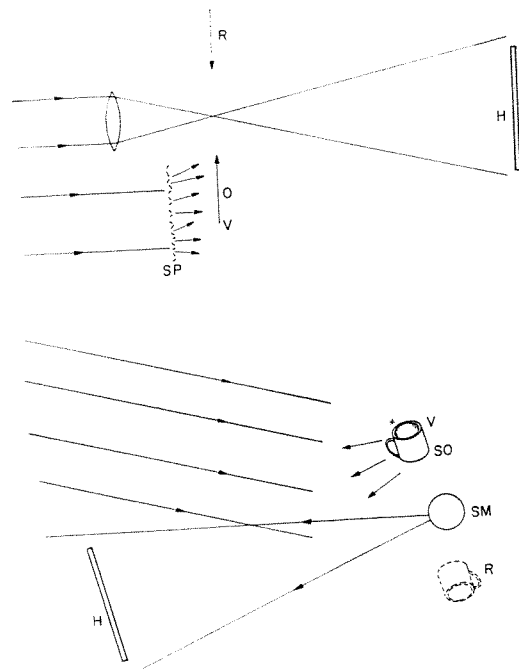


FIG. 7. Formation of hologram and reconstruction using spherical reference wave centered near the object *O*. In reconstruction the same reference beam is used and the object is reproduced as a virtual image *V*. In this case the second image *R* is also virtual. *H* is the photographic plate. *SP* is a scatter plate; *SO* is an opaque scattering object; *SM* is a spherical mirror.

³ E. N. Lieth and J. Upatnieks, *J. Opt. Soc. Am.* **54**, 1295 (1964).

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sion, and below is a corresponding situation in reflection. The distinguishing feature of these two situations is that the reference wave is now spherical, with the center of the sphere near the plane of the object. In transmission the spherical reference wave is easily produced by a lens, in the reflection case it may be produced by a spherical mirror (highly polished ball) as shown or by a lens and plane mirror.

Reconstruction in this case consists in illuminating the developed hologram with a spherical wavefront identical with the reference wave. The hologram then produces a virtual image of the object and in the same location. The other image, which in the earlier cases was real, is no longer real even though we have labeled it *R* on the figure. It is produced as before by a wavefront similar to the object wavefront except for the change of sign in the phase, but the reference wave relative to which the sign is reversed is now spherical rather than plane.

In the zone-plate concept, a simple calculation will show that the fringes formed by the reference wave and the wave from a specific point in the object are equally spaced rather than having their radii go as $n^{1/2}$. The zone plate therefore has infinite focal length or zero power. Under these conditions changing the sign of the focal length does not change the image distance.

Holograms such as these which carry information but have no focusing properties, i.e., which depend upon the radius of the reconstructing wavefront to provide curvature in the final wavefronts, are called Fourier transform holograms or sometimes Fraunhofer holograms.

One must also make the point that the reference wave need not be perfectly spherical, i.e., it may have aberrations.⁴ A perfect image (*V* not *R*) will be reproduced as long as the reference wave used in reconstruction is the same as the reference wave used in producing the hologram and the hologram is returned to its original position in the wavefront. An extension of this generality would indicate that if any two wavefronts, of arbitrary shape, interfere to form a hologram, and if the developed hologram is illuminated by one of those wavefronts, the other will be re-

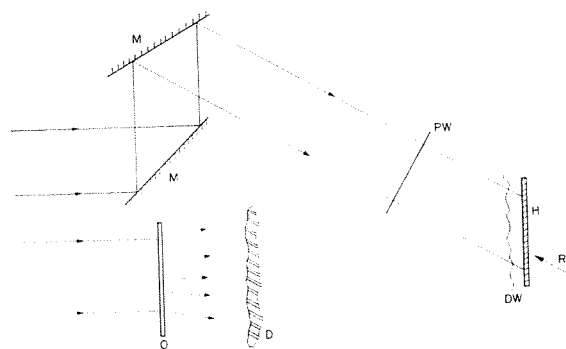


FIG. 8. Hologram made and reconstructed through a distorter (irregular glass plate), *D*; *M*, plane mirror; *O*, object; *PW*, plane reference wave; *DW*, distorted object wave. *RW*, the reconstructing wave is incident from the reverse direction.

constructed. There will also be another wavefront reconstructed, but only in the case of a simple reference wave will this represent an image of good quality.

