Physics 1B - Quiz 3 (12 Feb 2007)

Formulas for Electrostatics

The force on charge $q_1$from charge q_2 is $\vec{F}_{12} = k_e \frac{q_1 q_2}{r_{12}^2}\hat{r}_{12}$, where the direction vector $\hat{r}_{12}$ points from q_2 to q_1 and the proportionality constant is $k_e = 8.99x10^9$ Nm^2/C^2.

Note that the permittivity of free space is $\varepsilon_0 \equiv \frac{1}{4\pi k_e} = 8.85x10^{-12}$ $C^2/(Nm^2) = 8.85x10^{-12}$ $A^2s^4/(kg\ m^3)$.

Note that the unit of elemental electronic change is $e^- = -1.62x10^{-19}C$.

We note the Taylor's expansion $(1+x)^n = 1 + nx + \cdots$, which is useful when nx << 1. For example,

$$\frac{1}{(r+d)^2} = \frac{1}{r^2}\left(1+\frac{d}{r}\right)^{-2} = \frac{1}{r^2}\left(1 - 2\frac{d}{r} + \cdots\right) \simeq \frac{1}{r^2} - 2\frac{d}{r^3} \text{ for d << r.}$$

The force on a test charge q_0 induced by an electric field, denoted $\vec{E}$, is $\vec{F} = q_0\vec{E}$.

Formulas for Fields and Potentials

The electric flux through a surface is $\Phi_e \equiv \sum_{All\ Surfaces} EA_\perp = \sum_{All\ Surfaces} EA\cos\theta$, where $A_\perp = A\ cos\theta$ is the component of the area whose normal lies parallel to the electric field..

Gauss' Law relates the total flux through a closed surface to the total net charge enclosed by the surface, *i.e.*, $\Phi_e = 4\ \pi\ k_e\ Q_{Total}$.

The electric field produced by a point charge q at the origin, *i.e.*, $\vec{r} = 0$, is $\vec{E} = k_e\frac{q}{r^2}\hat{r}$ where $\hat{\mathbf{r}}$ is the radius vector in spherical coordinates.

The electric field produced by a line charge, with charge per unit length λ, is $\vec{E} = 2k_e\frac{\lambda}{r}\hat{r}$, where the line is defined to lie along the $\hat{z}$ axis and $\hat{\mathbf{r}}$ is the radius vector in cylindrical coordinates.

The electric field produced by a surface charge, with charge per unit area σ, is $\vec{E} = 2\pi k_e\sigma\ \hat{n}$, where the surface lies in the $\hat{x}$-$\hat{y}$ plane and $\hat{z}$ corresponds to the normal to the $\hat{x}$-$\hat{y}$ plane in Cartesian coordinates.

Work-Energy Theorem: $W = \Delta KE + \Delta PE$

Electric potential: $\Delta V = -E\ \Delta x\ cos\ \theta$, where $\Delta V = \frac{\Delta PE}{Q}$

$V = k_e\frac{q}{r}$ a distance r away from a point charge q.

Formulas on Current, Resistance and Capacitance

Current: $I = \frac{\Delta Q}{\Delta t}$

Capacitance: $Q = C\,\Delta V$ where $C = \dfrac{\kappa}{4\pi k_c}\dfrac{A}{d}$ for parallel plates and κ is the dielectric constant

$$I = C\frac{\Delta V}{\Delta t}$$

$$\text{Energy Stored} = \frac{1}{2}Q\ \Delta V = \frac{1}{2}C\ (\Delta V)^2 = \frac{1}{2C}Q^2$$

Resistance: $V = I\,R$ where $R = \rho\dfrac{l}{A}$ and ρ is the resistivity in Ohm-m.

	Series	Parallel
Capacitors	$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$	$C_{eq} = C_1 + C_2 + C_3 + \cdots$
Resistors	$R_{eq} = R_1 + R_2 + R_3 + \cdots$	$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$

Power Dissipated = $IV = I^2R = V^2/R$

Kirchoff's Laws: 1) Sum of voltage drops around any loop is zero, *i.e.*, gains = losses

2) Sum of current flow at a node is zero, *i.e.*, total current in = total current out

Finally! The Quiz

1. For current flow in a resistor, which statement is true?

A. The acceleration of the charge carrier is proportional to the voltage drop across the resistor
B. The velocity of the charge carriers in proportional to the voltage drop
C. The velocity of the charge carriers is a constant that depends solely on the type of material
D. The velocity of the charge carriers is a constant that depends solely on the temperature
E. The velocity of the charge carriers is a constant that depends solely on the geometry

* Students should be given $I = n q v_d A$ *

2. A 1.50 V battery is connected across a 4 F capacitor, as shown below in figure 1. What is the energy stored in the capacitor subsequent to removal of the battery?

