Week 5 Homework

Physics 1B

February 6, 2007

1 Serway C17.5

The car battery has some internal resistance (which doesn't change, since it's a property of the battery). The starter draws a large current from the battery, which causes a very large voltage drop across the internal resistance of the battery. This leaves less voltage for things like the headlights, since the voltage drop across an entire closed loop must still be zero. This reduced voltage across the headlights causes them to dim.

2 Serway C17.7

Batteries can be placed in many different circuits (remote controls, Furbies, Aquatine Hunger Force ads, etc.) which all may have different resistances. Because we don't know the resistance in the circuit the battery will be used, we don't know the current it will produce. Appliances, on the other hand, have a given resistance, so we can calculate the current.

3 Serway C17.9

Although connecting batteries in parallel doesn't increase the voltage, it does increase the ability to output current (and therefore, since P=IV, power).

4 Serway C17.19

So long as the parachutist does not bridge a gap between a high and low voltage, she will not be electrocuted. Therefore, when she holds only the power line, she is safe. However, if she holds the power line while touching the ground, she completes the circuit, and will be hurt.

5 Serway C17.21

b) When the switch is closed, current has an easy way to make it back to the battery (by bypassing lamp C), so that's the way the current goes. Correspondingly, lamp C goes out (zero intensity).

d) Since there is no current through lamp C, the voltage drop across lamp C is also 0. Therefore, the voltage drops across A and B must increase so that the net voltage drop for the circuit is still 0.

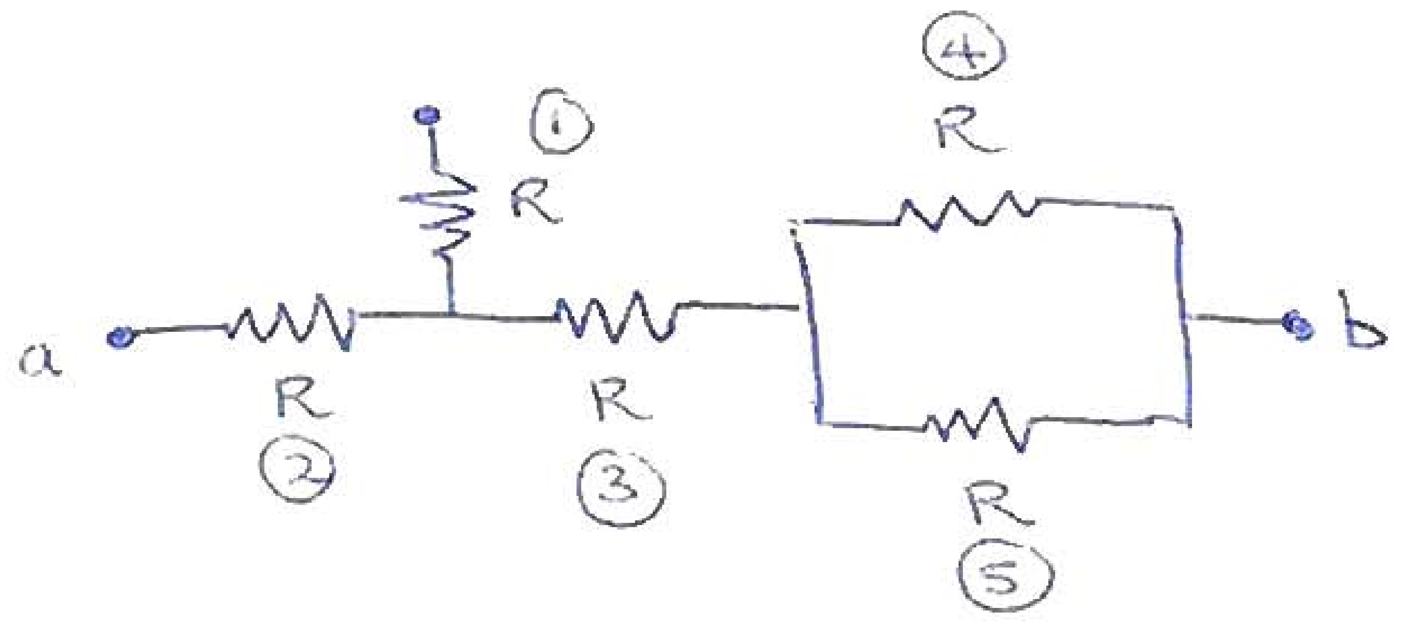
a) Because the voltages across A and B increase, the power dissipated by each of them increases. Therefore, their intensities increase as well.

c) Because the total resistance of the circuit goes down, but the voltage drop across the circuit is the same, the current in the circuit goes up.

e) We can tell that the power increases because $P = IV = V^2/R$. The voltage drop across the circuit doesn't change when closing the switch, but the net resistance decreases. Therefore the total power dissipated increases.

Chapter 18

7.

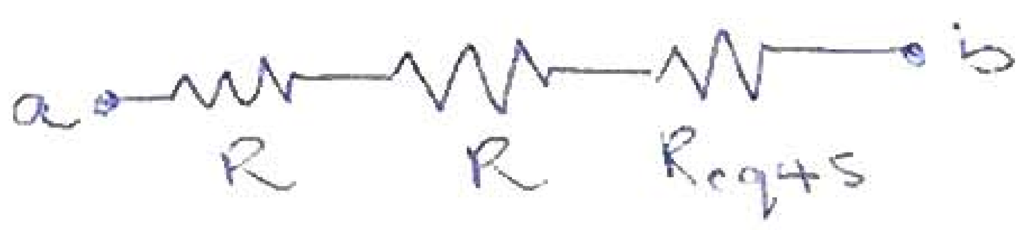


① Since this branch doesn't lead anywhere, no current passes through it. Therefore it doesn't add to the equivalent resistance.

