Electrostatics

The force on charge q_1 from charge q_2 is $\vec{F}_{12} = k_e \frac{q_1 q_2}{r_{12}^2}\hat{r}_{12}$, where the direction vector $\hat{r}_{12}$ points from q_2 to q_1 and the proportionality constant is $k_e = 8.99\text{x}10^9\ \text{Nm}^2/\text{C}^2$.

Note that the permittivity of free space is $\varepsilon_0 \equiv 1/(4\pi k_e) = 8.85\text{x}10^{-12}\ \text{C}^2/(\text{Nm}^2) = 8.85\text{x}10^{-12}\ \text{A}^2\text{s}^4/(\text{kg m}^3)$.

Note that the unit of elemental electronic change is $e^- = -1.60\text{x}10^{-19}\text{C}$.

The force on a test charge q_0 induced by an electric field, denoted $\vec{E}$, is $\vec{F} = q_0\vec{E}$.

Fields and Potentials

The electric flux through a surface is $\Phi_e \equiv \sum_{All\ Surfaces} E_\perp \Delta A$, where $E_\perp = E\ cos\theta$ is the component of the field that parallels the norm to the surface; θ is the angle between the direction of the field and the norm.
s
Gauss' Law relates the total flux through a closed surface to the net charge enclosed by the surface, *i.e.*, $\Phi_e = 4\ \pi\ k_e\ Q_{Total}$.

The electric field produced by a point charge q at the origin, *i.e.*, $\vec{r} = 0$, is $\vec{E} = k_e \frac{q}{r^2}\hat{r}$ where $\hat{r}$ is the radius vector in spherical coordinates.

The electric field produced by a line charge, with charge per unit length λ, is $\vec{E} = 2k_e \frac{\lambda}{r}\hat{r}$, where the line is defined to lie along the $\hat{z}$ axis and $\hat{r}$ is the radius vector in cylindrical coordinates.

The electric field produced by a surface charge, with charge per unit area σ, is $\vec{E} = 2\pi k_e \sigma\ \hat{n}$, where the surface lies in the $\hat{x}$-$\hat{y}$ plane and $\hat{z}$ corresponds to the normal to the $\hat{x}$-$\hat{y}$ plane in Cartesian coordinates.

Work-Energy Theorem: $W = \Delta KE + \Delta PE$

Electric potential: $\Delta V = -E\ \Delta x\ cos\ \theta$, where $\Delta V = \frac{\Delta PE}{Q}$

$V = k_e \frac{q}{r}$ a distance r away from a point charge q.

Current, Resistance and Capacitance

Current: $I = \frac{\Delta Q}{\Delta t}$ or $I = n\ e\ v_D\ A$ where n is the density of charge carriers, v_D is the drift velocity and A is the cross-section of the wire.

Capacitance: $Q = C\ \Delta V$ where $C = \frac{\kappa}{4\pi k_e}\frac{A}{d}$ for parallel plates and κ is the dielectric constant

$I = C\frac{\Delta V}{\Delta t}$

Energy Stored $= \frac{1}{2}Q\ \Delta V = \frac{1}{2}C\ (\Delta V)^2 = \frac{1}{2C}Q^2$

Resistance: V = I R where R = $\rho \frac{L}{A}$; ρ is the resistivity in Ohm-m and L is the length of the wire.

Kirchoff's Laws: 1) Sum of voltage drops around any loop is zero, *i.e.*, gains = losses

2) Sum of current flow at a node is zero, *i.e.*, total current in = total current out

Power Dissipated = $IV = I^2R = V^2/R$

A resistor/capacitor pair charges with a characteristic time, given by the product of R and C, i.e., $\tau = RC$.

Magnetostatics (Electrostatics in the Fast Lane)

Force on a test charge q_0 induced by electric field $\vec{E}$ and magnetic field $\vec{B}$, is $\vec{F} = q_0\vec{E} + q_0\vec{v}\times\vec{B}$. The cross product and has magnitude $|\vec{v}||\vec{B}|\sin\theta$ and a direction that is found from the "right hand rule".

