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The Zeeman Effect

Abstract

The splitting of the mercury 5461 Å line due to the application of a magnetic field was measured. This splitting, known as the Zeeman effect, enabled us to find an experimental value for the Bohr magneton. From observations of the lines polarized parallel and perpendicular to the applied magnetic field we were also able to verify the accuracy of the quantum mechanical treatment of the Zeeman effect.

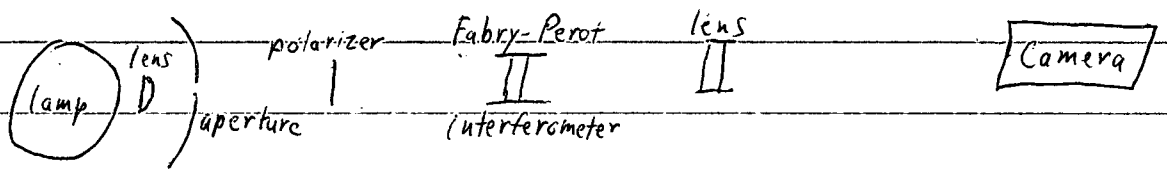
Introduction

A magnetic dipole of magnitude μ in a magnetic field $\vec{H}$ has a potential energy $-\vec{\mu} \cdot \vec{H}$. The atoms of many substances have a net magnetic dipole, and hence feel a potential energy due to the application of a magnetic field. Quantum mechanical calculation shows that the magnetic moment of an atomic electron is quantized, so that a given magnetic field will lead to several discrete electron energy levels, energy levels that are split from the degenerate energy level present before the magnetic field was applied. Since the wavelength of a photon is inversely proportional to it's energy, transitions between the closely spaced, split, energy levels lead to several closely spaced spectral lines. We measured these lines in order to deduce something about the energy level splitting.

In particular, we measured the splitting of mercury's 5461Å line for several values of the magnetic field using a Fabry-Perot interferometer. This line should split in a magnetic field into nine components, 3 polarized parallel to the field, and six polarized perpendicular to the field. We were able to measure the three lines parallel to the field by blocking the other six with a polarizer. When we turned the polarizer perpendicular to the field to view the six lines polarized in that direction our measurements were less successful because of the difficulty of resolving six components at once. We were, in fact, unable to prove that there are six separate lines, but our data did fit the theory of the six lines given assumptions about the overlap and non-resolution of lines.

Experimental Setup

The setup was as pictured below.



Our source of light was a mercury lamp placed between the poles of an electromagnet. The light went through the following train of optical devices: 1) a lens to collect the light 2) a small square aperture to block most light of the wrong frequency. 3) a polarizer 4) a Fabry-Perot interferometer, a device which is basically just two ~~plates~~ parallel plates of partially

reflecting glass 5) a lens to focus the parallel light emerging from the Fabry-Perot interferometer 6) a camera placed at the focal length of the lens to take pictures of the ring pattern.

The Fabry-Perot interferometer serves the same purpose as a grating in that it separates light of different frequencies into a circular fringe pattern. A Fabry-Perot can usually split lines with a small frequency difference with a higher resolution than a grating. This resolution is measured by the resolving power of the device, a number which is defined as $\lambda/\Delta\lambda_{(min)}$, where $\Delta\lambda_{min}$ is the minimum wavelength difference at which one can just resolve two lines, and λ is the mean wavelength of the two lines. For a Fabry-Perot interferometer the theoretical value for the resolving power is:

$$\frac{\lambda}{\Delta\lambda_{min}} = \frac{2\pi d r}{\lambda(1-r^2)}$$

r = reflectivity of glass
d = distance between parallel plates

Although we were not viewing under ideal conditions, we could resolve lines with a wavelength separation of about 7.5×10^{-10} cm. and a mean wavelength of $.55 \times 10^{-4}$ cm. for a resolving power of about $\frac{\lambda}{\Delta\lambda_{min}} \cong 10^5$.

As the wavelength separation of two spectral lines increases the point is reached where one component starts to overlap with the other component in the next spectral order. Since overlap is undesirable, an optical device in practice has a maximum wavelength separation for the measurement of two lines, and at

larger wavelength separations the lines will overlap. The maximum wavelength difference that can occur without any overlap is called the free spectral range, $\Delta\lambda_{fsr}$. The theoretical value of the free spectral range for a Fabry-Perot interferometer is:

$$\Delta\lambda_{fsr} = \frac{\lambda^2}{2d}$$

Clearly we want the free spectral range to be as large as possible, just as we want the minimum wavelength separation for resolution, $\Delta\lambda_{min}$, to be as small as possible. A number that is a measure of the quality of an optical device is the finesse;

$$\mathcal{F} = \frac{\Delta\lambda_{fsr}}{\Delta\lambda_{min}} = \frac{\pi r}{1-r^2}$$ (for a Fabry-Perot interferometer) r = reflectivity of glass

As the finesse increases, either the free spectral range must increase or $\Delta\lambda_{min}$ must decrease, so the quality of the optical device must improve.[1] The finesse, in fact, gives an idea of the maximum number of lines one can resolve without overlap. For instance, in our measurements we found the free spectral range to be, $\Delta\lambda_{fsr} \cong 3.2 \times 10^{-9}$ cm, and $\Delta\lambda_{min} \cong 7.5 \times 10^{-10}$ cm., for a finesse of about 4, which means we could not resolve any more than four evenly spaced lines without overlap. When we made measurements with the polarizer oriented perpendicular to the field we had six different components, so that it was impossible to resolve them without overlap.

Theory of the Zeeman Effect

In describing the theory of the Zeeman Effect we will use hydrogen as an example, since it is the simplest atom, comprised only of an electron and a proton. Our results will apply with some modification to the element we used in the experiment, mercury.

Several different operators are used to characterize the wave function of the electron in the hydrogen atom; $\vec{L}$, the orbital angular momentum; $\vec{S}$ the spin angular momentum; L_z and S_z, the z-components of the orbital and spin angular momentum; H, the Hamiltonian; $\vec{J} = \vec{L} + \vec{S}$, the total angular momentum; and J_z, the z-component of the total angular momentum. A wavefunction can be a simultaneous eigenfunction of J_z, $J^2 = \vec{J} \cdot \vec{J}$, L^2, and S^2, or it can be a simultaneous eigenfunction of L^2, S^2, S_z, L_z, but a wavefunction cannot be a simultaneous eigenfunction of all six operators. This is a result of a general theorem that a wavefunction cannot be a simultaneous eigenfunction of two operators that do not commute (i.e. A and B do not commute if $(AB - BA)\Psi \neq 0$). J^2 and S_z, and J^2 and L_z do not commute.