④⑤ in parallel:

$$\frac{1}{R_{eq45}} = \frac{1}{R_4} + \frac{1}{R_5}$$

$$R_{eq_{45}} = \frac{R}{2}$$



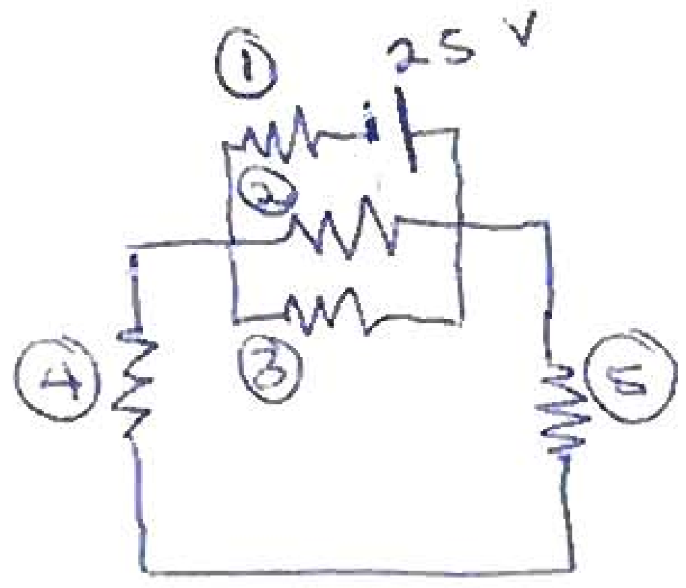
These resistors are all in series, so

$$R_{eq} = R + R + R_{eq45}$$

$$= R + R + \frac{R}{2}$$

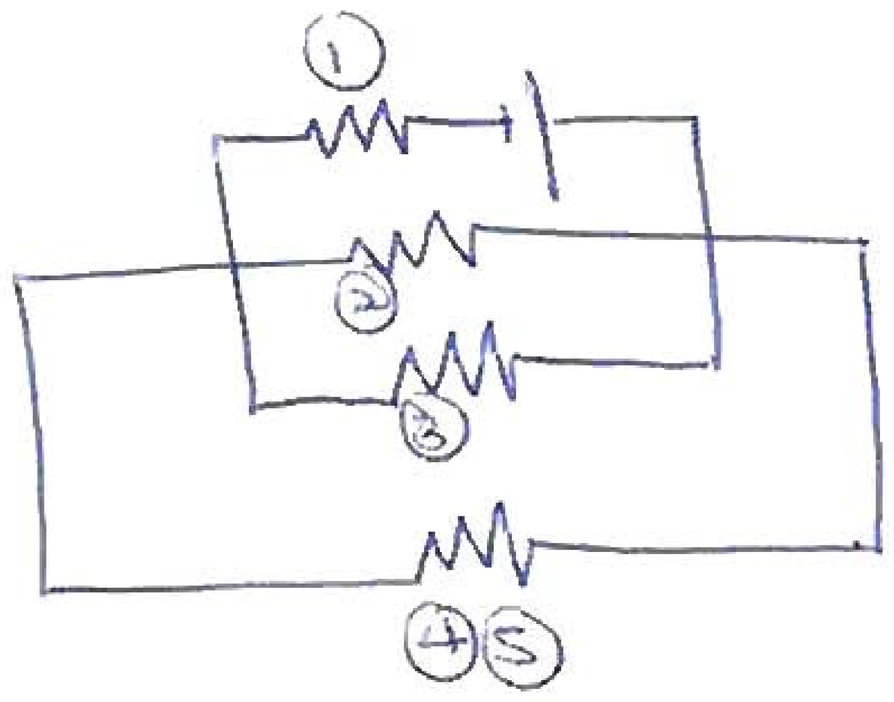
$$\boxed{R_{eq} = \frac{5}{2}R}$$

Week 5 HW

9. 

Find current through 5, V across ④⑤

- Resistors ④ and ⑤ are in series, so $R_{eq45} = R_4 + R_5$. We can redraw the picture



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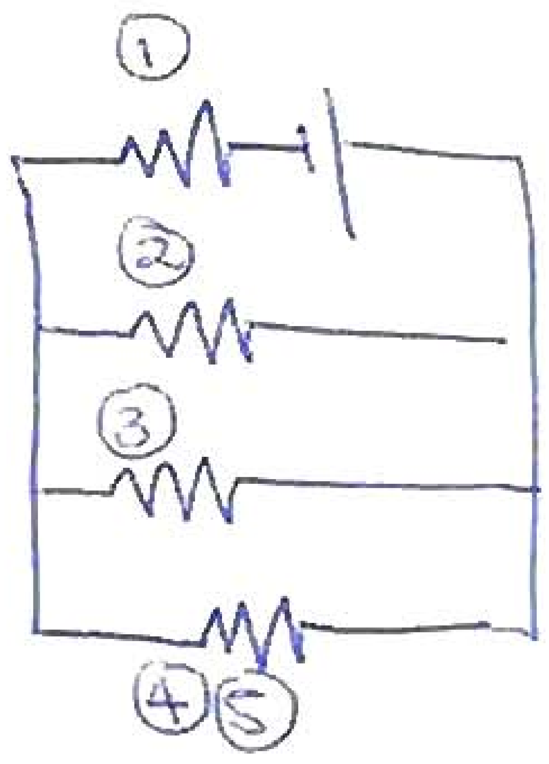
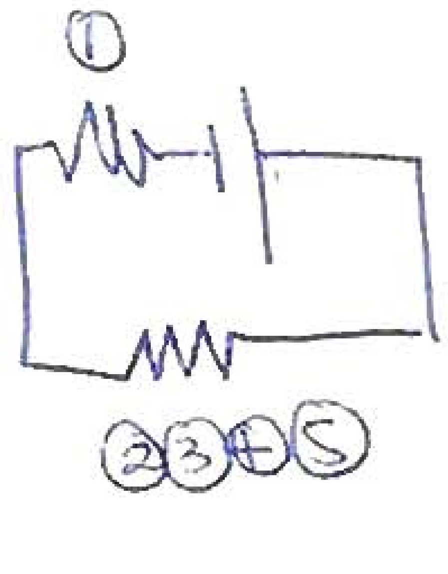