Force per unit length on a straight wire that carries a current I (where I has direction given by the motion of positive charge carriers) is $\vec{F}/l = \vec{I}\times\vec{B}$. The cross product is defined as above.

The force per unit length between two straight wires that carry current I_1 and I_2 respectively, and are separated by a distance d, is F/L = $\mu_0 \frac{I_1 I_2}{2\pi d}$, where $\mu_0 = 1.3\text{x}10^{-6}$ T m / A.

For completeness, $\mu_0\varepsilon_0 = 1/c^2$, where c is the speed of light.

The torque between a loop of cross-sectional area A that carries a current I and a uniform magnetic fields is $\vec{\tau} = \vec{\mu}\times\vec{B}$, where $\mu = IA$ is known as the magnetic moment.

When the loop contains multiple turns of wire, $\mu \rightarrow NIA$ where N is the number of turns.

Ampere's Law relates the magnetic field in a loop to the current, *i.e.*, $I = \frac{1}{\mu_0}\sum_{All\ Segments} B_{\|}\Delta L$

For a straight wire, $\vec{B} = \mu_0 \frac{I}{2\pi R}\left(\hat{\theta}\right)$, where the direction is given by the right-hand rule and R is the distance from the wire.

For a solenoid, $\vec{B} = \mu_0 \frac{N}{L} I\left(\hat{z}\right)$.

Faraday's Law relates the change in magnetic flux, $\phi_{\text{Magnetic}} \equiv \sum_{All\ Surfaces} B_{\perp}\Delta A$, to the potential V, or the current, V/R, induced in a loop, *i.e.*, $V = -\,N\,\frac{\Delta\phi_{\text{Magnetic}}}{\Delta t}$ where N is the number of turns in the loop and the minus sign implies that the induced field - caused by the induced current - opposes the original field.

Generator: $V = NBA\,\omega\sin\omega t$ where ω is the rotational frequency in radians/s (recall $\omega = 2\pi f$).

Inductance: $N\Delta\phi = L\,\Delta I$ where $L = \mu_o\left(\frac{N}{l}\right)^2 Al$ for a solenoid with N/l turns/length, cross-sectional area A, and length l.

$$V = L\frac{\Delta I}{\Delta t}$$

$$\text{Energy Stored} = \frac{1}{2} I\ N\varphi_{\text{Magnetic}} = \frac{1}{2} L\ I^2$$

A resistor/inductor pair has a characteristic time for a voltage change, given by the quotient of L and R, i.e., $\tau = L/R$.

AC Circuit Analysis

A resistor has an impedance of R and a voltage that faithfully tracks the current, *i.e.*, if $I_R(t) = I_0 \sin(2\pi ft)$, then $V_C(t) = I_0 R \sin(2\pi ft)$.

A capacitor has an impedance of $Z_C = 1/(2\pi fC)$ and a voltage that lags the current by 90°, *i.e.*, if $I_C(t) = I_0 \sin(2\pi ft)$, then $V_C(t) = I_0/(2\pi fC) \sin(2\pi ft-\pi/2)$.

An inductor has an impedance of $Z_L = 2\pi fL$ and a voltage that leads the current by 90°, *i.e.*, if $I_L(t) = I_0 \sin(2\pi ft)$, then $V_L(t) = 2\pi fL\ I_0 \sin(2\pi ft+\pi/2)$.

The current flow and voltage drops in a loop containing an inductor and capacitor has a resonance when the driving frequency satisfies

$$f_{\text{resonance}} = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$$

Electromagnetic Radiation

Dispersion relation in the vacuum is:

$$f\lambda = c,$$

where f is the frequency of the radiation, λ is the wavelength, and c is the speed of light, with $c = 2.99 \times 10^8$ m/s.