A. XXX J 6.0 J (E=CV)
B. YYY J 4.5 J correct
C. zero
D. XXX W 6.0 W
E. YYY W 4.5 W

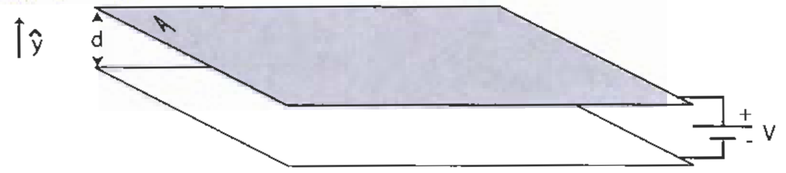


Figure 1

3. The resistivity of cytoplasm, the solution inside of a cell, is 100 Ω cm. What is the resistance of a cylinder of cytoplasm that is 1.0 mm long and 2.0 μm in diameter? Mind the units!

A. 3.2×10^{5} Ω (correct)
B. 3.1×10^{-15} Ω ($R = \rho L A$ w/correct units)
C. 3.2×10^{3} Ω (converting μm, not μm^2)
D. 8.0×10^{7} Ω (diameter, not radius)
E. 8.0×10^{3} Ω (errors from c and d)

4. A room heater operates at 220 Volts and consumes 2000 W of power. How much current does the heater draw?

A. 0.11 A (220 V/2000 W)
B. 4.1×10^{-3} A ($P = IV^2$)
C. 3.0 A ($P = I^2 V$)
D. 9.1 A (correct)
E. 4.4×10^{5} A (220 V · 2000 W)

5. In bacterial photosynthesis, the absorption of one photon causes a *pair* of electrons to move across the membrane. Under high light levels, *pairs* of *electrons* cross once every 100 μs. What is the current?

A. 1.6×10^{-21} A (1 e^-, s instead of μs)
B. 3.2×10^{-21} A (s instead of μs)
C. 2.0 A (2 e^-...)
D. 1.6×10^{-15} A (1 e^-, not 2)
E. 3.2×10^{-15} A (correct)

6. A battery supplies current to a circuit with 3 resistors, as shown below in figure 2. What is the correct expression for the total current I?

A. V/R
B. V/(2R)
C. 2V/R
D. 5V/4R
E. 3V/R

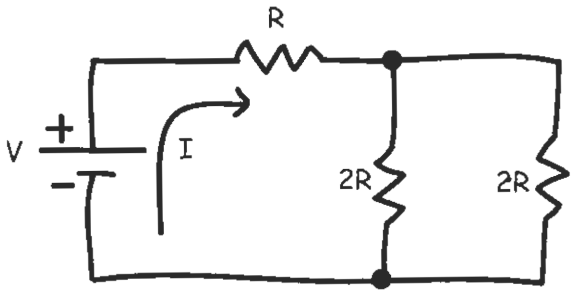


Figure 2

7. Find the equivalent resistance between points A and B for the circuit shown below in figure 3.

A. 50 Ω (correct)

B. 3.0×10^{-2} Ω (Req parallel = $\frac{1}{R_1}+\frac{1}{R_2}$)

C. 11 2 Ω (systematically confusing series & //)

D. 71 Ω (top in series)

E. 3.8 Ω (25 Ω in series w/ bottom)

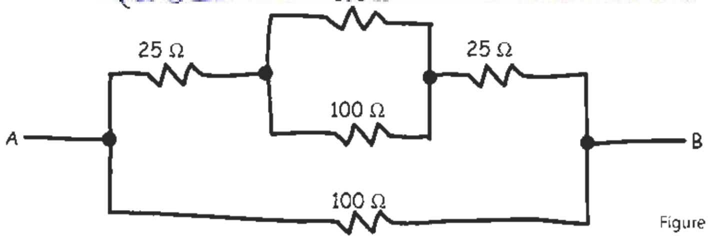


Figure 3

8. A battery supplies current to a circuit with 4 resistors, as shown below in figure 4. What is the correct expression for the potential V_1? Hint – you do not need to keep track of V_2 to find V_1.