In describing the wavefunction of hydrogen it is most useful to use eigenstates of J^2, J_z, L^2, and S^2 rather than of L^2, S^2, S_z, and L_z, since the quantum number of J^2 enters into the expression for the hydrogen energy levels (j is called the quantum number of J^2 if $J^2\Psi = j(j+1)\hbar^2\Psi$ where Ψ is the wave function). The energy of an electron in the hydrogen atom is, to the first order, $E = -\frac{1}{n^2}\left(1 + \frac{\alpha^2}{n}\left(\frac{1}{j+\frac{1}{2}} - \frac{3}{4n}\right)\right)$ Rydbergs, where $n = 1, 2, 3 \ldots$ is

$\vec{\mu} = \frac{-\mu_B}{\hbar}(\vec{L} + 2\vec{S})$.

Since $\vec{L}$ and $\vec{S}$ are not parallel vectors, $\vec{J} = \vec{L} + \vec{S}$ is not parallel to $\vec{\mu} = \frac{-\mu_B}{\hbar}(\vec{L} + 2\vec{S})$. However, it turns out that only the component of $\vec{\mu}$ parallel to $\vec{J}$ contributes to the net ~~magnetic~~ magnetic moment of the electron in an external field. The reason for this is that $\vec{\mu}$ is continually precessing about $\vec{J}$ whether an external field is present or not, due to the magnetic field of the proton (the spin-orbit effect). The precession of $\vec{\mu}$ about $\vec{J}$ is so rapid that on average the component of $\vec{\mu}$ perpendicular to $\vec{J}$ will be zero. Hence, the magnetic moment that the field will experience will effectively be only the component of $\vec{\mu}$ parallel to $\vec{J}$:

$$\vec{\mu}_j = \frac{\vec{\mu}\cdot\vec{J}}{|J|} = \frac{-\mu_B}{\hbar}(\vec{L} + 2\vec{S})\cdot\frac{(\vec{L}+\vec{S})}{|J|} = \frac{-\mu_B}{\hbar}(\vec{J}+\vec{S})\cdot\frac{\vec{J}}{|J|}$$

$$= -\frac{\mu_B}{\hbar}\left(1 + \frac{\vec{J}\cdot\vec{S}}{|J|^2}\right)\vec{J}$$

$$= -g_j \frac{\mu_B}{\hbar}\vec{J}$$

where g_j is called the Landé g-factor

$$g_j = \left\langle 1 + \frac{\vec{S}\cdot\vec{J}}{J^2} \right\rangle$$

$$S\cdot J = S^2 + \frac{1}{2}(J^2 - S^2 - L^2)$$

$$\therefore g_j = 1 + \left\langle \frac{J^2 + S^2 - L^2}{2J^2} \right\rangle = 1 + \frac{j(j+1) + s(s+1) - \ell(\ell+1)}{2j(j+1)}$$

We can assume the magnetic field is along the z-direction, hence the extra term in the Hamiltonian due to the Zeeman effect is:

$$H_{Zeeman} = -\mu_j \cdot H = g_j\left(\frac{\mu_B H}{\hbar}\right)J_z$$

The extra energy is:

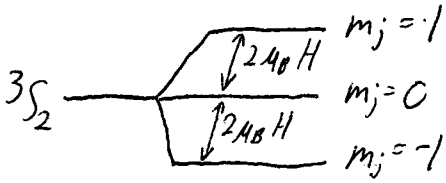
$$E_{Zeeman} = \langle H_{Zeeman} \rangle$$

$$= g_j \frac{\mu_B H}{\hbar} \langle J_z \rangle = g_j \mu_B H m_j$$

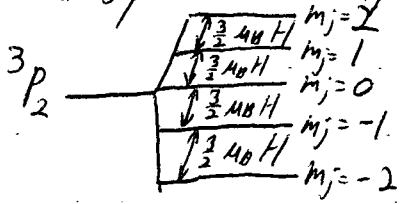
$$= \left[1 + \frac{j(j+1) + s(s+1) - \ell(\ell+1)}{2j(j+1)}\right] \mu_B H m_j$$

The energy now depends on the quantum number of J_z, m_j, so the original energy levels will be split into $2j+1$ energy levels corresponding to the $2j+1$ values of m_j.

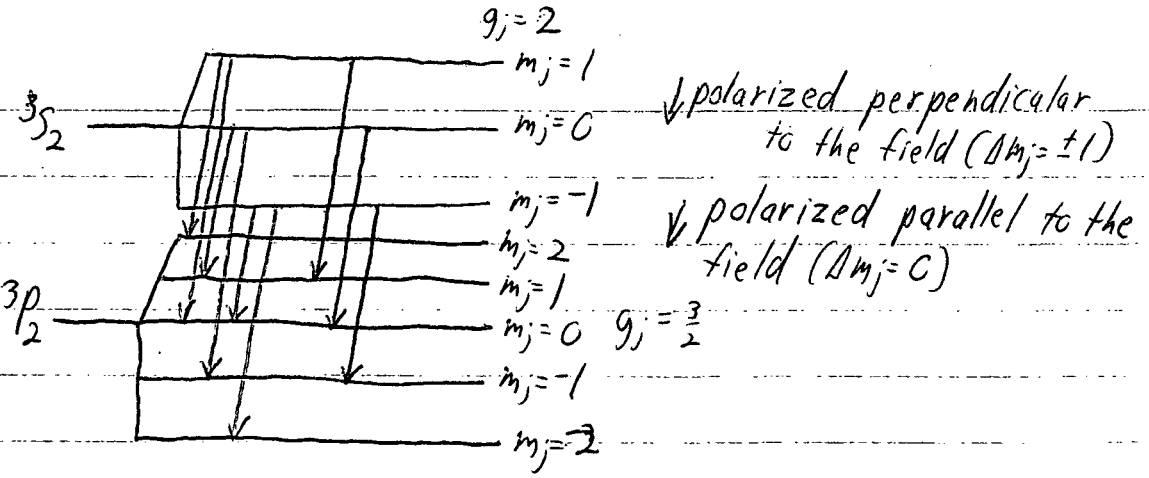
These results are directly applicable to mercury except that mercury has two valence electrons instead of one, so that s, j, and ℓ will take on values corresponding to the coupling of the two electrons. For instance, the transitions we were studying with the interferometer were between the $^3S_1$ and the $^3P_2$ level. For $^3S_1$ $s=1$, $\ell=0$, $j=1$, hence $g_j = 2$, and the original $^3S_2$ level will be split into three (see diagram below)