The ratio of the electric to magnetic field in an electromagnetic wave equals the speed of light, i.e.:

$$\frac{E(t)}{B(t)} = c$$

Electromagnetic waves carry energy as they travel; the average power per unit area is the intensity, denoted *I*, with:

$$I = \frac{1}{2\mu_0} E_{\max} B_{\max} = \frac{1}{2\mu_0 c} E_{\max}^2 = \frac{c}{2\mu_0} B_{\max}^2$$

	Series	**Parallel**
Capacitors	$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$	$C_{eq} = C_1 + C_2 + C_3 + \cdots$
Inductors	$L_{eq} = L_1 + L_2 + L_3 + \cdots$	$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots$
Resistors	$R_{eq} = R_1 + R_2 + R_3 + \cdots$	$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$

where $E_{\max}$ and $B_{\max}$ are the maximum values of the electric and magnetic fields.

Finally – the Final!

1. The circuit in figure 1 is a minimal model for the intracellular potential, $V_{Cardiac}$, of a cardiac cell during the plateau phase of the cardiac action potential. What is the expression for $V_{Cardiac}$ in terms of R_{Leak}, R_{Ca}, V_{Leak}, and V_{Ca}? Mind the signs!

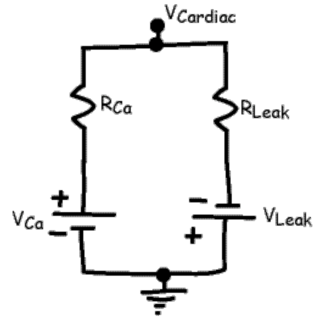

Figure 1

A. $(R_{Leak}V_{Ca} - R_{Ca}V_{Leak})/(R_{Leak}+ R_{Ca})$
B. $(R_{Ca}V_{Ca} - R_{Leak}V_{Leak})/(R_{Leak}+ R_{Ca})$
C. $V_{Ca} - V_{Leak}$
D. $(R_{Leak}V_{Ca} + R_{Ca}V_{Leak})/(R_{Leak}+ R_{Ca})$
E. $(R_{Ca}V_{Ca} + R_{Leak}V_{Leak})/(R_{Leak}+ R_{Ca})$

2. A circuit consists of a series combination of two capacitors, with values C_1 and C_2, and a resistor, with value R, as shown in figure 2. If the switch is closed at time t = 0, what is the value of the current immediately after the switch is thrown?

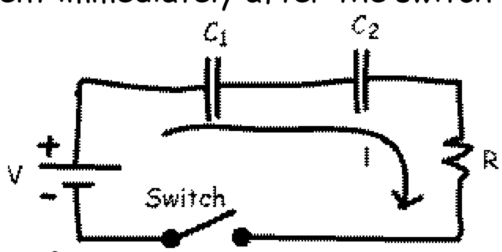

Figure 2

A. $(V/R)[(C_1C_2)/(C_1 + C_2)^2]$
B. V/R
C. 0
D. $(V/R)\ [1 – (C_1C_2)/(C_1 + C_2)^2]$
E. Insufficient information to answer

3. For the above circuit (Figure 3) with the switch is closed at time t = 0, what is the value of the current a very long time after the switch is thrown?
A. $V/R[(C_1C_2)/(C_1 + C_2)^2]$
B. V/R
C. 0
D. $(V/R)\ [1 – (C_1C_2)/(C_1 + C_2)^2]$
E. Insufficient information to answer

4. Two positive charges of strength $Q_1 = +1.0\times10^{-3}$ Coulombs sit along the x-axis as shown in figure 3, with L = 10 cm. What is the force on a negatively charged particle of strength $Q_2 = -1.0\times10^{-6}$ Coulombs that sits at (x, y) = (L, L)? Recall that $+\hat{x}$ points to the right along x and $+\hat{y}$ points up along y.