- Now we can easily see that ②, ③, and ④⑤ are in parallel, so R_{eq2345}:

$$\frac{1}{R_{eq2345}} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_{45}}$$

$$= \frac{1}{10\Omega} + \frac{1}{5\Omega} + \frac{1}{25\Omega}$$

$$R_{eq2345} = \frac{50}{17}\Omega = 2.94\,\Omega$$

- 

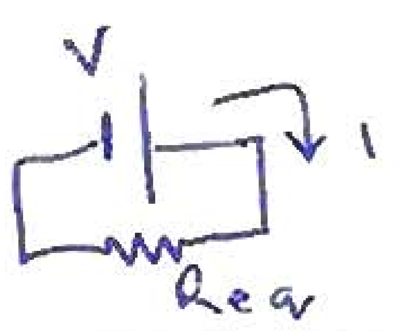
$$R_{eq} = R_1 + R_{eq2345}$$

$$= 10\Omega + \frac{50}{17}\Omega$$

$$= \frac{220}{17}\Omega = 12.9\,\Omega$$

$$I = \frac{25\,V}{\left(\frac{220}{17}\Omega\right)} = 1.93\,A$$

total current leaving battery



Week 5 HW

(9.) $\Delta V_{across\,①} = (1.93\ A)(10\ \Omega) = V_①$

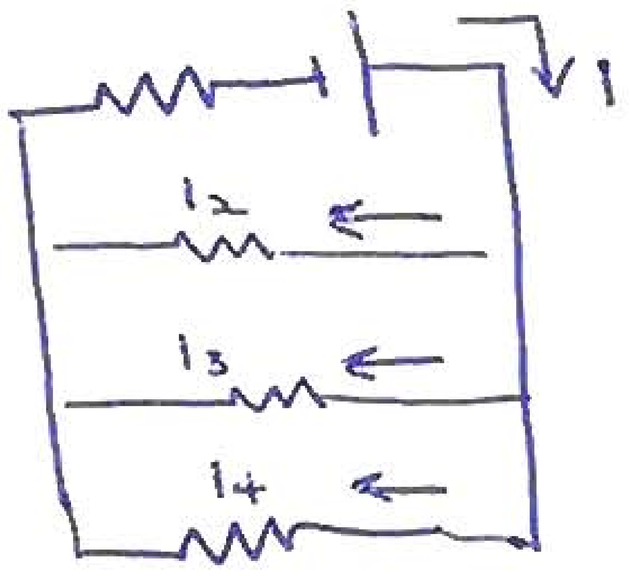
$= 19.3\ V$

$\Delta V_{across\,④⑤}$: $V_{④⑤} = V_① = V_② = V_③ = V - V_①$

In parallel, voltage drop same across all branches.

$$\boxed{\Delta V_{④⑤} = V - V_① = 25.0V - 19.3V = 5.7\ V}$$

- Current through ④⑤ $= I_4$:



$$I_2 = \frac{V_②}{R_②} = \frac{5.7V}{10.0\Omega} = 0.57\ A$$

$$I_3 = \frac{V_③}{R_③} = \frac{5.7V}{5.00\ \Omega} = 1.14\ A$$

$$\boxed{I_4 = \frac{V_{④⑤}}{R_{eq\,4 5}} = \frac{5.7V}{25.0\Omega} = 0.23\ A}$$

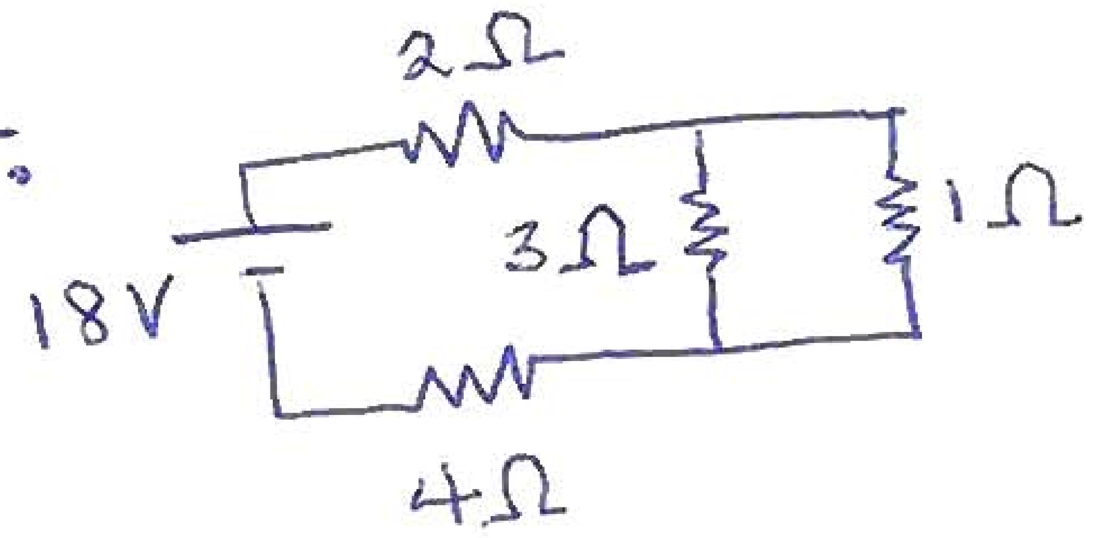
Current through branch 4 is current through 20.0 Ω resistor.

Check $I = I_2 + I_3 + I_4$ (junction rule)

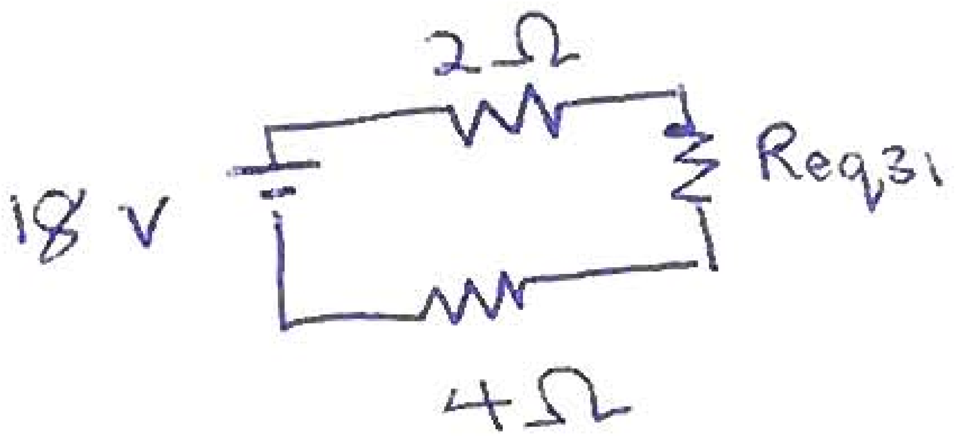
$1.93\ A = 0.57\ A + 1.14\ A + 0.23\ A$ ✓

Week 5 HW

14.