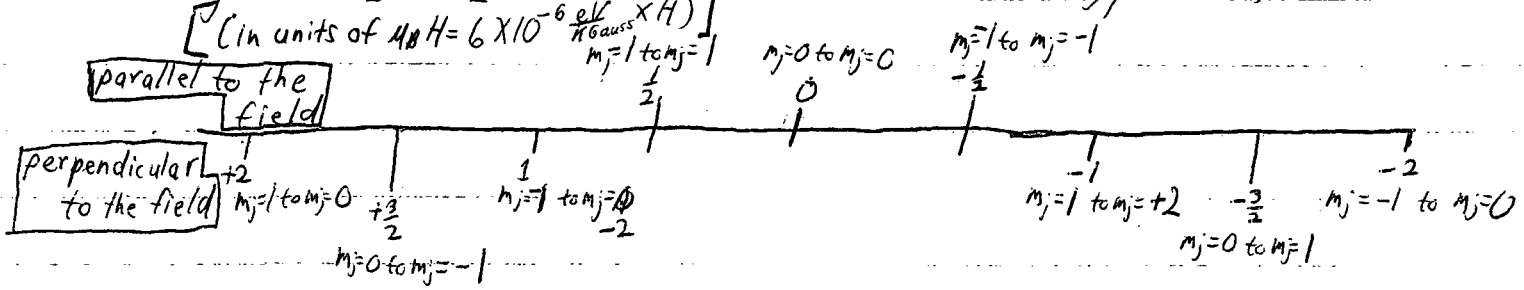
For $^3P_2$ $s=1$, $\ell=1$, $j=2$ so $g_j = \frac{3}{2}$, so in a magnetic field there will be five energy levels separated by $\frac{3}{2}\mu_B H$.



The transitions that give rise to the nine observed spectral lines are as follows:



From this energy transition diagram it can be seen that the nine spectral lines will have the following photon energies (calling the original $^3S_2 \rightarrow {}^3P_2$ transition the zero of the energy scale):



As can be seen from the above diagrams, only transitions between certain energy levels are allowed. The selection rules state that the only transitions possible are those for which:

$$\Delta j = \pm 1, 0$$
$$\Delta m_j = \pm 1, 0$$
$$\Delta \ell = \pm 1$$

These selection rules can be given a simple physical interpretation. A photon is a spin one particle and so has an angular momentum of $\hbar$, hence when an atom absorbs or emits a photon the atom's angular momentum should change by $\hbar$ ~~in order~~ to conserve angular momentum. This does not mean that the magnitude of the angular momentum need change by one; since angular momentum is a vector quantity a change in angular momentum

can manifest itself as a change in direction rather than magnitude. Hence we can have a change in the quantum numbers j, m_j, ℓ by either $-1, 0,$ or 1. The additional restriction on $\Delta\ell$, that is, $\Delta\ell \neq 0$ arises from a rule that the parity of a wavefunction must change during a transition.

As an example of how these selection rules come about quantum mechanically, we will prove the selection rule for m_j, that is $\Delta m_j = 0, \pm 1$. If we consider an electromagnetic wave perturbing an electron in an atom, to the first order the perturbing term in the Hamiltonian will be:

$$H' = \frac{e}{mc} \vec{A}_0 \cdot \vec{p} \cos\omega t$$

where $\vec{A}_0 \cos\omega t$ is the vector potential of the electromagnetic wave and $\vec{p}$ is the canonical momentum of the electron. The Fermi golden rule states that the transition probability between two states is proportional to the square of the matrix element of the perturbing term in the Hamiltonian which couples the two states. If we consider two states, $\psi_{j m_j \ell n s}$ and $\psi_{j' m_j' \ell' n' s'}$, the transition probability will therefore be proportional to:

$$\text{prob} \propto \left(\langle \psi_{j m_j \ell n s} | H' | \psi_{j' m_j' \ell' n' s'} \rangle \right)^2$$

$$\left(\langle \psi_{j m_j \ell n s} | H' | \psi_{j' m_j' \ell' n' s'} \rangle \right)^2 = \left(\cos\omega t \langle \psi_{j m_j \ell n s} | \vec{A}_0 \cdot \vec{p} | \psi_{j' m_j' \ell' n' s'} \rangle \right)^2$$

A transition will therefore be forbidden if:

$$\langle \psi_{j m_j \ell n s} | \vec{A}_0 \cdot \vec{p} | \psi_{j' m_j' \ell' n' s'} \rangle = 0$$

since the probability of the transition would then be zero.

If the electromagnetic wave is polarized in the z-direction (parallel to the field), then the transition will be forbidden if $\langle \psi_{j m_j l n s} | A_0 P_z | \psi_{j' m_j' l' n' s'} \rangle = 0$ or since A_0 is a constant we can remove it and the condition for the forbidden transition is $\langle \psi_{j m_j l n s} | P_z | \psi_{j' m_j' l' n' s'} \rangle = 0$. Since $[J_z, P_z] = J_z P_z - P_z J_z = 0$ we know that:

$$\langle \psi_{j m_j l n s} | J_z P_z - P_z J_z | \psi_{j m_j' l' n' s'} \rangle = 0$$

but $\langle \psi_{j m_j l n s} | J_z P_z | \psi_{j' m_j' l' n' s'} \rangle = +m_j' \hbar \langle \psi_{j m_j l n s} | P_z | \psi_{j' m_j' l' n' s'} \rangle$

$$\langle \psi_{j m_j l n s} | P_z J_z | \psi_{j' m_j' l' n' s'} \rangle = m_j \hbar \langle \psi_{j m_j l n s} | P_z | \psi_{j' m_j' l' n' s'} \rangle$$

$$\therefore \langle \psi_{j m_j l n s} | J_z P_z - P_z J_z | \psi_{j' m_j' l' n' s'} \rangle = (m_j' \hbar - m_j \hbar) \langle \psi_{j m_j l n s} | P_z | \psi_{j' m_j' l' n' s'} \rangle = 0$$

Therefore the matrix element of P_z must vanish unless $m_j = m_j'$, so the selection rule $\Delta m_j = 0$ applies. Since we have been assuming that the electromagnetic wave is polarized in the z-direction (parallel to the field) this means that transitions for which $\Delta m_j = 0$ will emit light polarized parallel to the field. Similarly one can show that if we have a circularly polarized electromagnetic wave $A_0(\hat{x} \pm i\hat{y})\cos\omega t$ the transition will be forbidden if the matrix element of $P_\pm = P_x \pm i P_y$ is zero. This matrix element, $\langle \psi_{j m_j l n s} | P_\pm | \psi_{j' m_j' l' n' s'} \rangle$, is zero unless $m_j - m_j' = \pm 1$, that is $\Delta m_j = \pm 1$. Thus the transitions for which $\Delta m_j = \pm 1$ will result in circularly polarized photons (if viewed along the z-direction; they will be linearly polarized perpendicular to the field when viewed along the x or y-directions, which is where we were viewing the photons from).[3]

Experimental Results.