A. 1/3 V_B

B. 1/2 V_B

C. 2/3 V_B

D. 4/5 V_B

E. 2 V_B

9. What is the correct expression for the potential V_2 in Figure 4? Think carefully before you start!

A. 1/5 V_B

B. 1/4 V_B

C. 1/3 V_B

D. 1/2 V_B

E. 2/5 V_B

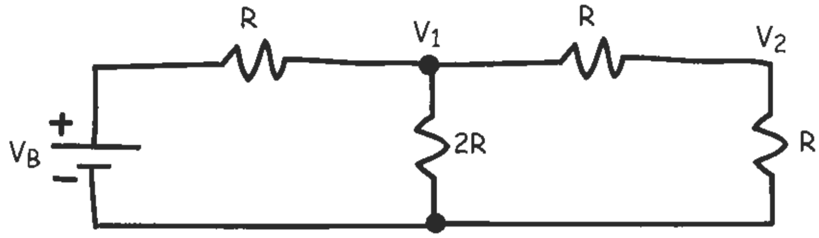


Figure 4

10. In a real battery with interval resistance (figure 5), the voltage drop across an external load resistor is

A. always less than the open circuit voltage, $V_{battery}$

B. equal to the open circuit voltage

C. unconstrained

D. zero

E. always greater than the open circuit voltage

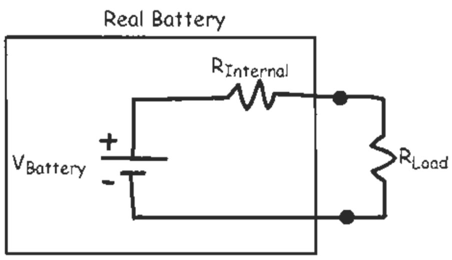


Figure 5

1B Quiz 3

1. Voltage is proportional to cunent and resistance ($V = IR$), and current is proportional to drift velocity ($I = nqv_dA$). Substituting,

$$V = (nqv_dA)R$$

$$v_d = \frac{V}{nqAR}$$

∴ charge carrier velocity is proportional to the voltage drop across the resistor.

IB Quiz 3

2. $E = \frac{1}{2}C(\Delta V)^2$

$= \frac{1}{2}(4\ F)(1.50\ V)^2$

$= 4.5\ J$

3. $\rho = 100\ \Omega \cdot cm = 1.00\ \Omega \cdot m$

$l = 1.0\ mm = 1.0 \times 10^{-3}\ m$

$d = 2.0\ \mu m$

$\hookrightarrow A = \pi r^2 = \pi(1.0 \times 10^{-6}\ m)^2 = 3.14 \times 10^{-12}\ m^2$

[redacted] $R = \rho \frac{l}{A}$

$= (1\ \Omega \cdot m)\frac{(1.0 \times 10^{-3}\ m)}{(3.14 \times 10^{-12}\ m^2)}$