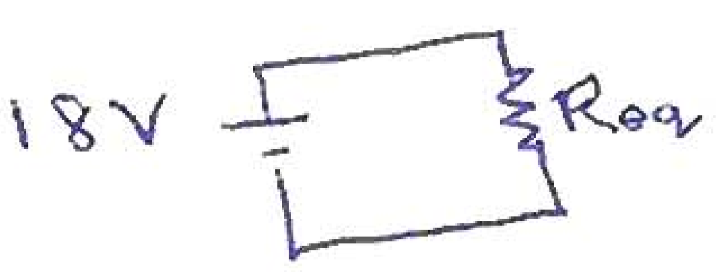
- $P = IV$
- Since $V = IR$, $P = V^2/R = I^2R$
- Find voltage across each resistor or current through it to calculate power dissipated by it.



$$\frac{1}{R_{eq31}} = \frac{1}{3\Omega} + \frac{1}{1\Omega}$$

$$R_{eq31} = \frac{3}{4}\Omega$$

⇓



$$R_{eq} = 6\tfrac{3}{4}\Omega$$

$$I = \frac{V}{R}$$

$$= \frac{18\text{ V}}{6\frac{3}{4}\Omega}$$

$$= 2.67\text{ A}$$

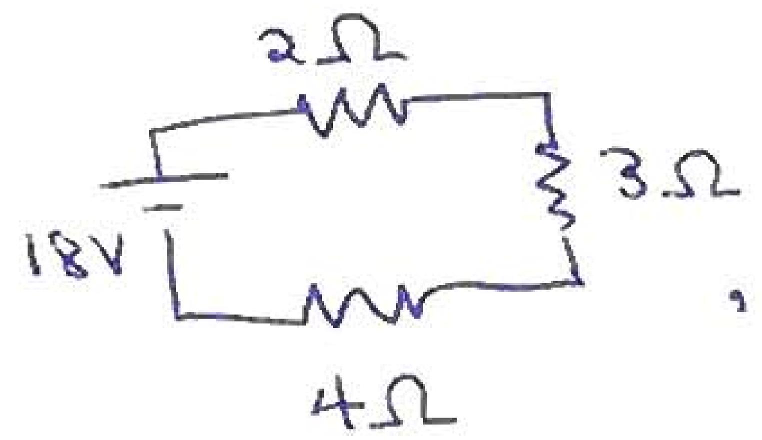
- This is the current that flows out of and back into the battery, so it is also the current through the 2 and 4Ω resistors:

$$\boxed{\begin{aligned} P_{4\Omega} &= (2.67\text{A})^2(4\Omega) = 28.4\text{W} \\ P_{2\Omega} &= (2.67\text{A})^2(2\Omega) = 14.2\text{ W} \end{aligned}}$$

Week 5 HW

(14.) $\Delta V_{loop} = 0$

$\Delta V_{resistor} = IR$

- Looking at the inner loop  ,

$\Delta V_{loop} = +18\ V + \Delta V_{2\Omega} + \Delta V_{3\Omega} + \Delta V_{4\Omega}$

$= 0$

$\Delta V_{3\Omega} = 18\ V - (2.67\ A)(2\ \Omega) - (2.67\ A)(4\ \Omega)$

$= 2\ V$

- Because the 3Ω and 1Ω resistor are in parallel, $\Delta V_{1\Omega} = 2\ V$ (same as $\Delta V_{3\Omega}$).

$\therefore\ P_{3\Omega} = \frac{V^2}{R}$

$= \frac{(2\ V)^2}{3\ \Omega}$

$\boxed{P_{3\Omega} = 1.33\ W}$

$P_{1\Omega} = \frac{V^2}{R}$

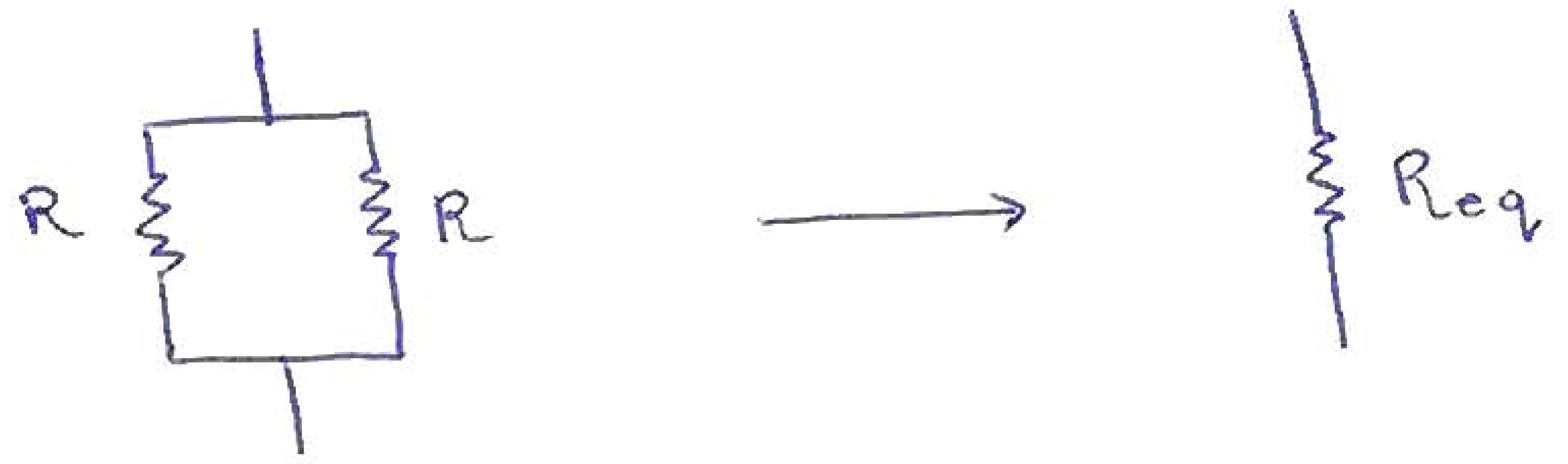
$= \frac{(2\ V)^2}{1\ \Omega}$

$\boxed{P_{1\Omega} = 4.00\ W}$

Week 5 HW

15. a)