Before taking our pictures of the ring pattern we had to carefully adjust the interferometer so that its plates were parallel. This is done by looking at the rings with the unaided eye and moving one's head back and forth or up and down to see if the ring pattern is uniform. If the ring pattern is not uniform there are screws that adjust the spacing of the interferometer plates.

We also had to ensure that the polarizer was oriented in the desired direction. This is done by looking through the polarizer at light reflected off a piece of glass at the brewster angle ($\cong 55°$). This reflected light will be polarized parallel to the surface of the glass, so if one turns the polarizer until the maximum intensity is reached, the polarizing axis will then be parallel to the glass surface.

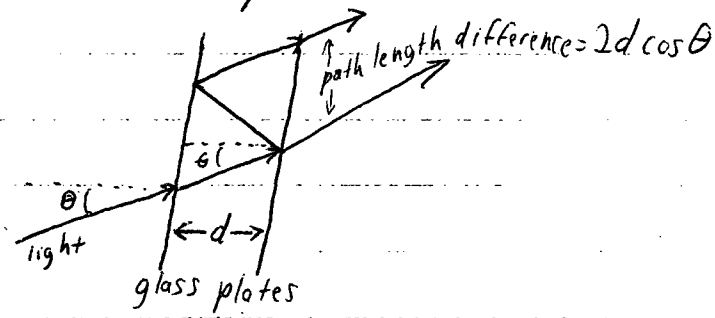
We took several pictures of the ring pattern at different values of the magnetic field, exposing each picture for between 30 and 50 seconds. Of course, all the optical devices had to be lined up as perfectly as possible, and we had to keep the room as dark as possible.

With the polarizing axis parallel to the magnetic field to get the $\Delta m_j = 0$ lines we found that only magnetic fields between about 6 kilogauss and 10 kilogauss gave satisfactory pictures, because lower fields did not split the lines enough for easy resolution and higher magnetic fields led to some overlapping of the spectral orders. For the six components polarized perpendicular to the magnetic field we tried a variety of fields, but it was impossible to resolve all six lines without overlap.

We measured the radii of the spectral lines on the pictures using a traveling microscope of 9X magnification. At this magnification we could not usually see the lines, although we could easily see the lines without the microscope. Using a pin, we carefully made a hole on each line, and we measured the distance between the holes with the microscope. We could find the distance between the pinholes with an error of only ±.005 inches, but perhaps the main problem with the technique was that we could not be sure the pinholes were on the middle of each lines.

To reduce the data we used an approximate method suggested in Melissinos[4]. The method is simple computationally, but it requires an explanation, which we will now give.

If a ray of light enters ~~the~~ a Fabry-Perot of plate separation d at an angle θ, the path length difference between successively transmitted rays will be $2d\cos\theta$ (see diagram



For constructive interference (the bright fringes) the path length difference must be a whole number of wavelengths, i.e. $n\lambda = 2d\cos\theta$ $n = 0, 1, 2 \cdots$.

At $\theta = 0$, which is the center of the pattern, the path length difference is $2d$, so the interference order at the center

is $n_o = \frac{2d}{\lambda}$, where n_o is not in general an integer, since there may not be a bright ring at the center. The bright ring closest to the center will appear at an angle θ_1 that satisfies the equation $n_1 = \frac{2d}{\lambda}\cos\theta_1 = n_o \cos\theta_1$ where n_1 is the closest integer to n_o that is smaller than n_o. Clearly, then, $n_1 = n_o - \epsilon$, where ϵ is some number between 0 and 1. The pth ring from the center will occur at an angle θ_p that satisfies $n_p = n_o \cos\theta_p$, where $n_p = (n_o - \epsilon) - (p-1)$. Since θ_p is always small we can expand $\cos(\theta_p)$ as a Taylor series to get:

$$n_p = (n_o - \epsilon) - (p-1) = n_o\left(1 - \frac{\theta_p^2}{2}\right)$$

$$\therefore \theta_p \cong \sqrt{\frac{2(p-1+\epsilon)}{n_o}} \quad \text{(Equation 1)}$$

The camera was placed at the focal length of the lens ($f = 121.9$ cm.), at which distance the radii of the rings are:

$$r_p = f\tan\theta_p \cong f\theta_p$$

Substituting from equation 1 for θ_p we get

$$r_p = \sqrt{\frac{2f^2}{n_o}[(p-1)+\epsilon]} \quad \text{(Equation 2)}$$

$$\therefore r_{p+1}^2 - r_p^2 = \frac{2f^2}{n_o} = \frac{\lambda f^2}{d} \quad \text{(Equation 3)}$$

It is clear from equation 3 that $r_{p+1}^2 - r_p^2$ should be a constant with nearly the same value whether the field is applied or not (since the change in wavelength due to the Zeeman effect is only about one part in 10^5). This we found to be the case. The table below gives the

mean and standard deviation of $r_{p+1}^2 - r_p^2$ for different values of the magnetic field.

Table 1

magnetic field H	$\langle r_{p+1}^2 - r_p^2 \rangle$	standard deviation
Lines polarized parallel to field		
0 Kilogauss	.1233 (inches)^2	.0083 (inches)^2
7.2 "	.1229 "	.003
8.1 K.G.	.1195 "	.0019
9.0 "	.1204 "	.006
9.2 "	.1202 "	.005
Lines polarized perpendicular to field		
0 K.G.	.1212 "	.0009 "
1.8 KG.	.1234 "	.0085 "
7.8 KG.	.1209 "	.004 "

One can find the thickness of the interferometer with the above data. From equation 3, $d = \lambda f^2 / \langle r_{p+1}^2 - r_p^2 \rangle$, so using $\lambda = 5461 \text{ Å}$, $f = 121.9$ cm, $\langle r_{p+1}^2 - r_p^2 \rangle = .1215$ (inches)^2 one gets $d = 1.035$, which is in fairly good agreement with the approximate known value of $d = 1.003$ cm (3.2% error).

