Homework 2: Solutions

[24] The average acceleration $\bar{a}$ of this car during the time interval Δt is the change in velocity Δv divided by Δt:

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

where $v_i = +10.0\ m/s$, $v_f = +6.00\ m/s$, $\Delta t = 3.00\ s$.

Plug the number into the function, then we can get:

$$\bar{a} = \frac{+6.00\ m/s - 10.0\ m/s}{3.00\ s} = \boxed{-1.3\ m/s^2}$$

[25] We assume that during their launch the acceleration of the space is constant. Then we know the average velocity as following:

$$\bar{v} = \frac{v_0 + v_f}{2}$$

where the initial velocity $v_0 = 0.0\ km/s$ and the final velocity $v_f = 10.97\ km/s$. So $\bar{v} = \frac{0.0\ km/s + 10.97\ km}{2} = 5.49\ km/s$.

Then use the function of $\Delta x = \bar{v} t$, where $\Delta x = 220\ m$, we can compute $t = \frac{\Delta x}{\bar{v}} = \frac{220\ m}{5.49\ km/s} = 0.04\ s$

Use the function $\Delta x = v_0 t + \frac{1}{2} a t^2$, we can get:

$$a = \frac{2(\Delta x - v_0 t)}{t^2} = \frac{2 \times 220\ m}{(0.04\ s)^2} = \boxed{2.74 \times 10^5\ m/s^2} >> g$$

2.27) $V_0 = 20\ m/s$ $V_f = 30\ m/s$

USING THE EQUATION

$$V_f^2 - V_0^2 = 2a\Delta x$$

SOLVING FOR a, WE GET $a = 1.3\ m/s^2$.

FOR PART b, USE

$$a = \frac{\Delta V}{t} \Rightarrow t = \frac{\Delta V}{a}$$

$$t = 8.0 \text{ SECONDS}$$

29 Assume the aircraft starts from rest ($v_0 = 0$) as it accelerates down the runway.

(a) Since the acceleration is uniform, we can use the function of $v^2 = v_0^2 + 2a\cdot\Delta x$ during the takeoff run to find:

$$a = \frac{v^2 - v_0^2}{2\cdot\Delta x} = \frac{(120\ km/h)^2 - 0}{2(240\ m)} \cdot \left(\frac{0.278\ m/s}{1\ km/h}\right)^2 = \boxed{2.32\ m/s^2}$$

Note: The problem 25 also can use this function to comput a, which is more convenient.

(b) The time required for the aircraft to reach lift-off speed is given by $v = v_0 + at$ as:

$$t = \frac{v - v_0}{a} = \frac{120\ km/h - 0}{2.32\ m/s^2} \cdot \left(\frac{0.278\ m/s}{1\ km/h}\right) = \boxed{14.4\ s}$$

2.30 (a) The time for the truck to reach 20 m/s is found from $v_f = v_i + at$ as

$$t = \frac{v_f - v_i}{a} = \frac{20\ \text{m/s} - 0}{2.0\ \text{m/s}^2} = 10\ \text{s}.$$

The total time is $t_{total} = 10\ \text{s} + 20\ \text{s} + 5.0\ \text{s} = \boxed{35\ \text{s}}$.

(b) The distance traveled during the first 10 s is

$$(\Delta x)_1 = \bar{v}_1 t_1 = \left(\frac{0 + 20\ \text{m/s}}{2}\right)(10\ \text{s}) = 100\ \text{m}.$$

The distance traveled during the next 20 s (with $a = 0$) is

$$(\Delta x)_2 = (v_i)_2 t_2 + \frac{1}{2} a_2 t_2^2 = (20\ \text{m/s})(20\ \text{s}) + 0 = 400\ \text{m}.$$

The distance traveled in the last 5.0 s is

$$(\Delta x)_3 = \bar{v}_3 t_3 = \left(\frac{20\ \text{m/s} + 0}{2}\right)(5.0\ \text{s}) = 50\ \text{m}.$$

The total displacement is then

$$\Delta x = (\Delta x)_1 + (\Delta x)_2 + (\Delta x)_3 = 100\ \text{m} + 400\ \text{m} + 50\ \text{m} = 550\ \text{m}$$

and the average velocity for the total motion is given by

$$\bar{v} = \frac{\Delta x}{t_{total}} = \frac{550\ \text{m}}{35\ \text{s}} = \boxed{16\ \text{m/s}}.$$

