Physics 1B - Quiz 5 (12 March 2007)

Electrostatics

The force on charge $q_1$from charge q_2 is $\vec{F}_{12} = k_e \frac{q_1 q_2}{r_{12}^2}\hat{r}_{12}$, where the direction vector $\hat{r}_{12}$ points from q_2 to q_1 and the proportionality constant is $k_e = 8.99 \times 10^9\ Nm^2/C^2$.

Note that the permittivity of free space is $\varepsilon_0 \equiv 1/(4\pi k_e) = 8.85 \times 10^{-12}\ C^2/(Nm^2) = 8.85 \times 10^{-12}\ A^2 s^4/(kg\ m^3)$.

Note that the unit of elemental electronic change is $e^- = -1.62 \times 10^{-19} C$.

The force on a test charge q_0 induced by an electric field, denoted $\vec{E}$, is $\vec{F} = q_0 \vec{E}$.

Fields and Potentials

The electric flux through a surface is $\Phi_e \equiv \sum_{All\ Surfaces} E_\perp \Delta A$, where $E_\perp = E \cos\theta$ is the component of the field that parallels the norm to the surface; θ is the angle between the direction of the field and the norm.
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Gauss' Law relates the total flux through a closed surface to the net charge enclosed by the surface, *i.e.*, $\Phi_e = 4\,\pi\, k_e\, Q_{Total}$.

The electric field produced by a point charge q at the origin, *i.e.*, $\vec{r} = 0$, is $\vec{E} = k_e \frac{q}{r^2}\hat{r}$ where $\hat{r}$ is the radius vector in spherical coordinates.

The electric field produced by a line charge, with charge per unit length λ, is $\vec{E} = 2k_e \frac{\lambda}{r}\hat{r}$, where the line is defined to lie along the $\hat{z}$ axis and $\hat{r}$ is the radius vector in cylindrical coordinates.

The electric field produced by a surface charge, with charge per unit area σ, is $\vec{E} = 2\pi k_e \sigma\, \hat{n}$, where the surface lies in the $\hat{x}$-$\hat{y}$ plane and $\hat{z}$ corresponds to the normal to the $\hat{x}$-$\hat{y}$ plane in Cartesian coordinates.

Work-Energy Theorem: $W = \Delta KE + \Delta PE$

Electric potential: $\Delta V = -E\, \Delta x \cos\theta$, where $\Delta V = \frac{\Delta PE}{Q}$

$V = k_e \frac{q}{r}$ a distance r away from a point charge q.

Current, Resistance and Capacitance

Current: $I = \frac{\Delta Q}{\Delta t}$ or $I = n\, e\, v_D\, A$ where n is the density of charge carriers, v_D is the drift velocity and A is the cross-section of the wire.

Capacitance: $Q = C\, \Delta V$ where $C = \frac{\kappa}{4\pi k_e}\frac{A}{d}$ for parallel plates and κ is the dielectric constant

$I = C\frac{\Delta V}{\Delta t}$

Energy Stored = $\frac{1}{2} Q\, \Delta V = \frac{1}{2} C\, (\Delta V)^2 = \frac{1}{2C} Q^2$

Resistance: $V = I\,R$ where $R = \rho \frac{l}{A}$ and ρ is the resistivity in Ohm-m.

Kirchoff's Laws: 1) Sum of voltage drops around any loop is zero, *i.e.*, gains = losses

2) Sum of current flow at a node is zero, *i.e.*, total current in = total current out

Power Dissipated = $IV = I^2R = V^2/R$

A resistor/capacitor pair charges with a characteristic time, given by the product of R and C, i.e., $\tau = RC$.

Magnetostatics (Electrostatics in the Fast Lane)

Force on a test charge q_0 induced by electric field $\vec{E}$ and magnetic field $\vec{B}$, is $\vec{F} = q_0\vec{E} + q_0\vec{v}\times\vec{B}$. The cross product and has magnitude $|\vec{v}||\vec{B}|\sin\theta$ and a direction that is found from the "right hand rule".

Force per unit length on a straight wire that carries a current I (where I has direction given by the motion of positive charge carriers) is $\vec{F}/l = \vec{I}\times\vec{B}$. The cross product is defined as above.

The force per unit length between two straight wires that carry current I_1 and I_2 respectively, and are separated by a distance R, is $F/L = \mu_0 \frac{I_1 I_2}{2\pi R}$, where $\mu_0 = 1.3\text{x}10^{-6}$ T m / A.

For completeness, $\mu_0\varepsilon_0 = 1/c^2$, where c is the speed of light.