If there are two closely spaced components of a spectral line with wavelengths λ_A and λ_B, we can derive an expression for their wave number difference ($\bar{\nu}_A - \bar{\nu}_B = \frac{1}{\lambda_A} - \frac{1}{\lambda_B}$) using the number introduced on the previous page, ϵ. The ϵ of the two components will be different, since:

$$\epsilon_A = n_{0(A)} - n_{1(A)} = \frac{2d}{\lambda_A} - n_{1(A)} = 2t\bar{\nu}_A - n_{1(A)}$$

$$\epsilon_B = n_{0(B)} - n_{1(B)} = \frac{2t}{\lambda_B} - n_{1(B)} = 2t\bar{\nu}_B - n_{1(B)}$$

If $n_{1(A)} = n_{2(A)}$, that is, if there is no overlap of the spectral orders then we get:

$$\bar{\nu}_A - \bar{\nu}_B = \frac{\epsilon_A - \epsilon_B}{2t} \quad \text{(Equation 4)}$$

Making use of equations 2 and 3 we find

$$\epsilon_A = \frac{n_0}{2f^2} r_{p(A)}^2 - p + 1 = \frac{r_{p(A)}^2}{r_{p+1(A)}^2 - r_{p(A)}^2} - p + 1 \quad \text{(Equation 5)}$$

$$\epsilon_B = \frac{n_0}{2f^2} r_{p(B)}^2 - p + 1 = \frac{r_{p(B)}^2}{r_{p+1(B)}^2 - r_{p(B)}^2} - p + 1 \quad \text{(Equation 6)}$$

From the argument made earlier we know that $r_{p+1}^2 - r_p^2$ is a constant nearly independent of the Zeeman effect so that $r_{p+1(A)}^2 - r_{p(A)}^2 \cong r_{p+1(B)}^2 - r_{p(B)}^2$) and ~~substituting~~ substituting Equations 5 and 6 into equation 4 we find:

$$\bar{\nu}_A - \bar{\nu}_B = \left(\frac{r_{p(A)}^2 - r_{p(B)}^2}{2d\left(r_{p+1}^2 - r_p^2\right)}\right)^* \quad \text{(Equation 7)}$$

To find the wave number difference between lines split by the magnetic field, then, all we have to do is take the average over all the measured radii (at a given magnetic field) of equation 7.

*Note: $r_{p(A)}^2 - r_{p(B)}^2$ is the difference of the squares of the radii of different components in the same spectral order, while $r_{p+1}^2 - r_p^2$ is the difference of the squares of the same component in successive spectral orders.

The values for $\langle r_{p+1}^2 - r_p^2 \rangle$ for the various values of the magnetic field are presented in Table 1 (page 15). Our values for $\langle r_{p(A)}^2 - r_{p(B)}^2 \rangle$ depend on which component of spectral line we take as component A, and which one we say is component B. The lines polarized parallel to the field are three evenly spaced lines with energy differences of $\frac{\mu_B H}{2}$ (see the bar graph on page 9). We took $\langle r_{p(A)}^2 - r_{p(B)}^2 \rangle$ as the absolute value of the difference in the squares of the radii of the lines ~~closest~~ adjacent to each other. In this case the wave number difference should be:

$$\bar{\nu}_A - \bar{\nu}_B = \frac{1}{\lambda_A} - \frac{1}{\lambda_B} = \frac{1}{hc}\left(\frac{hc}{\lambda_A} - \frac{hc}{\lambda_B}\right)$$

$$= \frac{\Delta E}{hc} = \frac{\mu_B H}{2hc} = 2.334 \times 10^{-5}\ cm^{-1} \times H_{gauss}$$

Table 2 presents the measurements of $\langle r_{p(A)}^2 - r_{p(B)}^2 \rangle$ for the various values of the magnetic field, as well as the experimental values for $\bar{\nu}_A - \bar{\nu}_B$, for the lines polorized parallel to the field

Table 2

Magnetic field in Kilo Gauss	$\langle r_{p(A)}^2 - r_{p(B)}^2 \rangle$	std. dev.	$\bar{\nu}_A - \bar{\nu}_B$ (experimental)	$\bar{\nu}_A - \bar{\nu}_B$ (theoretical)
0 (not applicable, since there is no splitting in zero field)				
7.2	.0367 (inches)^2	.003 (inches)^2	.1488 cm^{-1}	.1681 cm^{-1}
8.1	.0397 "	.003 "	.1656 cm^{-1}	.1891 cm^{-1}
9.0	.0425 "	.004 "	.1761 cm^{-1}	.2101 cm^{-1}
9.2	.0432 "	.0045 "	.1790 cm^{-1}	.2148 cm^{-1}