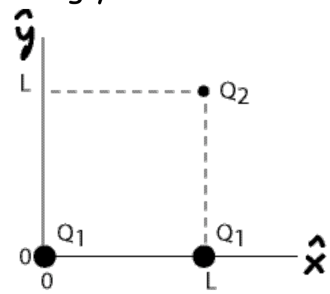

Figure 3

A) $-3.2\times10^2\ \hat{x}\ -1.2\times10^3\ \hat{y}$
B) $-3.2\times10^2\ \hat{x}\ -9.2\times10^2\ \hat{y}$
C) $-4.5\times10^2\ \hat{x}\ +1.3\times10^3\ \hat{y}$
D) $-4.5\times10^2\ \hat{x}\ + 0\ \hat{y}$
E) $-4.5\times10^2\ \hat{x}\ -1.3\times10^3\ \hat{y}$

5. A parallel plate capacitor is formed from two square plates with area A = 5000 cm^2, spaced d = 1.0 mm apart, as in figure 4. The gap between the plates is filled with air (dielectric constant $\kappa = 1$). What is the capacitance?

Figure 4

A. 2.2×10^{-9} F
B. 4.4×10^{-9} F
C. 2.2×10^{-7} F
D. 4.4×10^{-7} F
E. 8.8×10^{-7} F

6. A negative charged particle is placed in a uniform electric field that points to the left. It is initially at rest. Neglect gravity. What happens?

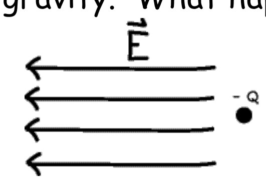

Figure 5

A. The particle accelerates to the right gaining both kinetic and potential energy
B. The particle accelerates to the right gaining kinetic but losing potential energy
C. Nothing – a particle at rest remains at rest
D. The particle accelerates to the left gaining kinetic but losing potential energy
E. The particle accelerates to the left gaining both kinetic and potential energy

7. A battery supplies current to a circuit with 4 resistors, as shown in figure 6. What is the correct expression for the potential V_1?

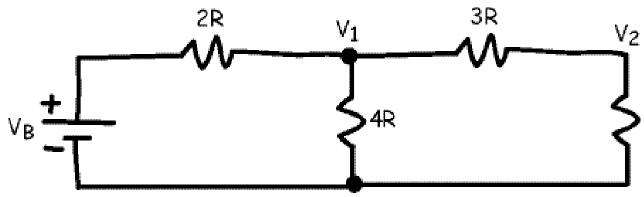

Figure 6

A. 1/4 V_B
B. 1/3 V_B
C. 1/2 V_B
D. 2/3 V_B
E. 3/4 V_B

8. What is the correct expression for the potential V_2 in figure 6?
A. 1/8 V_B
B. 1/4 V_B
C. 1/3 V_B
D. 1/2 V_B
E. 3/4 V_B

9. A positron, with elemental charge + e and velocity **v**, enters a region with both a uniform magnetic field, **B** = 0.5 T (⊗), and a uniform electric field, **E** = -1.0×10^4 V/m ($\hat{y}$), i.e., the particle enters from the left, the magnetic field points into the paper and the electric field points downward, as shown below in figure 7. Is there an initial nonzero velocity for which the electron is not deflected and, if so, what is that velocity?

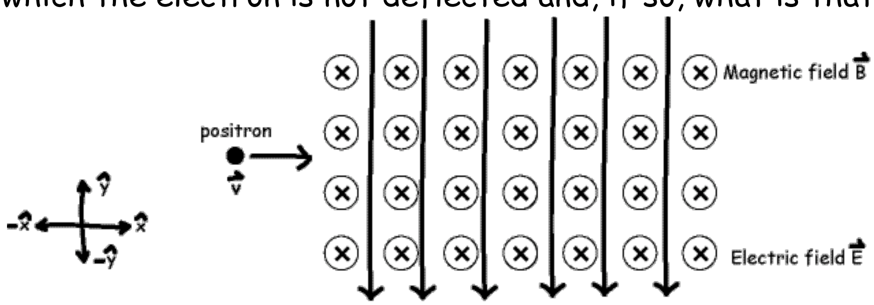

Figure 7

A. Yes: 0.5×10^4 m/s ($+\hat{x}$)
B. Yes: 1.0×10^4 m/s ($+\hat{x}$)
C. Yes: 2.0×10^4 m/s ($+\hat{x}$)
D. No: the electron always is deflected downward ($-\hat{y}$ direction).
E. No: the electron always is deflected upward ($+\hat{y}$ direction).

