Quiz 1 – Electrostatics

The force on charge q_1 from charge q_2 is $\vec{F}_{12} = k_e \frac{q_1 q_2}{r_{12}^2}\hat{r}_{12}$, where the direction vector $\hat{r}_{12}$ points from q_2 to q_1 and the proportionality constant is $k_e = 8.99\text{x}10^9\ \text{Nm}^2/\text{C}^2$.

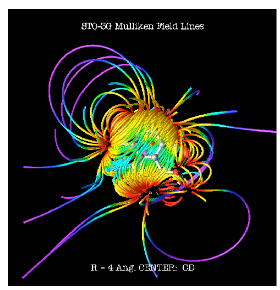
Note that the permittivity of free space is $\varepsilon_0 \equiv \frac{1}{4\pi k_e} = 8.85\text{x}10^{-12}\ \text{C}^2/(\text{Nm}^2) = 8.85\text{x}10^{-12}\ \text{A}^2\text{s}^4/(\text{kg m}^3)$.

Note that the unit of elemental electronic change is $e^- = -1.60\text{x}10^{-19}\text{C}$.

We note the Taylor's expansion $(1+x)^n = 1 + nx + \cdots$, which is useful when $nx << 1$. For example,

$$\frac{1}{(r+d)^2} = \frac{1}{r^2}\left(1+\frac{d}{r}\right)^{-2} = \frac{1}{r^2}\left(1 - 2\frac{d}{r} + \cdots\right) \simeq \frac{1}{r^2} - 2\frac{d}{r^3} \text{ for } d << r.$$

The force on a test charge q_0 induced by an electric field, denoted $\vec{E}$, is $\vec{F} = q_0\vec{E}$.



Quiz 2 – Fields and Potentials

The electric flux through a surface is $\Phi_e \equiv \sum_{All\ Surfaces} E_\perp \Delta A = \sum_{All\ Surfaces} E\Delta A\cos\theta$, where $\Delta A_\perp = \Delta A\ cos\theta$ is the component of the area whose normal lies parallel to the electric field; θ is the angle between the direction of the electric field and that of the normal to the surface.

Gauss' Law relates the total flux through a closed surface to the total net charge enclosed by the surface, *i.e.*, $\Phi_e = 4\pi k_e Q_{Total}$.

The electric field produced by a point charge q at the origin, *i.e.*, $\vec{\mathbf{r}} = 0$, is $\vec{E} = k_e \frac{q}{r^2}\hat{r}$ where $\hat{\mathbf{r}}$ is the radius vector in spherical coordinates.

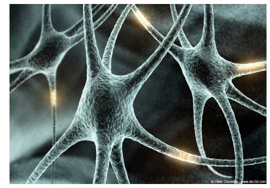
The electric field produced by a line charge, with charge per unit length λ, is $\vec{E} = 2k_e \frac{\lambda}{r}\hat{r}$, where the line is defined to lie along the $\hat{z}$ axis and $\hat{\mathbf{r}}$ is the radius vector in cylindrical coordinates.

The electric field produced by a surface charge, with charge per unit area σ, is $\vec{E} = 2\pi k_e \sigma\ \hat{n}$, where the surface lies in the $\hat{x}$-$\hat{y}$ plane and $\hat{z}$ corresponds to the normal to the $\hat{x}$-$\hat{y}$ plane in Cartesian coordinates.

Work-Energy Theorem: W = ΔKE + ΔPE

Electric potential: $\Delta V = -E\ \Delta x\ \cos\theta$, where $\Delta V = \dfrac{\Delta PE}{Q}$

$V = k_e \dfrac{q}{r}$ a distance r away from a point charge q.



Quiz 3 – Current, Resistance and Capacitance

Current: $I = \dfrac{\Delta Q}{\Delta t}$ or $I = n\ e\ v_D\ A$ where n is the density of charge carriers, v_D is the drift velocity and A is the cross-section of the wire.

Capacitance: $Q = C\ \Delta V$ where $C = \dfrac{\kappa}{4\pi k_e}\dfrac{A}{d}$ for parallel plates and κ is the dielectric constant

$I = C\dfrac{\Delta V}{\Delta t}$

Energy Stored $= \dfrac{1}{2}Q\ \Delta V = \dfrac{1}{2}C\ (\Delta V)^2 = \dfrac{1}{2C}Q^2$

Resistance: $V = I\ R$ where $R = \rho\dfrac{L}{A}$; ρ is the resistivity in Ohm-m and L is the length of the wire.

Power Dissipated = $IV = I^2R = V^2/R$

Kirchoff's Laws: 1) Sum of voltage drops around any loop is zero, *i.e.*, gains = losses

2) Sum of current flow at a node is zero, *i.e.*, total current in = total current out

A resistor/capacitor pair charges with a characteristic time, denoted t, that is given by the product of the resistance and membrane, i.e., τ = RC.