$\boxed{32}$ (a) When the plane experienced the maximum rate of acceleration, the time needed before it comes to rest is the minimum. Use the function $v_f = v_i + at$,

$$t_{min} = \frac{v_f - v_i}{a_{max}} = \frac{0 - 100\ \text{m/s}}{-5.00\ \text{m/s}^2} = \boxed{20\ \text{s}}$$

(b) The shortest runway required for the plane to rest is given by $v_f^2 = v_i^2 + 2a \cdot \Delta X$. So when a is the maximum,

$$\Delta X_{min} = \frac{v_f^2 - v_i^2}{2a_{max}} = \frac{0 - (100\ \text{m/s})^2}{2 \cdot (-5.00\ \text{m/s}^2)} = 1000\ \text{m} = 1.0\ \text{km} > 0.800\ \text{km}$$

Hence the answer is $\boxed{\text{No}}$.

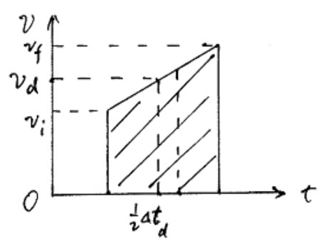
2.35 Using the uniformly accelerated motion equation $\Delta x = v_i t + \frac{1}{2}at^2$ for the full 40 s interval yields $\Delta x = (20\ \text{m/s})(40\ \text{s}) + \frac{1}{2}(-1.0\ \text{m/s}^2)(40\ \text{s})^2 = 0$, which is obviously wrong.

The source of the error is found by computing the time required for the train to come to rest. This time is $t = \frac{v_f - v_i}{a} = \frac{0 - 20\ \text{m/s}}{-1.0\ \text{m/s}^2} = 20\ \text{s}$. Thus, the train is slowing down for the first 20 s and is at rest for the last 20 s of the 40 s interval.

The acceleration is not constant during the full 40 s. It is, however, constant during the first 20 s as the train slows to rest. Application of $\Delta x = v_i t + \frac{1}{2}at^2$ to this interval gives the stopping distance as

$$\Delta x = (20\ \text{m/s})(20\ \text{s}) + \frac{1}{2}(-1.0\ \text{m/s}^2)(20\ \text{s})^2 = \boxed{200\ \text{m}}$$

$\boxed{40}$ From the $v-t$ picture of constant a, we can easily get the opinion that v_d, which is the average velocity, is equal to the instantaneous velocity of the glider when it is halfway through the photogate in terms of time, not in terms of distance.



Since the acceleration is constant,

$$v_d = \frac{v_i + v_f}{2} \quad \text{where } v_f = v_i + a \cdot \Delta t_d$$

$$\Rightarrow v_d = \frac{v_i + v_i + a \cdot \Delta t_d}{2} = v_i + \frac{1}{2} a \cdot \Delta t_d$$

$$= v_i + a \cdot \left(\frac{\Delta t_d}{2}\right) = v\Big|_{t=\frac{1}{2}\Delta t_d}$$

Hence, we $\boxed{\text{argue for (a), against (b)}}$.

In fact, $v\Big|_{x=\frac{1}{2}l} > v_d$ can be found easily from the pic. above (if $a>0$).

2.42 (a) If a car is a distance $s_{stop} = v_0 t_r - \frac{v_0^2}{2a}$ (See the solution to Problem 2.41) from the intersection of length s_i when the light turns yellow, the distance the car must travel before the light turns red is

$$\Delta x = s_{stop} + s_i = v_0 t_r - \frac{v_0^2}{2a} + s_i$$

Assume the driver does not accelerate in an attempt to "beat the light" (an extremely dangerous practice!). The time the light should remain yellow is then the time required for the car to travel distance Δx at constant velocity v_0. This is

$$t_{light} = \frac{\Delta x}{v_0} = \frac{v_0 t_r - v_0^2/2a + s_i}{v_0} = \boxed{t_r - \frac{v_0}{2a} + \frac{s_i}{v_0}}.$$

(b) With $s_i = 16$ m, $v_0 = 60$ km/h, $a = -2.0$ m/s^2, and $t_r = 1.1$ s,

$$t_{light} = 1.1\text{ s} - \frac{60\text{ km/h}}{2(-2.0\text{ m/s}^2)}\left(\frac{0.278\text{ m/s}}{1\text{ km/h}}\right) + \frac{16\text{ m}}{60\text{ km/h}}\left(\frac{1\text{ km/h}}{0.278\text{ m/s}}\right) = \boxed{6.2\text{ s}}.$$