10. Two parallel wires are separated by a distance d = 1.0 μm and carry equal but opposite currents of I = 1.0 mA each, as shown in figure 8. What is the value of the magnetic field – magnitude and sign – at point A?

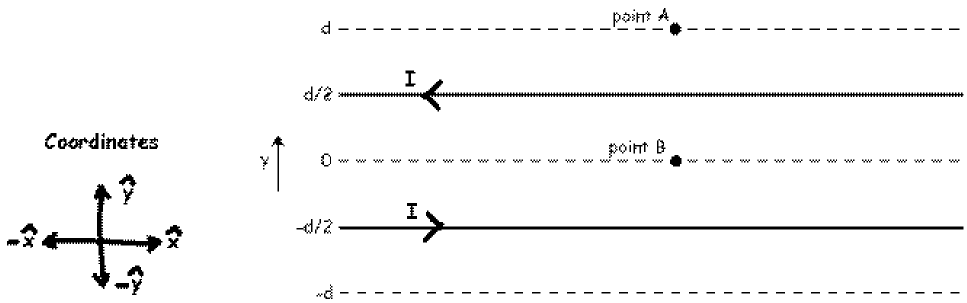

Figure 8

A. 2.7×10^{-4} T ($-\hat{y}$); downward
B. 2.7×10^{-4} T ($\otimes$); into page
C. 5.3×10^{-4} T ($\otimes$); into page
D. 2.7×10^{-4} T ($\odot$); out of page
E. 5.3×10^{-4} T ($\odot$); out of page

11. With reference to the same arrangement of current flow (figure 8 and parameters as in question 10), what is the value of the magnetic field – magnitude and sign – at point B?

A. 0 T
B. 4.0×10^{-4} T ($\otimes$); into page
C. 4.0×10^{-4} T ($\odot$); out of page
D. 8.0×10^{-4} T ($\otimes$); into page
E. 8.0×10^{-4} T ($\odot$); out of page

12. Two inductors, each with value L = 12 mH, are connected in series with a capacitor, with value C = 470 nF, and driven by a variable frequency sinusoidal voltage, as shown in figure 9. For what value of frequency, f, is the current flow in the circuit at resonance?

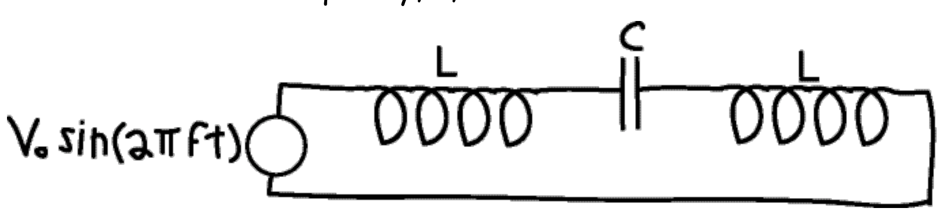

Figure 9

A. 1.5 Hz
B. 1.5 kHz
C. 3.0 kHz
D. 3.0 MHz
E. The value depends on V_0 as well as on L and C.

13. A sinusoidal voltage source, $V_0 \sin(2\pi ft)$ drives a circuit that consists of a resistor and capacitor in series, as shown in figure 10. A current I(t) flows through the circuit elements. Which statement is true under steady-state conditions?

