Physics 1B - Quiz 4 (26 Feb 2007)

Electrostatics

The force on charge q_1 from charge q_2 is $\vec{F}_{12} = k_e \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12}$, where the direction vector \hat{r}_{12} points from q_2 to q_1 and the proportionality constant is $k_e = 8.99 \times 10^9 \text{ Nm}^2/\text{C}^2$.

Note that the permittivity of free space is $\varepsilon_0 = \frac{1}{4\pi k_e} = 8.85 \times 10^{-12} \text{ C}^2 / (\text{Nm}^2) = 8.85 \times 10^{-12} \text{ A}^2 \text{s}^4 / (\text{kg m}^3).$

Note that the unit of elemental electronic change is $e^{-} = -1.62 \times 10^{-19}$ C.

The force on a test charge q_0 induced by an electric field, denoted \vec{E} , is $\vec{F} = q_0 \vec{E}$.

Fields and Potentials

The electric flux through a surface is $\Phi_{e} \equiv \sum_{All \ Surfaces} E_{\perp} \Delta A = \sum_{All \ Surfaces} E \Delta A \cos \theta$, where $\Delta A_{\perp} = \Delta A \ \cos \theta$ is

the component of the area whose normal lies parallel to the electric field; θ is the angle between the direction of the electric field and that of the normal to the surface.

Gauss' Law relates the total flux through a closed surface to the total net charge enclosed by the surface, *i.e.*, $\Phi_e = 4 \pi k_e Q_{Total}$.

The electric field produced by a point charge q at the origin, *i.e.*, $\vec{r} = 0$, is $\vec{E} = k_e \frac{q}{r^2} \hat{r}$ where \hat{r} is the radius vector in spherical coordinates.

The electric field produced by a line charge, with charge per unit length λ , is $\vec{E} = 2k_e \frac{\lambda}{r} \hat{r}$, where the line is defined to lie along the \hat{z} axis and \hat{r} is the radius vector in cylindrical coordinates.

The electric field produced by a surface charge, with charge per unit area σ , is $\vec{E} = 2\pi k_e \sigma \hat{n}$, where the surface lies in the \hat{x} - \hat{y} plane and \hat{z} corresponds to the normal to the \hat{x} - \hat{y} plane in Cartesian coordinates.

Work-Energy Theorem: $W = \Delta KE + \Delta PE$

Electric potential:
$$\Delta V = -E \Delta x \cos \theta$$
, where $\Delta V = \frac{\Delta P E}{Q}$

 $V = k_e \frac{q}{r}$ a distance r away from a point charge q.

Current, Resistance and Capacitance

Current: $I = \frac{\Delta Q}{\Delta t}$ or $I = n e v_D A$ where n is the density of charge carriers, v_D is the drift velocity and A is the cross-section of the wire.

Capacitance: $Q = C \Delta V$ where $C = \frac{\kappa}{4\pi k_e} \frac{A}{d}$ for parallel plates and κ is the dielectric constant $I = C \frac{\Delta V}{\Delta t}$

Energy Stored =
$$\frac{1}{2}Q \ \Delta V = \frac{1}{2}C (\Delta V)^2 = \frac{1}{2C}Q^2$$

Resistance: V = I R where R = $\rho \frac{1}{A}$ and ρ is the resistivity in Ohm-m.

	Series	Parallel
Capacitors	$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$	$C_{eq} = C_1 + C_2 + C_3 + \cdots$
Resistors	$\mathbf{R}_{\rm eq} = \mathbf{R}_1 + \mathbf{R}_2 + \mathbf{R}_3 + \cdots$	$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$

Power Dissipated = $IV = I^2R = V^2/R$

Kirchoff's Laws: 1) Sum of voltage drops around any loop is zero, *i.e.*, gains = losses

2) Sum of current flow at a node is zero, *i.e.*, total current in = total current out

A resistor/capacitor pair charges with a characteristic time, denoted t, that is given by the product of the resistance and membrane, i.e., $\tau = RC$.