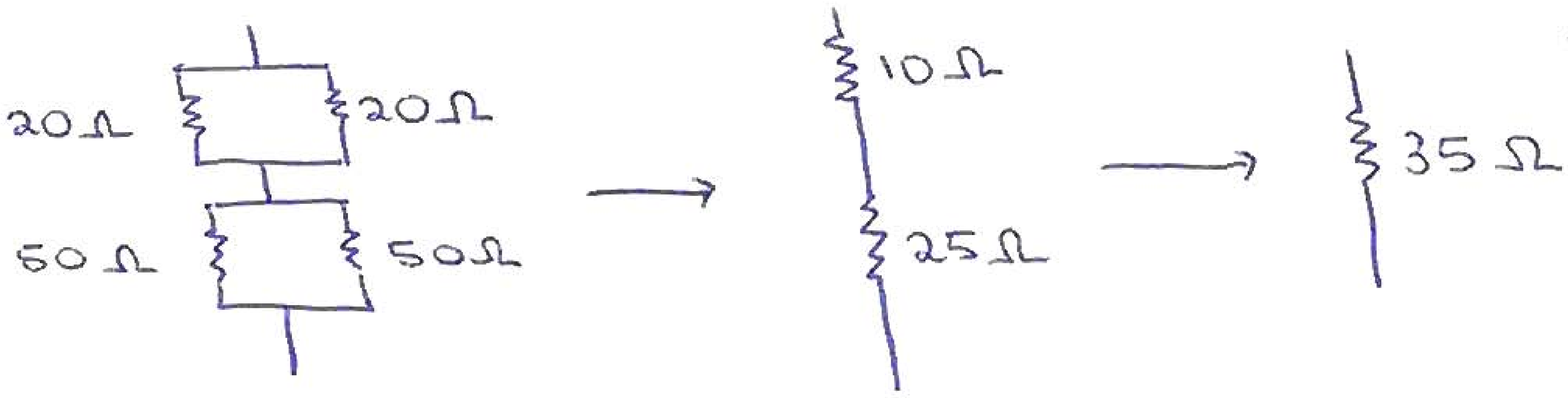
- If we put two resistors of equal resistance in parallel, the equivalent resistance is half of the original:



$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{R} = \frac{2}{R} \longrightarrow R_{eq} = \frac{R}{2}$$

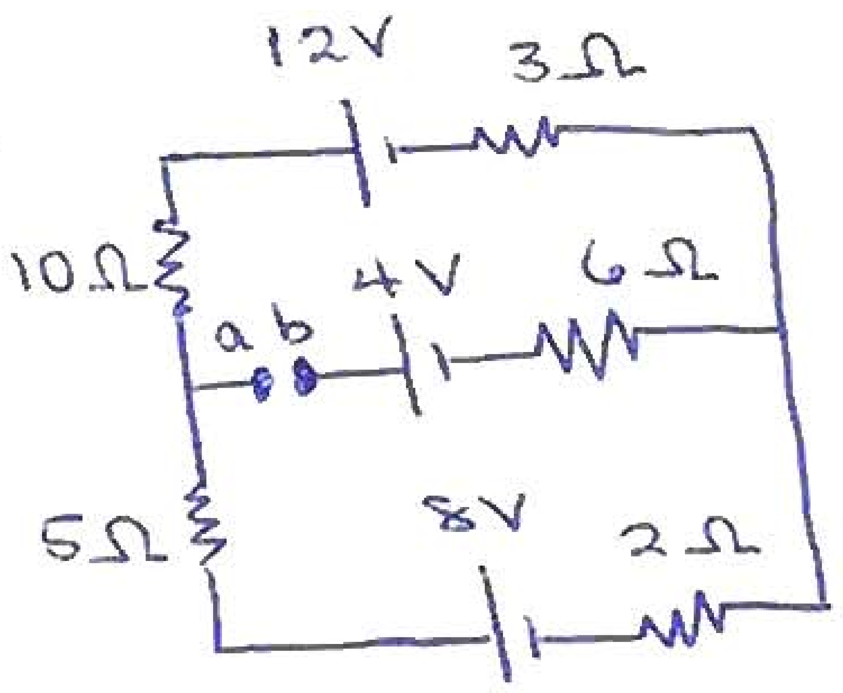
- Using this trick, we can make a 25 Ω resistor from two 50Ω resistors. Putting that in series with a 20Ω resistor gives the desired resistance:

b) - Like in (a), we can make a 10Ω resistor by putting two 20Ω resistors in parallel. Putting this in series with our 25 Ω resistor from part (a) yields $R_{eq} = 35\ \Omega$