43 (a) We know that freely falling objects have constant acceleration of $a = -g = -9.8\text{ m/s}^2$. We choose upward is the positive DIRECTION

$$h = \frac{v_f^2 - v_i^2}{2a} = \frac{0 - (25.0\text{ m/s})^2}{2\cdot(-9.8\text{ m/s}^2)} = \boxed{31.9\text{ m}}$$

(b) $$\Delta t = \frac{v_f - v_i}{a} = \frac{0 - 25.0\text{ m/s}}{-9.8\text{ m/s}^2} = \boxed{2.55\text{ s}}$$

(c) Since the acceleration is the same, the downward just the reverse process of the upward.

$$\Delta t' = \Delta t = \boxed{2.55\text{ s}}$$

(d) The same reason with (c), we can get:

$$v' = -v = \boxed{-25.0\text{ m/s}}$$

2.46 Use $\Delta y = v_i t + \frac{1}{2}at^2$, with $v_i = 0$, $a = -9.80$ m/s^2, and $\Delta y = -76.0$ m to find

$$t = \sqrt{\frac{2(\Delta y)}{a}} = \sqrt{\frac{2(-76.0\text{ m})}{-9.80\text{ m/s}^2}} = \boxed{3.94\text{ s}}.$$

[49] There are two periods, each of which has its own constant acceleration. Take upward the positive vertical direction. ⑦

(a) In the first period, $a_1 = 2.00\,m/s^2$ $\quad v_{01} = 50.0\,m/s$.

$h_1 = 150\,m$; $2a_1 h_1 = v_{f1}^2 - v_{i1}^2$ where $v_{f1} = v_{i2}$

$$\Rightarrow v_{f1} = \sqrt{v_{01}^2 + 2a_1 h_1} = \sqrt{(50.0\,m/s)^2 + 2\,(2.00\,m/s^2)\cdot(150\,m)} = 55.7\,m/s$$

In the second period, i.e. after engines stop, $a_2 = -g = -9.8\,m/s^2$.

$$h_2 = \frac{v_{f2}^2 - v_{i2}^2}{2a_2} = \frac{0 - (55.7\,m/s)^2}{2\cdot(-9.8\,m/s^2)} = 158\,m$$

$$h = h_1 + h_2 = 150\,m + 158\,m = \boxed{308\,m}$$

(b)
$$\Delta t_2 = \frac{v_{f2} - v_{i2}}{a_2} = \frac{0 - 55.7\,m/s}{-9.8\,m/s^2} = 5.7\,s$$

$$\Delta t_1 = \frac{v_{f1} - v_{i1}}{a_1} = \frac{55.7\,m/s - 50.0\,m/s}{2\,m/s^2} = 2.8\,s$$

$\Rightarrow$

$$\Delta t = \Delta t_1 + \Delta t_2 = 5.7\,s + 2.8\,s = \boxed{8.5\,s}$$

(c) After the rocket reached the highest, it spend Δt_3 to fall to the grand. $a_3 = -g = a_2$

Since $-h = v_{f2}\,\Delta t_3 + \frac{1}{2}a_3\,\Delta t_3^2 = 0 + \frac{1}{2}a_3\,\Delta t_3^2$

$$\Delta t_3 = \sqrt{\frac{-2h}{a_3}} = \sqrt{\frac{-2\cdot(308\,m)}{-9.8\,m/s^2}} = 7.9\,s$$

$$\Delta t' = \Delta t_3 + \Delta t = 8.5\,s + 7.9\,s = \boxed{16.4\,s}$$

[51] Taking upward as the positive vertical direction, the keys have a constant acceleration of $a = -g = -9.8\,m/s^2$ and undergo an upward displacement of $\Delta y = +4.00\,m$ during the 1.50 s time interval.

(a) Hence, $\Delta y = v_0 t + \frac{1}{2}at^2$ gives the velocity at the beginning of this interval as follows:

$$v_0 = \frac{\Delta y - \frac{1}{2}at^2}{t} = \frac{4.00\,m - \frac{1}{2}(-9.8\,m/s^2)(1.50\,s)^2}{1.50\,s} = \boxed{+10.0\,m/s}$$

(b) The velocity of the keys 1.50 s later (just before they are caught) is as follows:

$$v = v_0 + at = 10.0\,m/s + (-9.80\,m/s^2)(1.50\,s) = -\boxed{4.68\,m/s}$$

63 (a) The whole displacement is $\Delta x = 17500\text{ ft}$, during the time $\Delta t = t_1 + t_2 = 90\text{ s}$. ①