The torque between a loop of cross-sectional area A that carries a current I and a uniform magnetic fields is $\vec{\tau} = \vec{\mu}\times\vec{B}$, where $\mu = IA$ is known as the magnetic moment.

When the loop contains multiple turns of wire, $\mu \leftarrow NIA$ where N is the number of turns.

Ampere's Law relates the magnetic field in a loop to the current, *i.e.*, $I = \frac{1}{\mu_0} \sum_{All\ Segments} B_{\parallel}\Delta L$

For a straight wire, $\vec{B} = \mu_0 \frac{I}{2\pi R}\left(\hat{\theta}\right)$, where the direction is given by the right-hand rule.

For a solenoid, $\vec{B} = \mu_0 \frac{N}{L}\, I\left(\hat{z}\right)$.

Faraday's Law relates the change in magnetic flux, $\phi_{\text{Magnetic}} \equiv \sum_{All\ Surfaces} B_{\perp}\Delta A$, to the potential V, or the current, V/R, induced in a loop, *i.e.*, $V = -\,N\,\frac{\Delta\phi_{\text{Magnetic}}}{\Delta t}$ where N is the number of turns in the loop and the minus sign implies that the induced field - caused by the induced current - opposes the original field.

Generator: $V = NBA\,\omega \sin \omega t$ where ω is the rotational frequency in radians/s (recall $\omega = 2\,\pi\,f$).

Inductance: $N\Delta\phi = L\,\Delta I$ where $L = \mu_o \left(\frac{N}{l}\right)^2 Al$ for a solenoid with N/l turns/length, cross-sectional area A, and length l.

$$V = L\frac{\Delta I}{\Delta t}$$

$$\text{Energy Stored} = \frac{1}{2} I \, N\varphi_{\text{Magnetic}} = \frac{1}{2} L \, I^2$$

A resistor/inductor pair has a characteristic time for a voltage change, given by the quotient of L and R, i.e., $\tau = L/R$.

	Series	**Parallel**
Capacitors	$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$	$C_{eq} = C_1 + C_2 + C_3 + \cdots$
Inductors	$L_{eq} = L_1 + L_2 + L_3 + \cdots$	$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots$
Resistors	$R_{eq} = R_1 + R_2 + R_3 + \cdots$	$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$

Finally – Quiz 5!

1. The circuit in figure 1 is a minimal model for the intracellular potential, $V_{Membrane}$, of a neuron just at the top of the action potential. What is the expression for $V_{Membrane}$ in terms of R_{Leak}, R_{Na}, V_{Leak}, and V_{Na}?

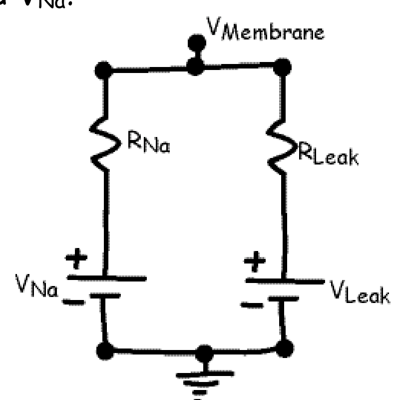

Figure 1

A. $(R_L V_{Na} + R_{Na} V_L)/(R_L + R_{Na})$
B. $(R_{Na} V_{Na} + R_L V_L)/(R_L + R_{Na})$
C. $V_{Na} - V_L$
D. $(V_{Na} - V_L)/(R_{Na}/R_L)$
E. $(V_L - V_{Na})/(R_L/R_{Na})$

2. The unit of magnetic flux is the Weber. One Weber is equal to which fundamental expression (in terms of mass, length, time, and charge)?
A. kg/(C s)
B. (kg s)/C
C. (kg m^2)/(C s)
D. (kg m^{2}s)/C
E. (kg m s)/C

3. A coil of consists of 10 turns of microwire, has a radius of 10 μm, and is immersed in a changing magnetic field (figure 2). What is the magnitude of the induced potential when the magnetic field is ramped higher as $\Delta B/\Delta t = +1.0\times10^{-2}$ T/s and the field points into the paper? Mind the units!

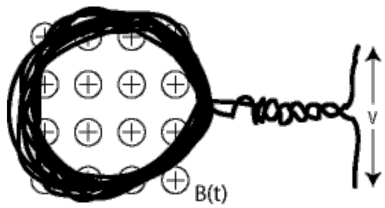

Figure 2

A. 3.1×10^{-11} V
B. 3.1×10^{-10} V
C. 3.1×10^{-10} A
D. 3.1×10^{-10} H
E. 3.1×10^{-11} H

4. The output of the above coil is connected to a resistor, as in figure 3. What is the direction of the induced current in the coil when the magnetic field is ramped higher and the field points into the paper?

