

NOTES ON EXPERIMENT 1

MEASURING THE RADIUS OF THE EARTH

We consider the sunset as viewed from two different heights, denoted h_b and h_c above the Earth's surface (figure). Our goal is to use the difference in height and the difference in sunset times, t_b and t_c respectively, to calculate the radius of the earth, R_E . This involves calculating the time for the earth to rotate so that the tangents from each height to the sun (the definition of sunset) coincide.

We note that $h_b \ll R_E$ and $h_c \ll R_E$.

The angle that the earth rotates between the observation of sunset at the two heights is just

$$\Delta \cos(\theta) \equiv \cos(\theta_c) - \cos(\theta_b) = \frac{2\pi}{T}(t_c - t_b) \quad (1)$$

where T is the period of the Earth's rotation ("1 day"). Thus

$$t_c - t_b = \frac{T}{2\pi} [\cos(\theta_c) - \cos(\theta_b)] \quad (2)$$

We now consider an expression for the angles θ_b and θ_c , that is valid for measurements performed at the equator (and we are not at the equator, but the geometric correction is given below)

$$\cos(\theta_b) = \frac{R_E}{R_E + h_b} = \frac{1}{1 + \frac{h_b}{R_E}} \simeq 1 - \frac{h_b}{R_E} \quad (3)$$

But when $\cos(\theta_b) \simeq 1$, we can use the Taylor expansion to write

$$\cos(\theta_b) \simeq 1 - \frac{\theta^2}{2} \quad (4)$$

Thus

$$1 - \frac{\theta_b^2}{2} \simeq 1 - \frac{h_b}{R_E} \quad (5)$$

or

$$\theta_b \simeq \sqrt{\frac{2h_b}{R_E}} \quad (6)$$

Similarly,

$$\theta_c \simeq \sqrt{\frac{2h_c}{R_E}} \quad (7)$$

Combining terms gives

$$t_c - t_b = \frac{T}{2\pi} [\cos(\theta_c) - \cos(\theta_b)] = \frac{T}{2\pi} \frac{\sqrt{2h_c} - \sqrt{2h_b}}{\sqrt{R_E}} \quad (8)$$

This is rearranged to give

$$R_E = \frac{T^2}{2\pi^2} \left(\frac{\sqrt{h_c} - \sqrt{h_b}}{t_c - t_b} \right)^2 \quad (9)$$

We add a geometrical factor, denoted " C ", to account for the latitude of La Jolla, denoted " λ_{LJ} ", and for the tilt of the Earth's rotation relative to the orbital plane around the sun, denoted " λ_{tilt} ".

$$R_E = \frac{1}{C} \frac{T^2}{2\pi^2} \left(\frac{\sqrt{h_c} - \sqrt{h_b}}{t_c - t_b} \right)^2 \quad (10)$$

where

$$C = \cos^2 \lambda_{LJ} \cos^2 \lambda_{tilt} - \sin^2 \lambda_{LJ} \sin^2 \lambda_{tilt} \quad (11)$$

with

$$\lambda_{LJ} = 32.87^\circ \quad (12)$$

and

$$\lambda_{tilt} = -23.4^\circ \sin \left(\frac{2\pi d}{365} \right) \quad (13)$$

where d is the number of days since 22 September or 20 March.

Lastly, we consider the issue of error analysis. There are three measured quantities in the expression for R_E so there are three main contributions to the error in R_E , denoted as σ_{R_E} . These three terms, which must be summed in quadrature to obtain the final error, are

$$\left| \frac{\partial R_E}{\partial \Delta t} \right| \sigma_{\Delta t} = \frac{2R_E}{\Delta t} \sigma_{\Delta t} = \frac{4\pi}{T(\sqrt{h_c} - \sqrt{h_b})} \sqrt{\frac{CR_E}{2}} \sigma_{\Delta t} \quad (14)$$

