The force on charge \( q_1 \) from charge \( q_2 \) is \( \vec{F}_{12} = k_e \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12} \), where the direction vector \( \hat{r}_{12} \) points from \( q_2 \) to \( q_1 \) and the proportionality constant is \( k_e = 8.99 \times 10^9 \text{ Nm}^2/\text{C}^2 \).

Note that the permittivity of free space is \( \varepsilon_0 \equiv \frac{1}{4\pi k_e} = 8.85 \times 10^{-12} \text{ C}^2/(\text{Nm}^2) \).

Note that the unit of elemental electronic change is \( e^- = -1.62 \times 10^{-19} \text{ C} \).

We note the Taylor’s expansion \( (1 + x)^n = 1 + nx + \ldots \), which is useful when \( nx \ll 1 \). For example,

\[
\frac{1}{(r + d)^2} = \frac{1}{r^2} \left( 1 + \frac{d}{r} \right)^{-2} = \frac{1}{r^2} \left( 1 - 2 \frac{d}{r} + \ldots \right) = \frac{1}{r^2} - 2 \frac{d}{r^3} \text{ for } d \ll r.
\]

The force on a test charge \( q_0 \) induced by an electric field, denoted \( \vec{E} \), is \( \vec{F} = q_0 \vec{E} \).

1. Two positive charges of strength \( Q_1 = +1.0 \times 10^{-3} \text{ Coulombs} \) sit along the x-axis as shown in figure 1 (+\( \hat{x} \) points to the right along x & +\( \hat{y} \) points to the up along y), with \( L = 20 \text{ cm} \). What is the x-direction of the force on a negative charge of strength \( Q_2 = -1.0 \times 10^{-6} \text{ Coulombs} \) that sits at \( (x, y) = (L, L) \)?

   A) -3.0 \times 10^2 \text{ N } \hat{x}
   B) -2.3 \times 10^3 \text{ N } \hat{x}
   C) -7.9 \times 10^1 \text{ N } \hat{x}
   D) +3.0 \times 10^2 \text{ N } \hat{x}
   E) +7.9 \times 10^1 \text{ N } \hat{x}

2. With reference again to figure 1, and \( Q_1, Q_2 \), and \( L \) defined as above, what is the y-direction of the force on \( Q_2 \)?

   A) -3.0 \times 10^2 \text{ N } \hat{y}
   B) -2.3 \times 10^3 \text{ N } \hat{y}
   C) -7.9 \times 10^1 \text{ N } \hat{y}
   D) -6.1 \times 10^1 \text{ N } \hat{y}
   E) -3.0 \times 10^0 \text{ N } \hat{y}
3. Two charged balls with identical mass, M, form a double pendulum as shown in figure 2. The charge on one ball is $Q_1 = +1.0 \times 10^{-6} \text{ C}$ and the other is $Q_2 = +3.0 \times 10^{-6} \text{ C}$. The strings have the same length, with $L = 20 \text{ cm}$ and the angle formed by the two balls is $2\theta = 60^\circ$ ($\theta = 30^\circ$). What is the mass?

A) $3.0 \times 10^{-2} \text{ kg}$
B) $1.2 \times 10^{-1} \text{ kg}$
C) $4.8 \times 10^{-1} \text{ kg}$
D) $1.2 \text{ kg}$
E) This system is unstable as the two values of the charge are different.

4. Four charges, two positive with strength $+|e|$ and two negative with strength $-|e|$, are symmetrically configured along the x-axis as shown in figure 3. What is the force on the test charge $q_o$?

A) $k \frac{q_o e d}{r^3} \hat{x}$
B) $k \left( \frac{q_o e}{r^2} - \frac{q_o e}{(r + d)^2} \right) \hat{x}$
C) $k \frac{q_o e d}{8 r^3} \hat{x}$
D) zero
E) $k \left( \frac{2 q_o e}{r^2} - \frac{2 q_o e}{(r + d)^2} \right) \hat{x}$

5. A charged sphere with mass $M = 1.0 \times 10^{-15} \text{ kg}$ and unknown charge $Q$ is suspended in a uniform electric field of $1.0 \times 10^7 \text{ N/C}$, as shown in figure 4. Recall that the gravitational acceleration is $9.8 \text{ m/s}^2$ and that $1 \text{ N} = 1 \text{ kg m/s}^2$. What is the strength of the charge (sign and magnitude of $Q$) that is required to suspend the particle?

A) $+8.8 \times 10^{-12} \text{ C}$
B) $+9.8 \times 10^{-22} \text{ C}$
C) $+9.8 \times 10^{-25} \text{ C}$
D) $-9.8 \times 10^{-22} \text{ C}$
E) $-9.8 \times 10^{-22} \text{ C}$