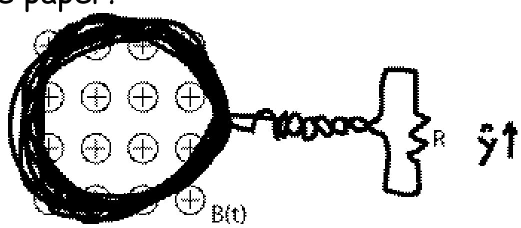

Figure 3

A. Clockwise around the coil
B. $-\hat{y}$ as current only flows through the resistor
C. Counterclockwise around the coil
D. $+\hat{y}$ as current only flows through the resistor
E. Insufficient information

5. A circuit consists of a series combination of 2 inductors, with values L_1 and L_2, and a resistor, with value R, as shown in figure 4. If the switch is closed at time t = 0, what is the value of the current immediately after the switch is thrown?

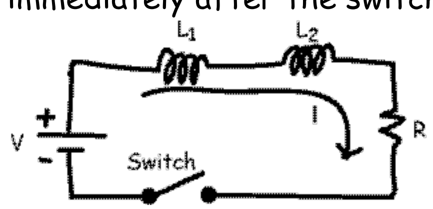

Figure 4

A. $(V/R)\,[(L_1L_2)/(L_1 + L_2)]$
B. V/R
C. 0
D. $(V/R)\,[1 - (L_1L_2)/(L_1 + L_2)^2]$
E. Insufficient information to answer

6. For the above circuit (figure 4) with the switch is closed at time t = 0, what is the value of the current a very long time after the switch is thrown?

A. $(V/R)\,[(L_1L_2)/(L_1 + L_2)]$
B. V/R
C. 0
D. $(V/R)\,[1 - (L_1L_2)/(L_1 + L_2)^2]$
E. Insufficient information to answer

7. For the above circuit (figure 4) with $L_1 = 10\ \mu H$, $L_2 = 5\ \mu H$ and $R = 8\ \Omega$, what is the inductive time-constant? Mind the units!

A. 1.9×10^{-6} s
B. $5.3 \times 10^{5}\ s^{-1}$
C. 1.9×10^{6} s
D. 4.2×10^{-7} s
E. $2.4 \times 10^{6}\ s^{-1}$

8 (Hard – but from HW). A conducting rectangular loop of mass m, width w, length l, and resistance R, falls from rest into a magnetic field, $\vec{B}$, where $\vec{B}$ points into the paper as shown in figure 5. Note that (1) there is a time interval when the bottom of the loop is just outside the magnetic field and the top just enters the magnetic field, and (2) the loop is stiff, so that the area of the loop remains constant. Is there a constant velocity at which the loop could fall?

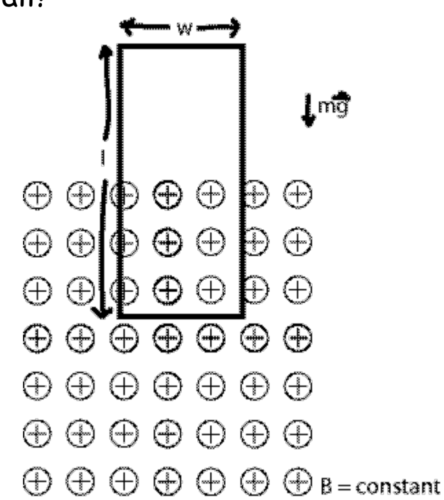

Figure 5

A. No
B. Yes, with value $mg(R/Bw)^2$
C. Yes, with value $(Bw)^2\ mg/R$
D. Yes, with value $mgR/(Bw)^2$
E. Insufficient information – we require that $\vec{B}$ change with time.

9. A flat square coil, 1.0 cm on a side, consists of 1000 turns of wire. The coil is rotated 60 cycles/s. It's axis of rotation is perpendicular to a magnetic filed with B = 0.4 T. What is the maximum voltage induced in the coil?

A. 7.5 V
B. 2.4 V
C. 1.5×10^{-2} V
D. 30 V
E. 15 V

10. A solenoid that is 10 cm in diameter and 100 cm long has 1000 turns and carries a current of 2.5 A. Calculate the magnetic flux, $\phi_{magnetic}$, through the circular cross-sectional area of the solenoid.

A. 2.5×10^{-1} Wb
B. 2.5×10^{-5} Wb
C. 2.5×10^{-5} H
D. 2.5×10^{-1} H
E. None of the above

Fini!