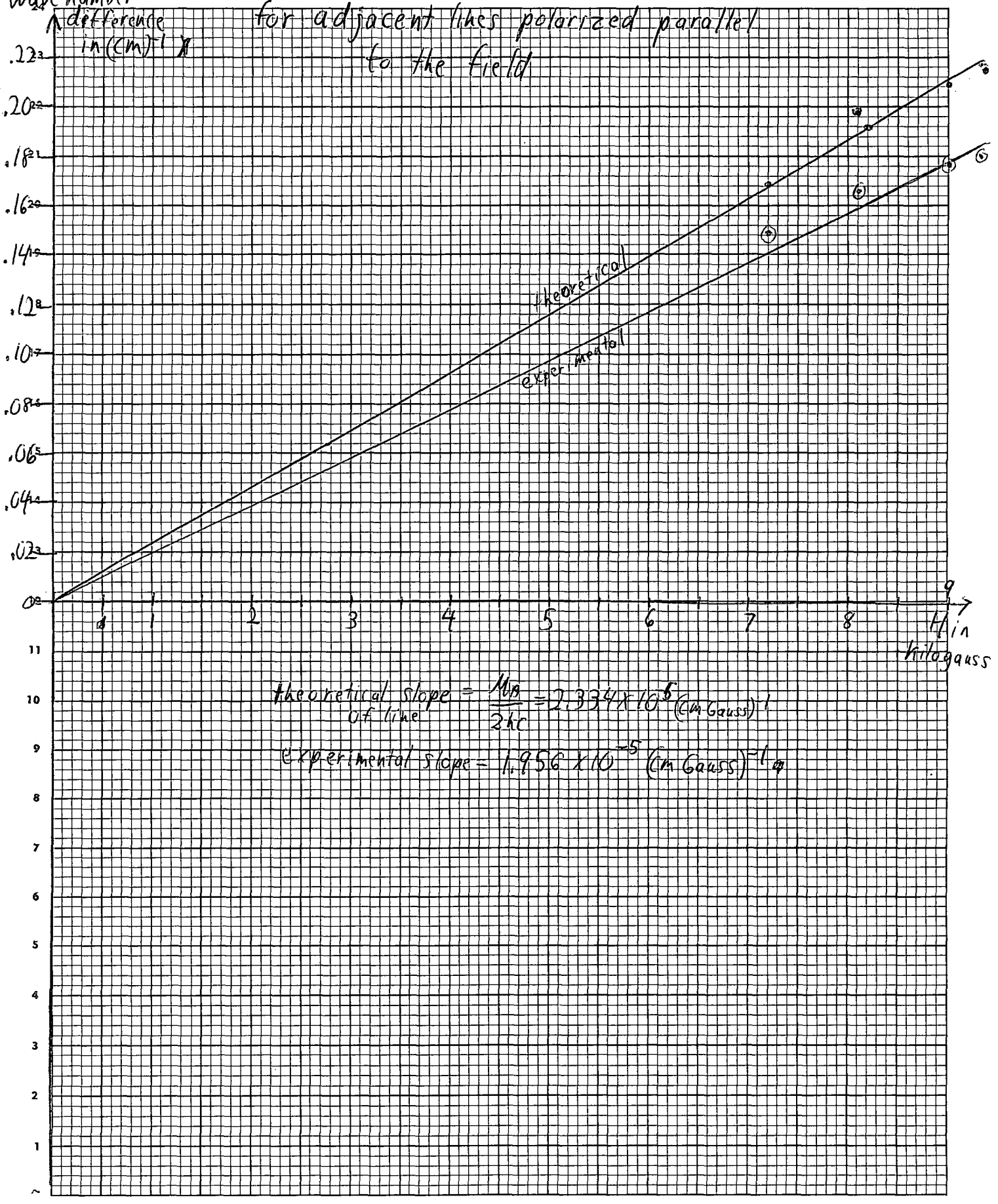
As one can see, all experimental values for $\bar{\nu}_A - \bar{\nu}_B$ are low by between 13% and 20%. Our standard deviations for $\langle r_{p+1}^2 - r_p^2 \rangle$ and $\langle r_{p(A)}^2 - r_{p(B)}^2 \rangle$ would indicate an error in $\bar{\nu}_A - \bar{\nu}_A$ of between 10% and 12%, but of course, the errors should be random rather than all tending to make

$\bar{\nu}_A - \bar{\nu}_B$ too low. The source of bias in the measurements is most likely in the measurement of the magnetic field. In measuring the magnetic field we were careful to move the probe of the gauss-meter around between the poles of the electromagnet until the maximum field was found, and then we recorded this maximum field. The mercury lamp may not feel the maximum field, in fact it certainly doesn't on the edges. The mercury lamp probably feels an average field that is significantly lower than the maximum. This would account for the experimental values of $\bar{\nu}_A - \bar{\nu}_B$ being low, since our theoretical value assumes a higher field than the lamp probably felt. The graph on the next page presents the theoretical versus experimental values of $\bar{\nu}_A - \bar{\nu}_B$ for the different field strengths. The slope of the experimental line should be $\frac{\mu_B}{2hc} = \frac{e}{4\pi mc}$. Using $h = 6.6 \times 10^{-27}$ dyne-cm, $c = 3 \times 10^{10} \frac{cm}{sec}$, and our experimental slope of $1.956 \times 10^{-5} (cm\ Gauss)^{-1}$ we find ~~for~~ our experimental value of the Bohr magneton to be $\mu_B = 2hc(1.956 \times 10^{-5} (cm\ Gauss)^{-1}) \cong 7.56 \times 10^{-21} \frac{ergs}{Gauss}$. This is about 19% below the accepted value of $9.27 \times 10^{-21} \frac{ergs}{Gauss}$, further corroboration that we were probably ~~using~~ recording higher values for the magnetic field than the mercury lamp actually felt.

The lines polarized perpendicular to the magnetic field are harder to interpret. According to theory, there should ~~be~~ not be a middle line, but rather there should be three lines on either side of the original unsplit line. The split lines should have photon energies with respect to the original, unsplit line of $\pm \mu_B H$, $\pm \frac{3}{2} \mu_B H$, and $\pm 2 \mu_B H$ (see graph on page 9).

Our first attempt at measuring these lines was at a low field of 1.7 kilogauss. At this field strength it appeared that the

Wave number difference vs. field strength for adjacent lines polarized parallel to the field



$$\text{theoretical slope of line} = \frac{\mu_B}{2hc} = 2.334 \times 10^{-5} \ (cm\ Gauss)^{-1}$$

$$\text{experimental slope} = 1.956 \times 10^{-5} \ (cm\ Gauss)^{-1}$$

5 SQUARES TO THE CENTIMETER

original, unsplit line had separated into two rather fuzzy lines. This is subject to the obvious interpretation that ~~the~~ each of the two fuzzy lines was actually three components that were unresolved because the field was too low. The separation into two ~~fuzzy~~ smeared lines instead of one large smeared line with all six components is explained by the fact that the two inner lines are split in photon energies by $2\mu_B H$, while the ~~three~~ two three ~~outer~~ lines on either side are separated in energy by only $\frac{\mu_B H}{2}$, hence the field was probably large enough to resolve the two inner lines, but not the closer spacing of the three lines on either side. To test this hypothesis we called the inner radius of each ring ~~the~~ component A, and the outer radius of the ring component B, and we measured $\langle r_{p(A)}^2 - r_{p(B)}^2 \rangle$ and found the wavenumber difference. Theoretically, we would expect that the lines would be split in photon energy by $4\mu_B H$, since we were taking the ~~very~~ outermost and innermost line ~~ring~~ in each spectral order (see graph page 9). The wave number difference should be:

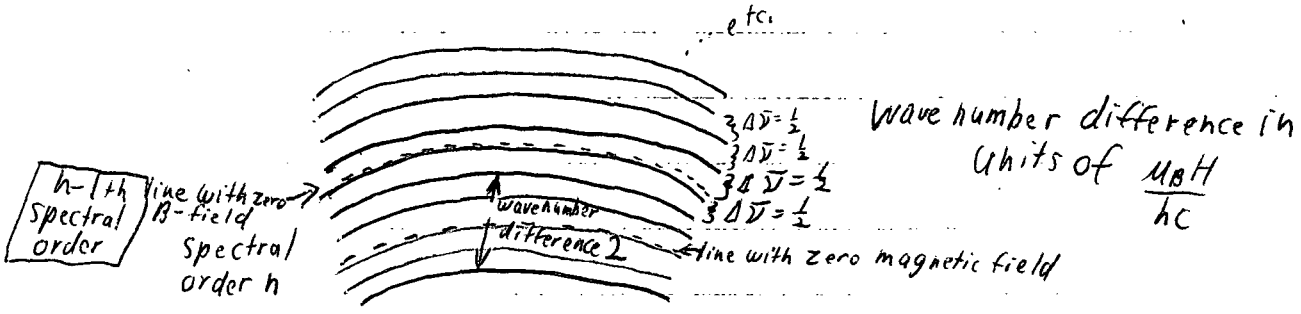
$$\bar{\nu}_A - \bar{\nu}_B = \frac{1}{\lambda_A} - \frac{1}{\lambda_B} = \frac{E_A}{hc} - \frac{E_B}{hc} = \frac{4\mu_B H}{hc}$$

$$H = 1.7 \text{ kilo gauss}$$

$$\therefore \bar{\nu}_A - \bar{\nu}_{B(\text{theoretical})} = 3.176 \times 10^{-4} (\text{cm})^{-1}$$

Our experimental value for $\bar{\nu}_A - \bar{\nu}_B$ was $2.12 \times 10^{-4} (\text{cm})^{-1}$, which is 33% below the theoretical value. This discrepancy is probably explained mostly by the problem of measuring the magnetic field discussed earlier. Since the field strength was so much lower in this measurement than in the measurements of the lines parallel to the field, any error made in measuring the magnetic field, ~~or the~~ is magnified. The inhomogeneity of the field can also have a larger relative effect.

We took other pictures of the lines polarized perpendicular to the field for field strengths between 7.2 k.G. and 9.2 k.G., but most of the pictures were unusable for measurements because the overlapping was difficult to determine. At a field strength of 7.8 k.G., though, the overlapping seemed fairly regular. There were three lines in each spectral order, all evenly spaced, and in fact the spacing between the last component of one spectral order and the first component of the next spectral order was the same as the spacing of all other adjacent lines. The obvious interpretation of this picture is that the three components on either side of the original ~~loop~~ unsplit line are separated by just the right amount for them to overlap with the lines in the next spectral order. Pictorially we can see the effect as this:



With the regular overlapping depicted in the above diagram one can see that the wave number difference ~~with~~ will effectively be $\frac{\mu_B H}{2hc}$ for all adjacent lines. This sort of regular overlapping can occur only if $\langle r^2_{p(A)} - r^2_{p(B)} \rangle$ is a multiple of $\langle r^2_{p+1} - r^2_p \rangle$ which we found to be true for the picture at 7.8 kilogauss. We found $\langle r^2_{p(A)} - r^2_{p(B)} \rangle = .0394$, and $\langle r^2_{p+1} - r^2_p \rangle = .1209$. Our experimental value for the wave number

difference was therefore $\Delta\bar{\nu} = .1625 cm^{-1}$, which is 11% below the theoretical ($\Delta\bar{\nu} = \frac{4\pi H}{2hc}$) value of $.1821 = \Delta\bar{\nu}$. This is in line with our ~~pxcu~~ measurements of the lines parallel to the magnetic field, where we were consistently low by 13-20%. This, of course, does not mean that our interpretation of the overlap on this picture is correct, but at least our interpretation is consistent with our data. Better measurements of the six lines polarized ~~perpl~~ perpendicular to the field would require a Fabry-Perot interferometer of higher finesse ($\mathcal{F} \gtrsim 8$), so that one could resolve the components without overlap.

Conclusion

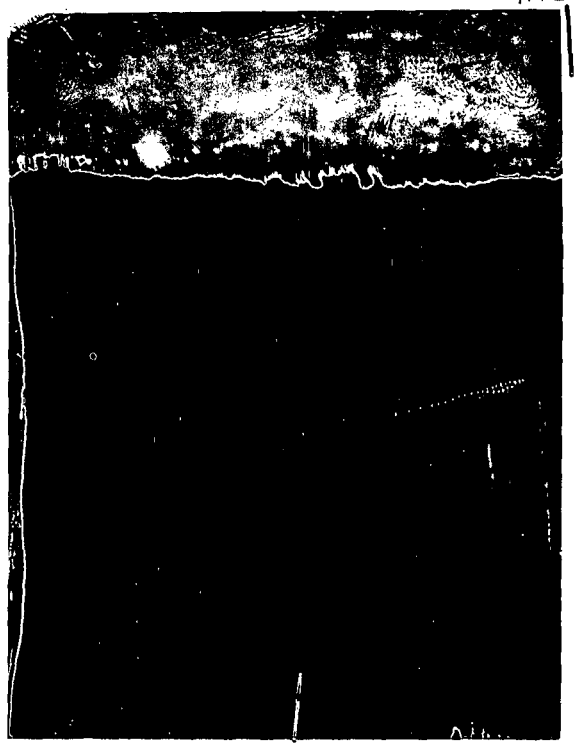
All of our measurements ~~were~~ ~~lower~~ ~~yield~~ yielded lower values of the wavenumber difference than expected. The most likely explanation for this downward bias was that we recorded the value for the maximum magnetic field, while the mercury lamp feels a magnetic field substantially weaker. Measuring the magnetic field felt by the mercury lamp would be difficult since the mercury lamp fills a finite volume over which the magnetic field varies greatly. We can use our values for the wavenumber difference to deduce an approximate value for the actual field felt by the lamp. Since our wavenumber differences were about 15% low, we can say the $\vec{H}$ felt by the mercury lamp is about 15% less than the $\vec{H}$ recorded (which is the maximum field.).