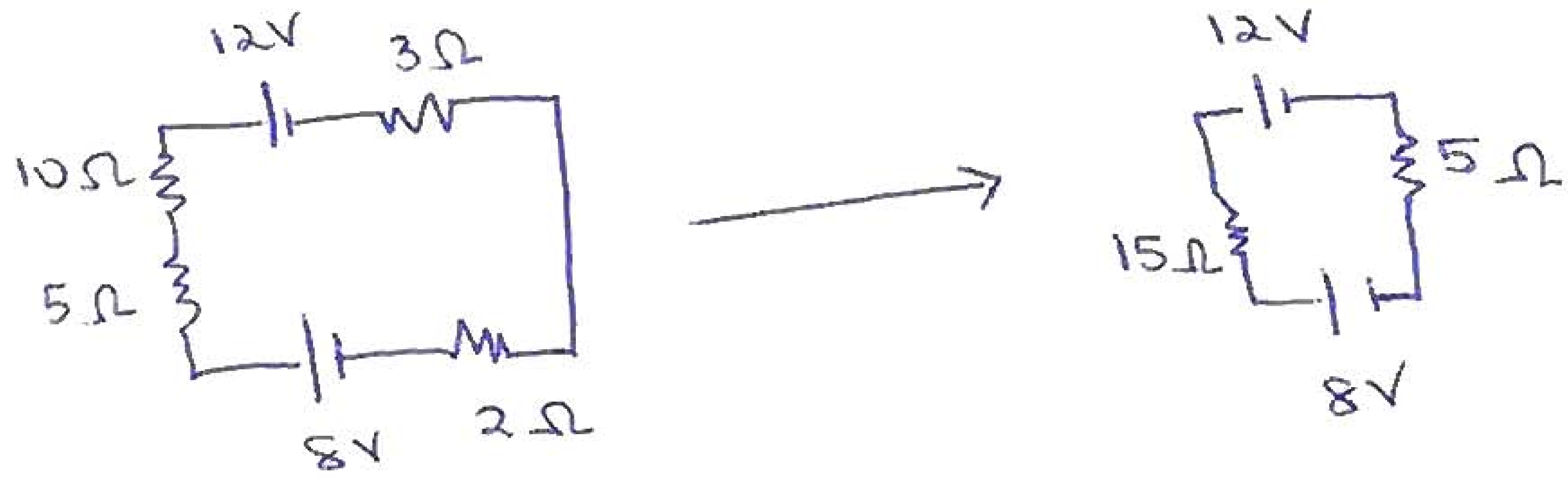


Week 5 HW

18.



Because there is a break in the wire between a and b, there is no current through the middle branch. Examining the outside loop,



going through battery backwards drops voltage

$\circ\ \Delta V_{loop} = 0$

$$= 12V - (15\Omega)(I) - 8V - (5\Omega)(I)$$

$I = 0.2A$

$\therefore\ \Delta V$ across 3Ω resistor at top of original picture is $\Delta V_{3\Omega} = IR = (0.2A)(3\Omega) = 0.6\ V$

$\circ\ \Delta V$ across 5Ω, 10Ω, and 2Ω resistors in original picture are

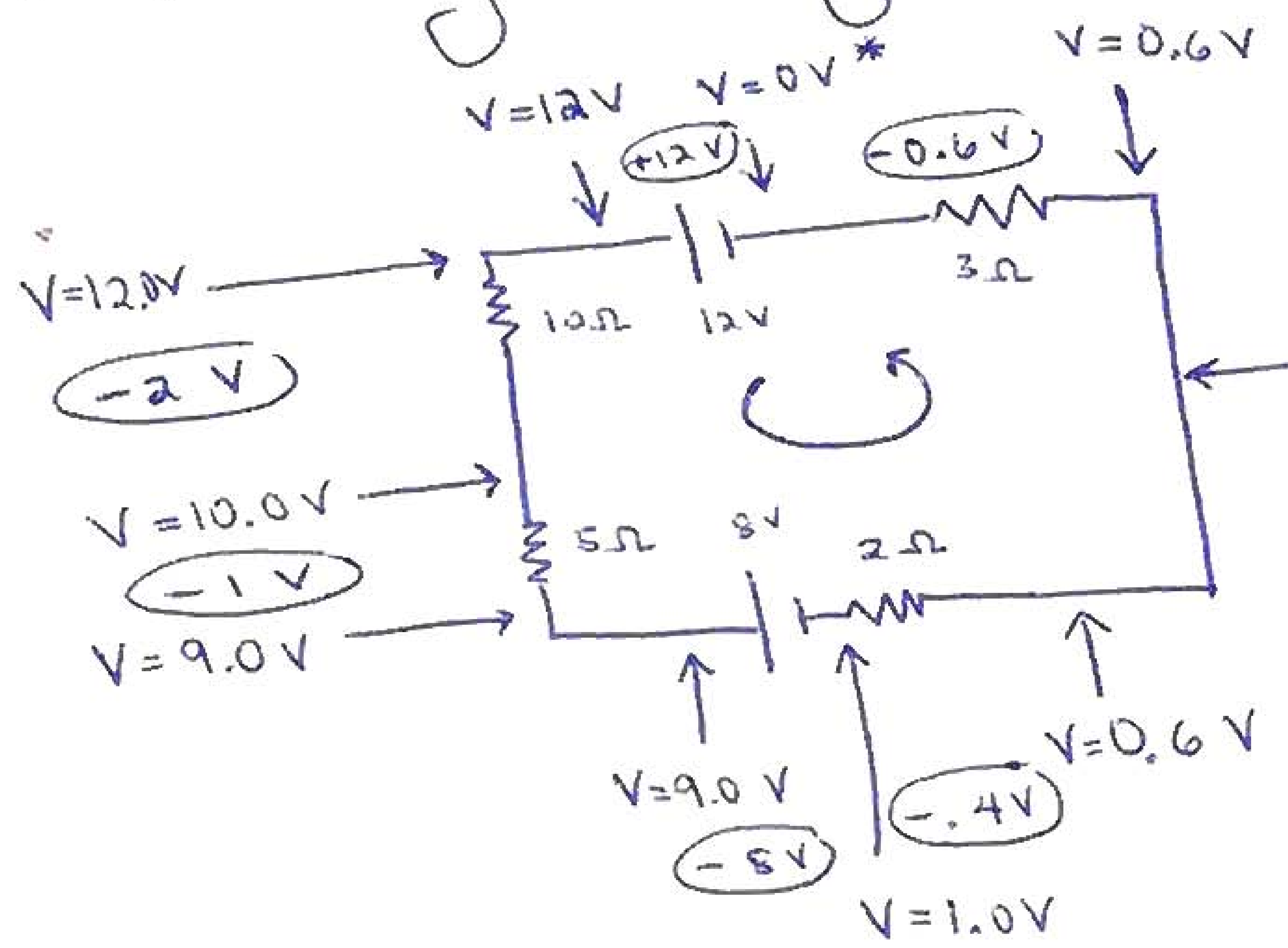
$\Delta V_{5\Omega} = (0.2A)(5\Omega) = 1.0\ V$

$\Delta V_{2\Omega} = (0.2A)(2\Omega) = 0.4\ V$

$\Delta V_{10\Omega} = (0.2A)(10\Omega) = 2\ V$

Week 5 HW

(18.) Going through the circuit:



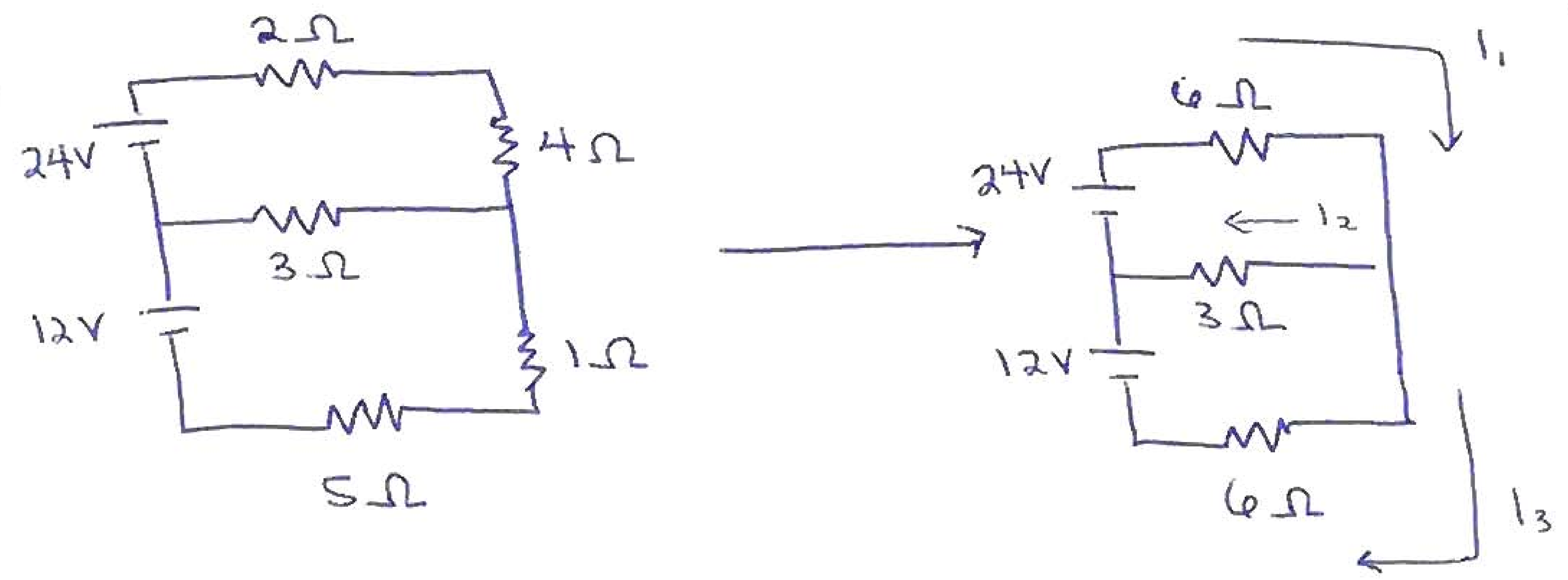
* The choice to fix one voltage value is arbitrary. Here we choose V=0 at *.

∴ The voltage across the middle branch is $10.0\,V - 0.6\,V = 9.4\,V$. But between the left side of the middle branch and point b, there is a battery which raises the voltage from 0.6 V to 4.6 V. ∴ $V_{ab} = 10.0V - 4.6V$

$$\boxed{V_{ab} = 5.4\ V}$$

Week 5 HW

25.



- Looking at outer loop of right circuit,

$\Delta V_{loop} = 0$ ← true for any closed loop in a circuit

$= 24\text{ V} - (I_1)(6\,\Omega) - (I_3)(6\,\Omega) + 12\text{ V}$

$I_1 + I_3 = 6\text{ A} \qquad (1)$

- Looking at top loop of right circuit,

$\Delta V_{loop} = 0$

$= 24\text{V} - (I_1)(6\,\Omega) - (I_2)(3\,\Omega)$

$I_1 + \frac{1}{2} I_2 = 4\text{ A}$

$2I_1 + I_2 = 8\text{ A} \qquad (2)$

- Applying Kirchoff's junction rule,

$I_1 = I_2 + I_3 \qquad (3)$

- Substituting (3) into (1) and (2)

$(I_2 + I_3) + I_3 = 6\text{ A}$

$I_2 + 2I_3 = 6\text{ A} \qquad (1)'$

$2(I_2 + I_3) + I_2 = 8\text{A}$

$3I_2 + 2I_3 = 8\text{A} \qquad (2)'$

- Subtracting (1)' from (2)',

$2I_2 = 2\text{ A}$

$\boxed{I_2 = 1.0\text{A}}$

Week 5 HW

(25). • Substituting into (2')

$$3(1\ A) + 2I_3 = 8\ A$$

$$\boxed{I_3 = 2.5\ A}$$

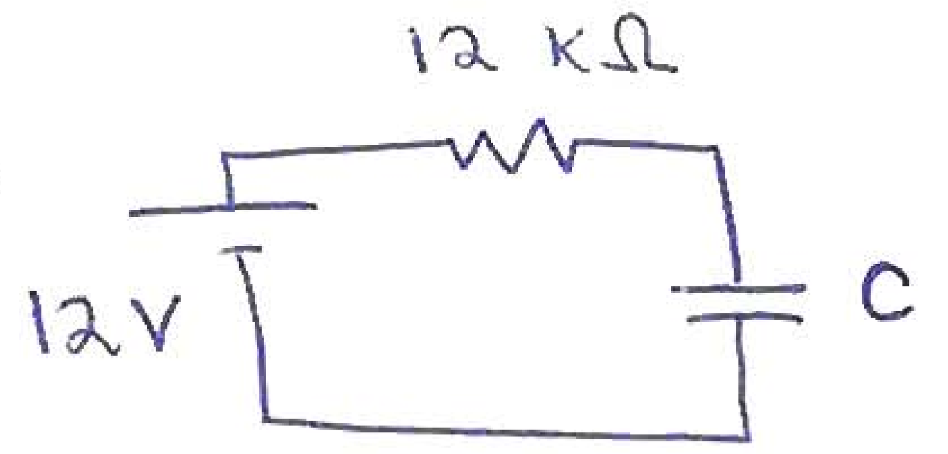
• Substituting into (3),

$$I_1 = (1.0\ A) + (2.5\ A)$$

$$\boxed{I_1 = 3.5\ A}$$

Week 5 HW

34.