Magnetostatics (Electrostatics in the Fast Lane)

The force on a test charge q_0 induced by an electric field, denoted \vec{E} , and a magnetic field, denoted B, is $\vec{F} = q_0\vec{E} + q_0\vec{v}\times\vec{B}$. The cross product $\vec{v}\times\vec{B}$ points normal to the plane defined by \vec{v} and \vec{B} , and has magnitude $|\vec{v}||\vec{B}|\sin\theta$ and a direction that is found from the "right hand rule".

The force per unit length on a straight wire that carries a current I (where I has both magnitude and direction, given by the motion of positive charge carriers) is given by $\vec{F}/l = \vec{I} \times \vec{B}$. The cross product ids defined as above.

The force per unit length between two straight wires that carry current I₁ and I₂ respectively, and are separated by a distance R, is $F/L = \mu_0 \frac{I_1 I_2}{2\pi R}$, where $\mu_0 = 1.3 \times 10^{-6}$ Tm/A.

For completeness, $\mu_0 \varepsilon_0 = 1/c^2$, where c is the speed of light.

The torque between a loop of cross-sectional area A that carries a current I and a uniform magnetic fields is $\vec{\tau} = \vec{\mu} \times \vec{B}$, where $\mu = IA$ is known as the magnetic moment.

When the loop contains multiple turns of wire, $\mu \leftarrow \text{NIA}$ where N is the number of turns.

Ampere's Law relates the magnetic field in a loop to the current, *i.e.*, $I = \frac{1}{\mu_0} \sum_{All Segments} B_{\parallel} \Delta L$

For a straight wire, $\vec{B} = \mu_0 \frac{I}{2\pi R} (\hat{\theta})$, where the direction is given by the right-hand rule. For a solenoid, $\vec{B} = \mu_0 \frac{N}{L} I(\hat{z})$.

Finally! Quiz Four

1. Two resistors ($R_1 = 500 \Omega$, $R_2 = 200 \Omega$), a capacitor ($C = 7 \mu F$), a battery ($V_B = 1.5 V$) and a switch are connected as shown below in figure 1. The switch is initially open and the capacitor is initially uncharged. At time t = 0 the switch is closed. What is the final (t $\rightarrow \infty$) voltage across the capacitor?



- A. 0 V
- B. 0.50 V
- C. 0.75 V
- D. 1.5 V
- E. Insufficient information to answer

2. With reference to the same circuit (figure 1 and parameters as in question 1), what is the time-constant for the change in voltage across the capacitor?

- A. 4.9×10^{-8} s
- B. $4.9 \times 10^{-3} \text{ s}$
- C. $1.0 \times 10^{-3} \text{ s}$
- D. 1.4x10⁻³ s
- E. 0.7 s

3. An electron, with elemental charge - $e = -1.6 \times 10^{-19}$ C and velocity $\mathbf{v} = 1.0 \times 10^6$ m/s ($-\hat{\mathbf{x}}$), enters a region with a uniform magnetic field $\mathbf{B} = 1.0$ T (\otimes), i.e., the particle moves from right to left and the field points into the paper, as shown below in figure 2. What is the force (magnitude and direction) on the electron?



- A. 0 N
- B. 1.6×10^{-13} N ($-\hat{y}$); downward
- C. 1.6×10^{-13} N (+ \hat{y}); upward
- D. 1.6×10^{-13} N (\otimes); into page
- E. $1.6 \times 10^{-13} \text{ N} (\Theta)$; out of page

4. An electron, with elemental charge - e and velocity v, enters a region with both a uniform magnetic field, $\mathbf{B} = 1.0 \text{ T} (\otimes)$, and a uniform electric field, $\mathbf{E} = 2.0 \text{ V/m} (+\hat{\mathbf{y}})$, i.e., the particle enters from the left, the magnetic field points into the paper and the electric field points upward, as shown below in figure 3. Is there an initial nonzero velocity for which the electron is not deflected and, if so, what is that velocity?