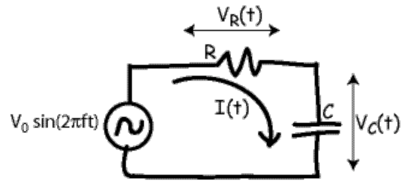

Figure 10

A. The current through the capacitor and the resistor are in phase with each other.
B. The current through the capacitor and the resistor vary 90^0 out of phase with respect to each other.
C. The voltage drop across the capacitor, $V_C(t)$, and the resistor, $V_R(t)$, are in phase with each other.
D. The voltage drops $V_C(t)$ and $V_R(t)$ are 180^0 out of phase with respect to each other.
E. No current can flow through the capacitor under steady-state conditions.

14. A square loop of wire moves at constant speed across a uniform magnetic field that is confined to a square region of space, as shown in figure 11. The sides of the loop are one-half the length of the sides of the region with the field. Which of the following best describes the behavior of the induced current?

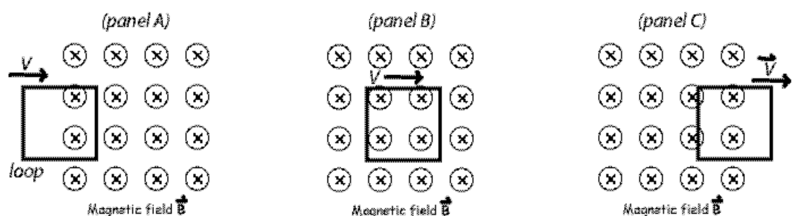

Figure 11

A. The current flow is zero (panel A), then clockwise (CW) (panel B), then zero (panel C).
B. The current flow is zero (panel A), then counterclockwise (CCW) (panel B), then zero (panel C).
C. The current flow is CW (panel A), then zero (panel B), then CCW (panel C).
D. The current flow is CCW (panel A), then zero (panel B), then CW (panel C).
E. The current flow is CW (panel A), then zero (panel B), then CW (panel C).

15. Each wire of a high-voltage transmission line (700,000 V with respect to ground) carries 1,000 A for a distance of 100 km. If the resistance of the wire is 0.5 Ω/km, what is the voltage drop caused by resistive losses?
A. 50 V
B. 500 V
C. 50,000 V
D. 650,000 V
E. 750,000 V

16. For the circuit in figure 12, each of the resistors has a value of $R = 2.0\ \Omega$, while the light bulb has a resistance of $R_{bulb} = 1.0\ \Omega$. The battery has a potential difference of $V_B = 14$ V. Which of the following statements best corresponds to the potential drop across the bulb?

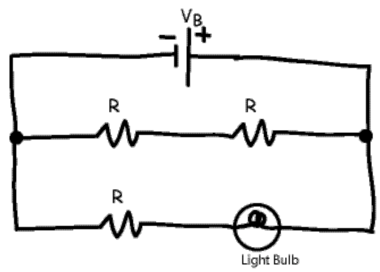

Figure 12

A. 4.7 V
B. 7.0 V
C. 9.4 V
D. 12.0 V
E. 14.0 V

17. Consider an electron that moves within a plane that lies normal to a uniform magnetic field, as denoted in figure 13. Neglect the force of gravity. Which of the following choices best describes the trajectory of the electron?

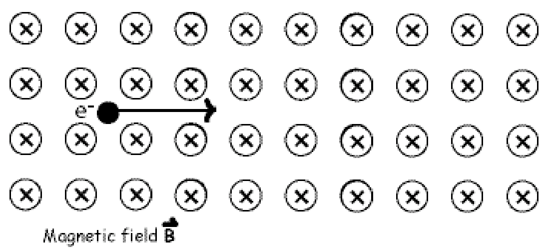

Figure 13

A. The electron will move in a straight line.
B. The electron will slow down and come to a stop.
C. The electron will move in a clockwise (CW) circle.
D. The electron will move in a counterclockwise (CCW) circle.
E. Without knowing the precise speed and value of the magnetic field, the motion cannot be specified.