As a trick let us consider Fig. 8 in which a deliberately poor piece of glass is placed between the object and the hologram plate.⁵ This glass may be so bad (e.g., a bathroom window) that the object as viewed from the plate position is unrecognizable. A beam of light, coherent with the one illuminating the object, is brought around the distorter and serves as reference beam. This reference beam and the distorted object beam interfere to form a hologram which is developed and returned to its original position. For reconstruction the reference beam is reversed in direction so that it falls upon the hologram from the back, as indicated by the arrow in the figure. The hologram reconstructs the distorted wave which fell upon it, but projected backward toward the distorter. If everything has been perfectly placed, the wavefront will go backward through the distorter along just the correct paths to undo the distortion and reproduce at the object position a clear, recognizable, real image which may be photographically recorded.

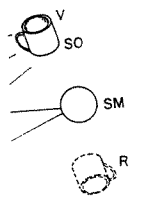
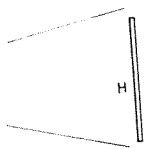
In order to execute the experiment just described, it is clear that the object and reference beams will intersect at larger angles than were used in the earlier experiments. This leads us to

⁴ R. W. Meier, *J. Opt. Soc. Am.* **55**, 987 (1965).

⁵ H. Kogelnik, *Bell System Tech. J.* **44**, 2451 (1965).

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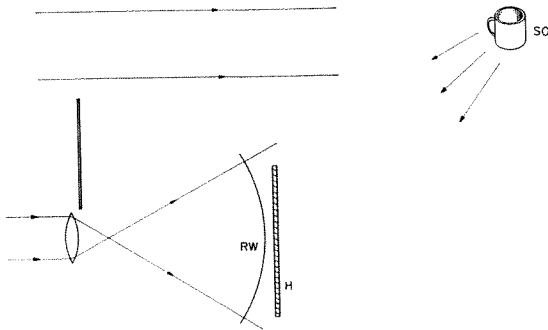


FIG. 9. Hologram made with object and reference waves impinging upon opposite sides of the plate. *SO* is an opaque scattering object. *H* is the photographic plate. *RW* is the spherical reference wave.

question the possibility of using the set-up shown in Fig. 9. The distinguishing feature of this arrangement is that the object and reference beams fall upon opposite sides of the photographic plate. There are minor problems here but not insurmountable ones.

The basic fringe spacing Δy on the earlier hologram methods is given by the equation

$$\Delta y = \lambda / \sin \theta,$$

where θ is the angle between the object and reference beams. As long as Δy is large as compared with the emulsion thickness, we can ignore thickness and treat the emulsion as two dimensional. If θ is of the order of 45° as suggested in Fig. 8, then $\Delta y \approx 1.4\lambda$ which is less than 1μ . The emulsion is 15 or 20 μ thick so that the fringes are three dimensional using the thickness of the emulsion. In this case simple fringes would

somewhat resemble a venetian blind. Not only is the spacing important but also the angle which the "slats" of the venetian blind make with the surface. The slats or surfaces of maximum silver deposit will lie along a direction which bisects the angle between the object and reference beams. In Fig. 8 this direction will be about 22° from the normal to the surface. In Fig. 9 the direction is nearer 70° from the normal or approaching parallel to the plate surface. Under these conditions, of fringe spacing less than the emulsion thickness, the hologram considered as a grating is effectively blazed. In addition to behaving according to the usual grating rules, it will put most of the incident light into the image only if the ordinary laws of reflection are obeyed for the 'slats' of the venetian blind.

In Fig. 9, where the 'slats' are almost parallel to the over-all surface, the hologram becomes effectively an interference filter (such as found in the Lippman process of color photography) and will reproduce the image only if the reconstructing wavelength is properly matched to the fringe spacing. Since the emulsion thickness generally shrinks 12% to 15% during processing, the original wavelength is no longer satisfactory for reconstruction. However, the wavelength selectivity is often sufficient to enable white light (spatially coherent, of course) to be used in reconstruction. The hologram then selects the appropriate wavelength and reconstructs the image. The eye is placed on the same side of the hologram as the source and views the image using light reflected or back scattered by the hologram.

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