- How much charge a capacitor has on its plates is proportional to the voltage across it at any given time. We know that the charge on the capacitor at time t is given by

$$q = Q(1 - e^{-t/RC})$$

where Q is the capacitor's charge once it is fully charged. When it is fully charged, current no longer flows, and the voltage across the capacitor is the same as the battery's voltage. $\therefore$ $Q = C(12\text{ V})$.

- Let V = voltage across capacitor at time t. $\therefore$ $q = CV$. Substituting,

$$CV = C(12\text{ V})(1 - e^{-t/RC})$$

$$\frac{V}{12\text{V}} = 1 - e^{-t/RC}$$

$R = 12 \times 10^3\ \Omega$

At $t = 1\text{ s}$, $V = 10\text{ V}$.

- Substituting,

$$\frac{10\text{ V}}{12\text{V}} = 1 - e^{-(1\text{ s})/(12\times10^3\Omega)C}$$

$$1 - \frac{5}{6} = e^{-(1\text{ s})/(12\times10^3\Omega)C}$$

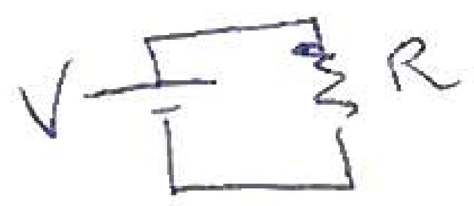
$$\ln\left(\frac{1}{6}\right) = \frac{-(1\text{ s})}{(12\times10^3\ \Omega)C} \longrightarrow \boxed{C = 4.7\times10^{-5}\text{ F}}$$

Week 5 HW

36. a) Since the emf of the battery is 48.0V, the maximum voltage across the capacitor will be 48.0V (when it is fully charged).

$\therefore Q = C\,(48.0\ V)$

- We are given $\tau = 0.960s$, where $\tau = RC$
- The maximum current ($I = 0.500$ mA) flows the after the switch is closed. Here all the voltage drop is across the resistor. We use this to find R:



at start of problem, capacitor acts like a wire.

$$R = \frac{V}{I} = \frac{48.0V}{0.500\times10^{-3}\ A} = 96\ k\Omega$$

$$\therefore \tau = RC \Rightarrow C = \frac{\tau}{R} = \frac{0.960s}{9.6\times10^{4}\Omega}$$

$$\boxed{C = 1.0\times10^{-5}\ F}$$

b) $q = Q(1-e^{-t/RC})$

$$= (1.0\times10^{-5}F)(48.0V)(1-e^{-1.92s/0.960s})$$

$$\boxed{q = 4.2\times10^{-4}\ C}$$

Week 5 HW

60.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \qquad C_{eq} = C_1 + C_2$$

$$R_{eq} = 1.2\ k\Omega \qquad C_{eq} = 5.0\ \mu F$$

Looking at right circuit, as in # 34, 36,

$$q = Q\left(1 - e^{-t/RC}\right)$$

$$= C_{eq}\,\mathcal{E}\left(1 - e^{-t/R_{eq}C_{eq}}\right)$$

$$= (5.0\ \mu F)(120\ V)\left(1 - e^{-t/(1.2\times10^{3}\,\Omega)(5.0\times10^{-6}\,F)}\right)$$

$$= (6\times10^{-4}\ C)\left(1 - e^{-t/6\times10^{-3}\,s}\right)$$

$$q = CV \Rightarrow V = q/C_{eq} = \frac{(6\times10^{-4}\ C)}{(5.0\ \mu F)}\left(1 - e^{-t/6\times10^{-3}\,s}\right)$$

This is the voltage across the capacitors at any time t.

$$V = (120\ V)\left(1 - e^{-t/6\times10^{-3}\,s}\right)$$

Using $q = CV$ for each capacitor,

$$\boxed{\begin{aligned} q_1 &= (2.0\times10^{-6}\ F)(120\ V)\left(1 - e^{-t/6\times10^{-3}\,s}\right) \\ q_2 &= (3.0\times10^{-6}\ F)(120\ V)\left(1 - e^{-t/6\times10^{-3}\,s}\right) \end{aligned}}$$

Restatement of 18.41

A mylinated axon can be modeled as a parallel plate cylindrical capacitor with the following parameters:

$l = 10$ cm

$d = 1.0 \times 10^{-8}$ m

$K = 3.0$

$r = 10$ μm

Recall that for a parallel plate capacitor, $C = \frac{1}{4\pi K_e} \cdot \frac{A}{d}$. Here the area of a plate is the surface area of the cylinder.

a) From fig. 18.28, when the ion is not conducting an electric pulse, the potential difference across it is 70mV. How many K+ ions are outside the axon? Is this a large charge per unit area? (see hint in book).

See book for parts (b) through (d)

Week 5 HW

41. a) $Q = CV$

$$C = \frac{K}{4\pi K_e} \cdot \frac{A}{d} \quad \text{where } A = 2\pi r \ell = 6.3\times10^{-6}\ m^2$$

$$Q = \frac{AVK}{4\pi K_e d}$$

$$= \frac{2\pi(10\times10^{-6}\ m)(10\times10^{-2}\ m)(70\times10^{-3}\ V)(3.0)}{4\pi(8.99\times10^{9}\ N\,m^2/C^2)(1.0\times10^{-8}\ m)}$$

$$\boxed{Q = 1.2\times10^{-9}\ C}$$

- Potassium ion has a net charge of $+e$, so this is $\frac{1.2\times10^{-9}}{1.6\times10^{-19}} = \boxed{7.3\times10^{9}\ \text{ions.}}$
- $6.3\times10^{-6}\ m^2 = 6.3\times10^{14}\ \text{Å}^2$

→ 1.2×10^{-5} ions / square angstrom

$\boxed{\text{Not large}}$

b) Same as part a), except $V = 30\,mV - (-70\,mV) = 100\,mV$

$$\boxed{Q = 1.7\times10^{-9}\ C}$$

$$\frac{1.7\times10^{-9}}{1.6\times10^{-19}} = \boxed{1.0\times10^{10}\ \text{ions}}$$

c) $I = \frac{Q}{t}$

$Q = 1.7\times10^{-9}\ C$

$t = 2.0\times10^{-3}\ s$

$$\boxed{I = 8.5\times10^{-7}\ A}$$

d) $PE = qV$

$V = (30 - (-70))\,mV = 100\,mV$