18. An AC circuit with a frequency of $f = 100$ Hz includes an inductor. The inductive impedance (reactance) is $Z_L = 50\ \Omega$. What is the value of the inductance (L)?

A. 20 mH
B. 80 mH.
C. 240 mH.
D. 500 mH.
E. 740 mH.

19. A resistive network consists of a battery and 4 resistors arranged as a parallel combination of 3 resistors in series with the fourth resistor, as shown in figure 14. What is the current that flows through the 10 Ω resistor?

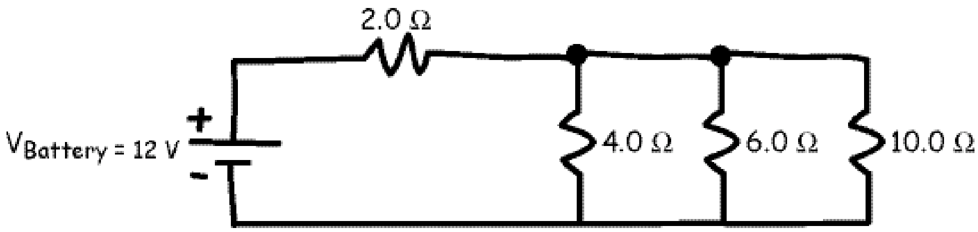

Figure 14

A. 0.18 A
B. 0.25 A
C. 0.59 A
D. 1.0 A
E. 1.2 A

20. The inductance of an inductor depends upon:
A. The amount of current, I, that flows through the inductor.
B. The time rate of change of current, dI/dt, in the inductance.
C. The geometry of the inductor.
D. The frequency, f, of the alternating current or voltage source that drives the circuit.
E. The overall impedance, Z, of the circuit that contains the inductor.

21. The capacitance of a capacitor depends upon:
A. The drop in voltage, V, across the inductor.
B. The time rate of change of voltage, dV/dt, across the capacitance.
C. The geometry of the capacitor.
D. The frequency, f, of the alternating current or voltage source that drives the circuit.
E. The overall impedance, Z, of the circuit that contains the capacitor.

22. An airplane with a wingspan of 60.0 m flies parallel to the Earths surface at a point where the vertical (downward) component of the Earth's magnetic field is 0.4×10^{-4} T. If the induced potential between the tips of the wings is 0.90 V, what is the speed of the plane relative to the ground?

A. 250 m/s
B. 338 m/s
C. 375 m/s
D. 417 m/s
E. 750 m/s

23. A 100 m long conducting wire used for animal tracking has a radius of 25 μm and a resistance of 150 Ω. What is the resistivity of the metal used to make the wire?

A. 2.9×10^{-9} Ω/m
B. 7.6×10^{12} Ω/m
C. 2.9×10^{-9} $\Omega \bullet$m
D. 2.9×10^{3} $\Omega \bullet$m
E. 7.6×10^{12} $\Omega \bullet$m

24. An MRI machine for rodents has a solenoid coil that produces a field of 5.0 T with a 10 cm diameter bore (5.0 cm solenoid radius) and a 50 cm length. If the maximum current that NbTn superconducting wire can tolerate in 800 A, how may turns of wire are necessary to achieve the required field.

A. 25
B. 2500
C. 5000
D. 5.0×10^{4}
E. 2.5×10^{5}

25. What is the electric field at the origin, where q_o is located, for the configuration of positive and negative elemental changes ($|e| = 1.6 \times 10^{-19}$ Coulombs) along a line shown in figure 15?