Quiz 4 –Magnetostatics (Electrostatics in the Fast Lane)

The force on a test charge q_0 induced by an electric field, denoted $\vec{E}$, and a magnetic field, denoted B, is $\vec{F} = q_0\vec{E} + q_0\vec{v}\times\vec{B}$. The cross product $\vec{v}\times\vec{B}$ points normal to the plane defined by $\vec{v}$ and $\vec{B}$, and has magnitude $|\vec{v}||\vec{B}|\sin\theta$ and a direction that is found from the "right hand rule".

The force per unit length on a straight wire that carries a current I (where I has both magnitude and direction, given by the motion of positive charge carriers) is given by $\vec{F}/l = \vec{I} \times \vec{B}$. The cross product ids defined as above.

The force per unit length between two straight wires that carry current I_1 and I_2 respectively, and are separated by a distance R, is F/L = $\mu_0 \frac{I_1 I_2}{2\pi R}$, where $\mu_0 = 1.3 \times 10^{-6}$ Tm/A.

For completeness, $\mu_0 \varepsilon_0 = 1/c^2$, where c is the speed of light.

The torque between a loop of cross-sectional area A that carries a current I and a uniform magnetic fields is $\vec{\tau} = \vec{\mu} \times \vec{B}$, where μ = IA is known as the magnetic moment.

When the loop contains multiple turns of wire, $\mu \leftarrow$ NIA where N is the number of turns.

Ampere's Law relates the magnetic field in a loop to the current, *i.e.*, $I = \frac{1}{\mu_0} \sum_{All\ Segments} B_{||} \Delta L$

For a straight wire, $\vec{B} = \mu_0 \frac{I}{2\pi R} \left(\hat{\theta}\right)$, where the direction is given by the right-hand rule.

For a solenoid, $\vec{B} = \mu_0 \frac{N}{L} I \left(\hat{z}\right)$.

Faraday's Law relates the change in magnetic flux, $\phi_{\text{Magnetic}} \equiv \sum_{All\ Surfaces} B_{\perp} \Delta A$, to the potential V, or the current, V/R, induced in a loop, *i.e.*, $V = -N \frac{\Delta \phi_{\text{Magnetic}}}{\Delta t}$ where N is the number of turns in the loop and the minus sign implies that the induced field - caused by the induced current - opposes the original field.

Generator: $V = NBA\,\omega \sin \omega t$ where ω is the rotational frequency in radians/s ($\omega = 2\pi f$).

Inductance: $N\Delta\phi = L\,\Delta I$ where $L = \mu_o \left(\frac{N}{l}\right)^2 Al$ for a solenoid with N/l turns/length, cross-sectional area A, and length l. Thus

$$V = L \frac{\Delta I}{\Delta t}$$

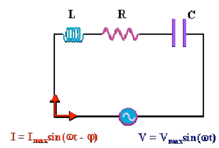
$$\text{Energy Stored} = \frac{1}{2} I\ N\varphi_{\text{Magnetic}} = \frac{1}{2} L\ I^2$$

A resistor/inductor pair has a characteristic time for a voltage change, given by the quotient of L and R, i.e., $\tau = L/R$.

Formulas for Physics 1B **Prof. David Kleinfeld** **Winter 2007**

	Series	Parallel
Capacitors	$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$	$C_{eq} = C_1 + C_2 + C_3 + \cdots$
Inductors	$L_{eq} = L_1 + L_2 + L_3 + \cdots$	$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots$
Resistors	$R_{eq} = R_1 + R_2 + R_3 + \cdots$	$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$

AC Circuit Analysis



A resistor has an impedance of R and a voltage that faithfully tracks the current, *i.e.*, if $I_R(t) = I_0 \sin(2\pi ft)$, then $V_C(t) = I_0 R \sin(2\pi ft)$.

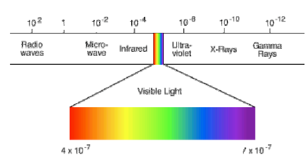
A capacitor has an impedance of $Z_C = 1/(2\pi fC)$ and a voltage that lags the current by 90º, *i.e.*, if $I_C(t) = I_0 \sin(2\pi ft)$, then $V_C(t) = I_0/(2\pi fC) \sin(2\pi ft - \pi/2)$.

An inductor has an impedance of $Z_L = 2\pi fL$ and a voltage that leads the current by 90º, *i.e.*, if $I_L(t) = I_0 \sin(2\pi ft)$, then $V_L(t) = 2\pi fL\, I_0 \sin(2\pi ft + \pi/2)$.

The current flow and voltage drops in a loop containing an inductor and capacitor has a resonance when the driving frequency satisfies

$$f_{resonance} = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$$

Electromagnetic Radiation



Dispersion relation in the vacuum is:

$$f\lambda = c,$$

where f is the frequency of the radiation, λ is the wavelength, and c is the speed of light, with $c = 2.99 \times 10^8$ m/s.

The ratio of the electric to magnetic field in an electromagnetic wave equals the speed of light, i.e.:

$$\frac{E(t)}{B(t)} = c$$

Electromagnetic waves carry energy as they travel; the average power per unit area is the intensity, denoted *I*, with:

$$I = \frac{1}{2\mu_0} E_{max} B_{max} = \frac{1}{2\mu_0 c} E_{max}^2 = \frac{c}{2\mu_0} B_{max}^2$$