$\Delta x = x_1 + x_2$ $\quad a_1 = 40\text{ ft/s}^2$ ② $\quad v = v_0 + a_1 t_1 = a_1 t_1$

where $x_1 = v_0 t + \frac{1}{2}a_1 t_1^2 = 0 + \frac{1}{2}a_1 t_1^2 = \frac{1}{2}a_1 t_1^2$

$x_2 = v t_2 = a_1 t_1 t_2$

$\Rightarrow \quad \Delta x = x_1 + x_2 = \frac{1}{2}a_1 t_1^2 + a_1 t_1 t_2 = 17500\text{ ft}$ ③

Combine equation ①② and ③, we can get:

$t_1 = \boxed{5.0\text{ s}}$, $t_2 = \boxed{85\text{ s}}$

(b) $v = a_1 t_1 = 40\text{ft/s}^2 \cdot (5.0\text{s}) = \boxed{200\text{ ft/s}}$ from part (a).

(c) The displacement from the point where sled started to slow down is $x_3 = \frac{v_f^2 - v^2}{2a_3}$ where $v_f = 0$, $a_3 = -20\text{ft/s}^2$.

Thus, $x_3 = \frac{v_f^2 - v^2}{2a_3} = \frac{0 - (200\text{ ft/s})^2}{2(-20\text{ft/s}^2)} = 1000\text{ ft}$.

Hence the total displacement from the starting point is

$x = \Delta x + x_3 = 17500\text{ ft} + 1000\text{ ft} = \boxed{18500\text{ ft}}$.

(d) From starting to slow down, the time spent is

$t_3 = \frac{v_f - v}{a_3} = \frac{0 - 200\text{ ft/s}}{-20\text{ ft/s}^2} = \boxed{10\text{ s}}$

So total trip time should be:

$t = t_1 + t_2 + t_3 = 90\text{ s} + 10\text{ s} = \boxed{100\text{ s}}$.

2.65 (a) Since the sound has constant velocity, the distance it traveled is

$$\Delta x = v_{sound}t = (1100\text{ ft/s})(5.0\text{ s}) = \boxed{5.5\times10^3\text{ ft}}$$

(b) The plane travels this distance in a time of $5.0\text{ s} + 10\text{ s} = 15\text{ s}$, so its velocity must be

$$v_{plane} = \frac{\Delta x}{t} = \frac{5.5\times10^3\text{ ft}}{15\text{ s}} = \boxed{3.7\times10^2\text{ ft/s}}$$

(c) The time the light took to reach the observer was

$$t_{light} = \frac{\Delta x}{v_{light}} = \frac{5.5\times10^3\text{ ft}}{3.00\times10^8\text{ m/s}}\left(\frac{1\text{ m/s}}{3.281\text{ ft/s}}\right) = 5.6\times10^{-6}\text{ s}.$$

$\boxed{\text{During this time the plane would only travel a distance of 0.002 ft}}$

70.) IN 10 MIN, SHE HAS RUN 1143 METER

$$\frac{500 \text{ yards}}{} \left| \frac{.94 \text{ meter}}{\text{yard}} \right| = 457 \text{ METERS LEFT TO GO.}$$

HER INITIAL VELOCITY IS

$$\frac{1143 \text{ m}}{} \left| \frac{}{10 \text{ min}} \right| \frac{1 \text{ min}}{60 \text{ sec}} = 1.9 \text{ m/s}$$

IF SHE ACCELERATES AT .15 m/s², THEN TO MAKE IT THE FOLLOWING STATEMENT MUST BE TRUE:

$$① \quad 457\text{m} \leq 1.9\text{m/s}\,(120 \text{ sec}) + \frac{1}{2}(.15\text{m/s}^2)(120^2 s^2)$$

FROM $x = V_0 t + \frac{1}{2}at^2$ AND 2 MIN = 120 SEC.

SHE WILL EASILY MAKE IT; EQ ① = 1308 m.
HOWEVER, IT RELIES ON A BAD ASSUMPTION.
HER FINAL VELOCITY, ASSUMING SHE ACCELERATED THE ENTIRE TIME, WOULD BE

$$1.9\text{m/s} + 18\text{m/s} = 19.9\text{m/s}.$$

FOR COMPARISON, MAURICE GREEN (THE FASTEST HUMAN) RAN 100m IN 9.79 s, AT A SPEED OF 10.2 m/s. SOMEHOW I DOUBT THE GIRL IN THE PROBLEM CAN MAINTAIN HER ACCELERATION OF .15 m/s² FOR THE DURATION.