Lines polarized parallel to field

#11

H = 7.2 KG

#12

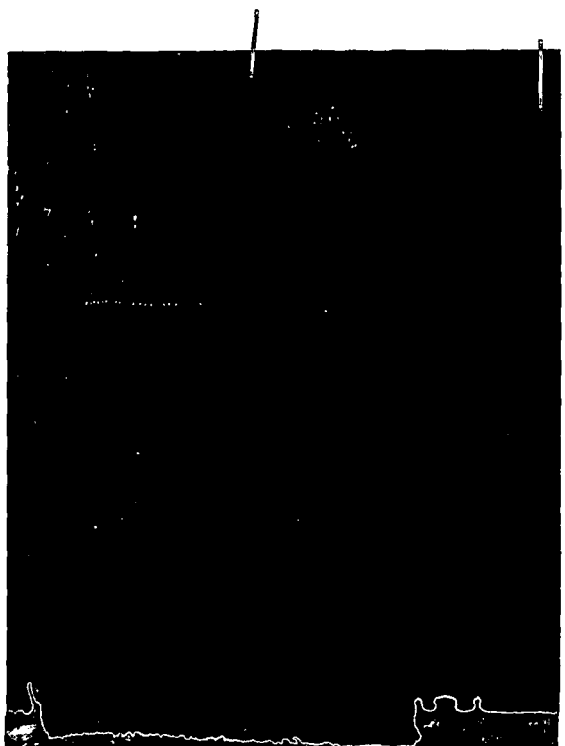


H = 9.0 KG

H = 8.1 KG

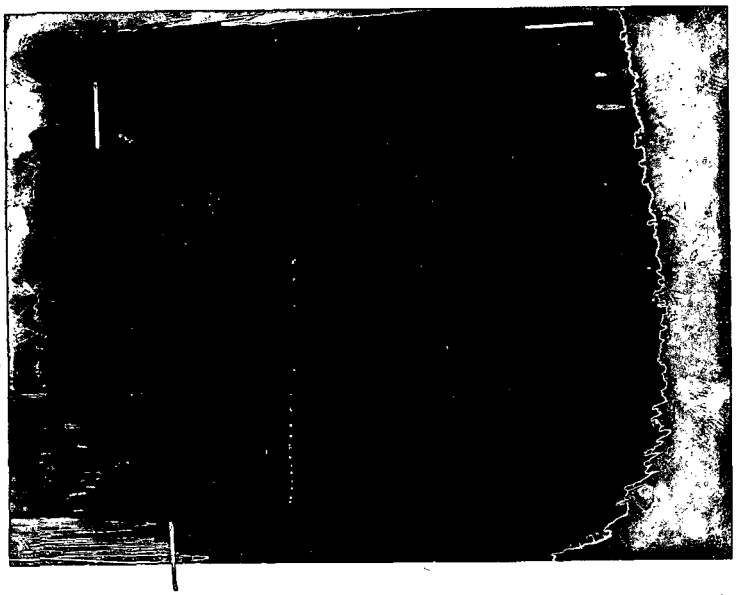
#10

H = 9.2 KG

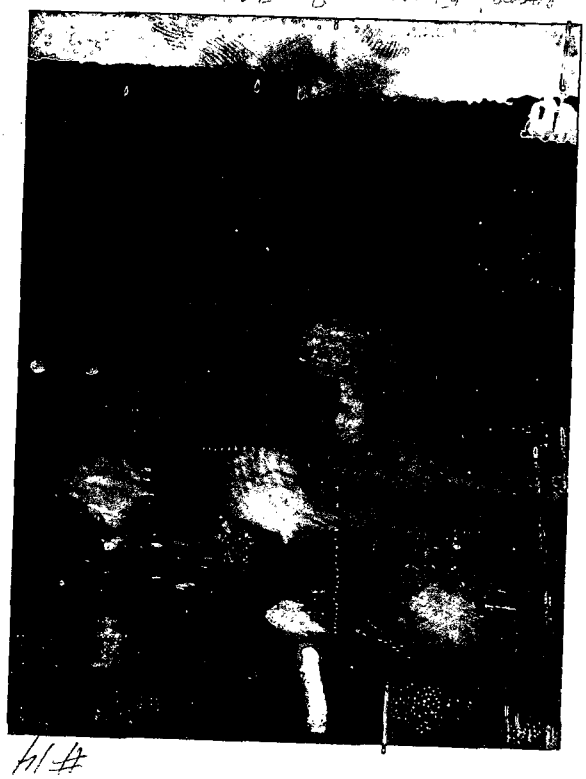


Lines polarized perpendicular to field

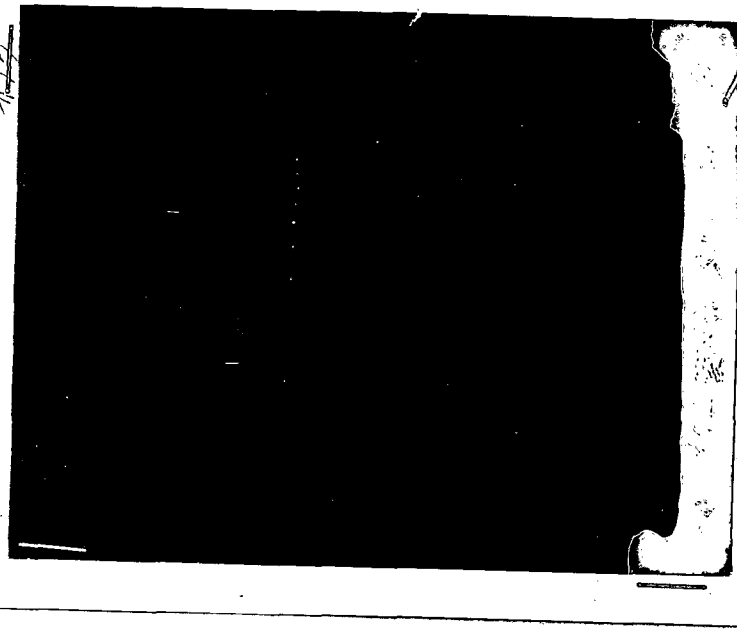
H = 1.7 kG



H = 7.8 kG.



H = 0 (polarizer removed)



References

1. Optics by Hecht and Zajac, (1974); Chapter 9 pages 301-311

2. Lecture Notes for Physics 130B by Professor Sham; (on reserve) Chapter 10

3. Introduction to Quantum Mechanics by R.H Dicke and J.P. Wittke (1960) Chapter 15 pages 272-281

4. Experiments in Modern Physics by Melissinos (1966); Chapter 7