- A. Yes $1.0 \text{ m/s} (+ \hat{x})$
- B. Yes $-4.0 \text{ m/s} (+\hat{\mathbf{x}})$
- C. Yes $-0.25 \text{ m/s} (+\hat{\mathbf{x}})$
- D. No the electron always is deflected downward ($-\hat{y}$ direction).
- E. No the electron always is deflected upward ($+\hat{y}$ direction).

5. A wire that is 3.0 m long runs East to West so that it lies perpendicular to the Earth's magnetic field. If the magnitude of the Earth's field in La Jolla is 5.0×10^{-4} T, and the current in the wire is 200 A, what is the magnitude of the total force on the wire?

- A. 0
- B. 1.5x10⁻³ N
- C. 0.3 N
- D. 3.0×10^4 N
- E. Insufficient information to answer

6. A wire with mass m = 1.0 g and length l = 10 cm is supported so that it can move freely in the ydirection but is fixed in the x and z (into, \otimes , and out of, \odot , the page) directions, as below shown in figure 4. The wire further sits in a region with uniform magnetic field $\mathbf{B} = 2.0$ T (\otimes). What current, denoted I, must pass through the wire to keep it from falling? Ignore the magnetic field that is generated by current that flows in the fixed, support wires and recall that $\mathbf{g} = 9.8$ m/s ($-\hat{\mathbf{y}}$). Mind the units!



A. 2.5x10⁻² A

- B. 4.9x10⁻² A
- C. 9.8x10⁻² A
- D. 0.2 A

E. The current flows in a direction that cannot keep the wire from falling

7. The magnetic field featured in problem 6 (figure 4) is now rotated, so that it points to the right $(+\hat{x} \text{ direction})$ as shown below in figure 5. For this new geometry, what current, denoted I, must pass through the wire to keep it from falling? Again, ignore the magnetic field that is generated by current that flows in the fixed, support wires.



- A. 2.5x10⁻² A
- B. 4.9x10⁻² A
- C. 9.8x10⁻² A
- D. 0.2 A
- E. The current flows in a direction that cannot keep the wire from falling

8. Two parallel wires carry currents that are equal in magnitude and direction, with I = 10 A, as shown below in figure 6. The wires are separated by a distance d = 2 mm. What is the force per unit length – direction and magnitude – between the two wires?



- A. Attractive, with $F/l = 1.0 \times 10^{-2} \text{ N/m}$
- B. Attractive, with $F/l = 1.0 \times 10^{-3} \text{ N/m}$
- C. Zero since the two currents have the same magnitude.
- D. Repulsive, with $F/l = 1.0 \times 10^{-3} \text{ N/m}$
- E. Repulsive, with $F/l = 1.0 \times 10^{-2} \text{ N/m}$

9. Two parallel wires are separated by a distance $d = 1 \mu m$ and carry equal but opposite currents of $I = 100 \mu A$ each, as shown in figure 7. What is the value of the magnetic field – magnitude and sign – at point A?



- A. 0 N
- B. $4.0x10^{-5}$ N (\otimes); into page
- C. 4.0×10^{-5} N (\odot); out of page
- D. $8.0 \times 10^{-5} \text{ N} (\otimes)$; into page
- E. 8.0×10^{-5} N (\odot); out of page

10. With reference to the same arrangement of wires and currents (figure 7 and parameters as in question 9), value of the magnetic field – magnitude and sign – at point B?

- A. 2.7×10^{-5} N ($-\hat{y}$); downward
- B. 2.7×10^{-5} N (\otimes); into page
- C. 5.3×10^{-5} N (\otimes); into page
- D. 2.7×10^{-5} N (\odot); out of page
- E. 5.3×10^{-5} N (\odot); out of page

Fini!