$\boxed{R = 3.18 \times 10^{8}\ \Omega}$

4. $V = 220\ V$

$P = 2000\ W$

$P = IV$

$I = P/V$

$= \frac{2000W}{220V}$

$\boxed{I = 9.1\ A}$

1B Quiz 3

5. Current $(I) \equiv \frac{\text{charge } (Q)}{\text{time } (t)}$

$Q = 2e = 2 \times 1.60 \times 10^{-19}\ C$

$t = 100\ \mu s = 1.00 \times 10^{-4}\ s$

$$I = \frac{3.2 \times 10^{-19}\ C}{1.0 \times 10^{-4}\ s}$$

$$\boxed{I = 3.2 \times 10^{-15}\ A}$$

6. Find R_{eq}

$$\frac{1}{R_{eq1}} = \frac{1}{2R} + \frac{1}{2R}$$

$$R_{eq1} = R$$

$$R_{eq} = R + R_{eq1}$$
$$= 2R$$

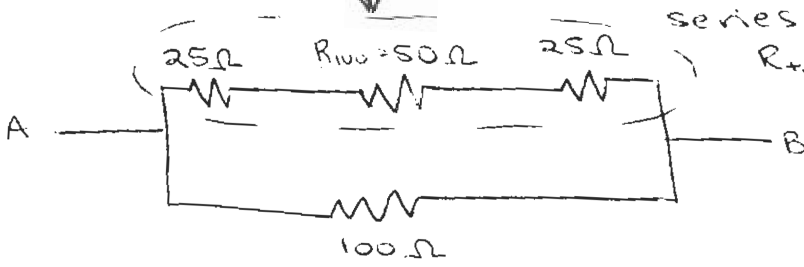
Using $V = IR$, $I = V/R$ where resistance $= 2R$

$$\therefore \boxed{I = \frac{V}{2R}}$$

IB Quiz 3

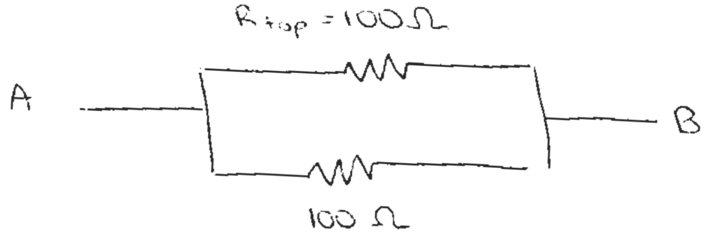
7. Find R_{eq}

parallel $\frac{1}{R_{100}} = \frac{1}{100\Omega} + \frac{1}{100\Omega} \rightarrow R_{100} = 50\Omega$



series

$R_{top} = 25\Omega + 50\Omega + 25\Omega$
$= 100\Omega$

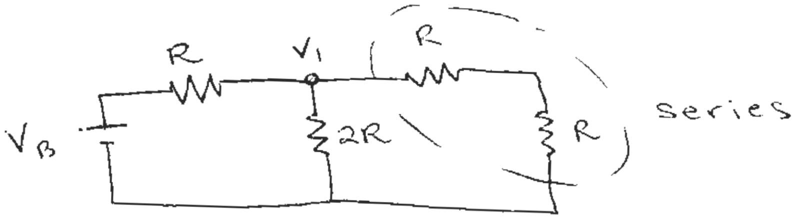


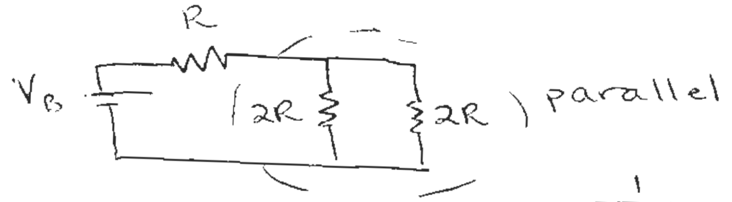
$$\frac{1}{R_{eq}} = \frac{1}{R_{top}} + \frac{1}{100\Omega}$$

$$\boxed{R_{eq} = 50\Omega}$$

1B Quiz 3

8.





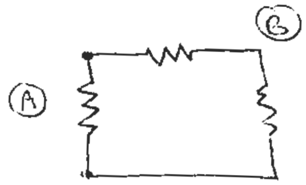
$$\frac{1}{R_{par}} = \frac{1}{2R} + \frac{1}{2R}$$

$R_{par} = R$

Same resistance, so (by symmetry) same voltage drop across both: $\frac{V_B}{2}$.

$\therefore$ $\boxed{V_1 = \frac{V_B}{2}}$

9.



Branches Ⓐ and Ⓑ are in parallel, so the same voltage drops across both branches. In #7, we found that this was $\frac{V_B}{2}$. The two resistors in series in branch B have the same resistance, so half of the voltage in branch B drops across each.

$\therefore$ $\boxed{V_2 = \frac{V_B}{4}}$

IB Quiz 3

10. Any time the circuit is closed, current from the battery will flow across the battery's internal resistance. Therefore there is a voltage drop across the internal resistance, leaving less voltage to drop across the load resistance than the open circuit voltage. Therefore the voltage drop across the external load is

always less than $V_{battery}$.