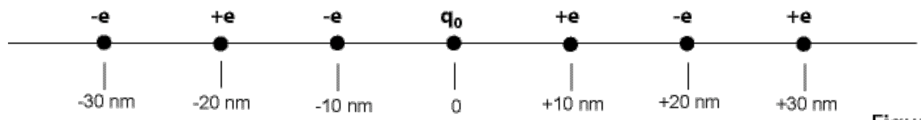

Figure 15

A. 1.2×10^7 N/C; to the right ($+\hat{x}$)
B. 2.5×10^7 N/C; to the right ($+\hat{x}$)
C. 0
D. 1.2×10^7 N/C; to the left ($-\hat{x}$)
E. 2.5×10^7 N/C; to the left ($-\hat{x}$)

26. A dish antenna with a radius of 30.0 m receives, at normal incidence to the dish, a radio signal from a distant star, as shown in figure 16. The radio signal is a continuous sinusoidal wave whose electric field has an amplitude of E_0 = 0.10 V/m. Assume that the antenna absorbs all the radiation that falls on the dish. What is the intensity, I, of the radiation received by the antenna?

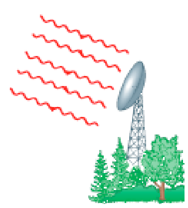
Figure 16

A. 1.3×10^{-5} W/m^2
B. 4.0×10^3 W/m^2
C. 1.3×10^6 W/m^2
D. 1.3×10^{-5} V/m
E. 4.0×10^3 V/m

27. The switch in the LR circuit shown in figure 17 to closed at time t = 0. How much time is required for the current to reach 86.5 % of its final value? Hint: 1-exp{-2} = 0.865.

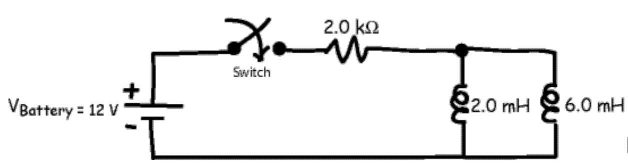

Figure 17

A. 1.5×10^{-6} s
B. 3.0×10^{-6} s
C. 1.6×10^{-5} s
D. 3.2×10^{-5} s
E. 3.0 s

28. The switch in the RC circuit shown in figure 18 to closed at time t = 0. What is the time constant, τ, for the decay of the current that flows in the circuit?.

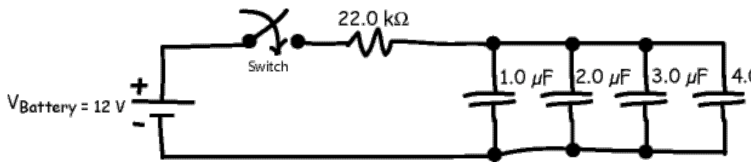

Figure 18

A. 0.45 ns
B. 46 ms
C. 220 ms
D. 2.2 s
E. 11 s

29. A square coil, comprised of N= 100 turns of wire and L = 10 cm on edge, rotates about a vertical axis at 1500 revolutions/min, as in figure 19, at a point where the horizontal (tangential) component of the Earth's magnetic field is 0.2×10^{-4} T. Calculate the maximum voltage drop (EMF) induced in the coils as it rotes in the Earth's field. Mind the units!

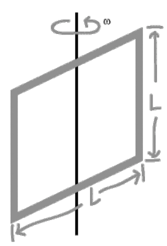

Figure 19

A. 0.5 mV
B. 3.14 mV
C. 30 mV
D. 190 mV
E. 1.3 V

30. A small charged ball of mass M = 0.5 g is suspended by a string of length L = 20 cm in a uniform electric field of E = 1000 N/C $\hat{x}$ (Figure 20). The string makes an angle of $\theta = 15^{\circ}$ relative to the vertical axis. What is the net charge, Q, on the ball? Take $|g|$ = 9.8 m/s^2. Mind the units!

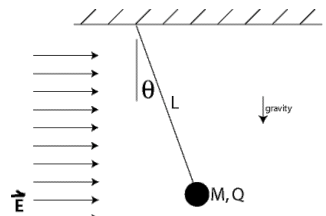

Figure 20

A. -1.3×10^{-6} C
B. -1.8×10^{-6} C
C. $+1.3\times10^{-6}$ C
D. $+1.8\times10^{-6}$ C
E. $+5.0\times10^{-6}$ C