and

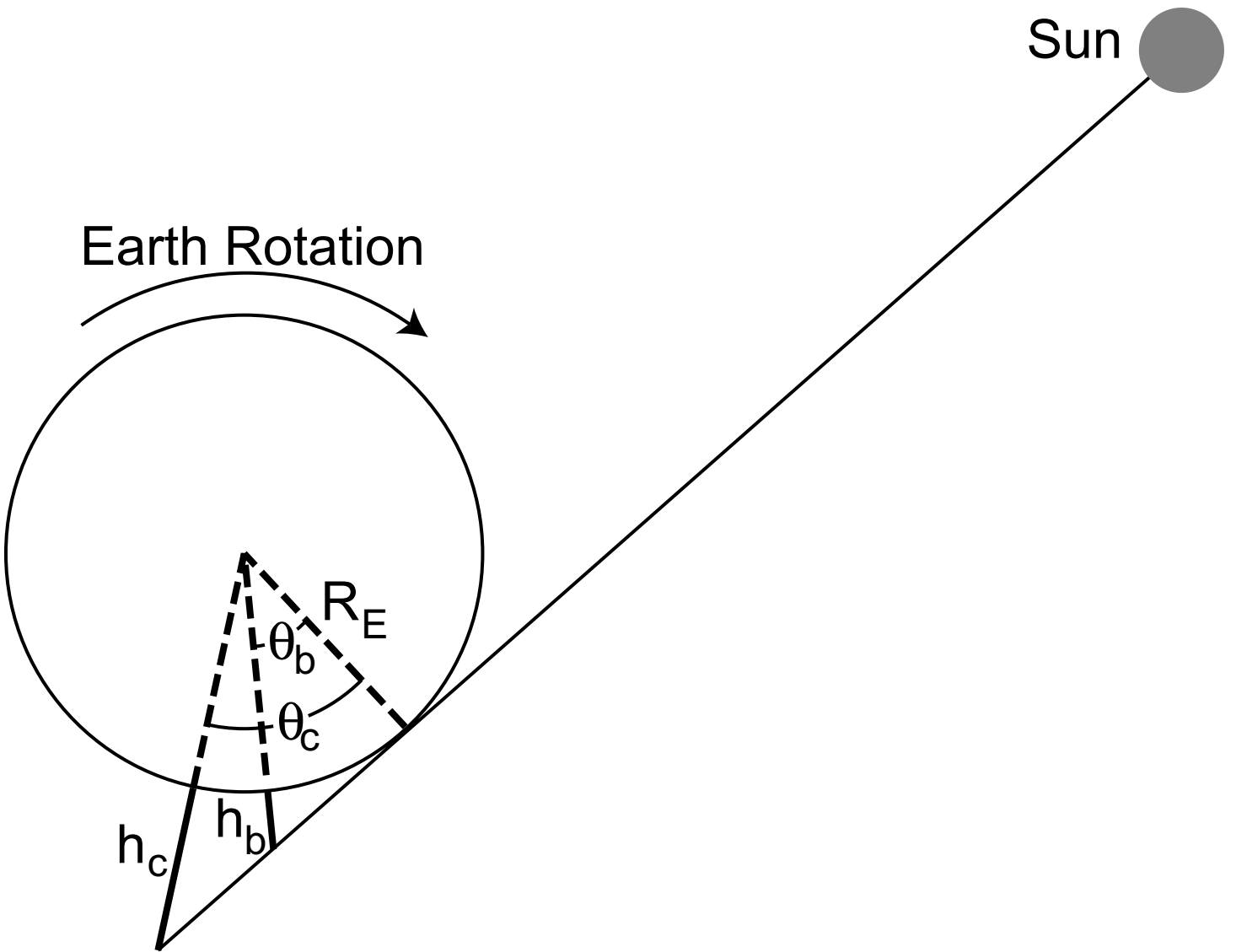
$$\left| \frac{\partial R_E}{\partial h_c} \right| \sigma_{h_c} = \frac{R_E}{\sqrt{h_c}(\sqrt{h_c} - \sqrt{h_b})} \sigma_{h_c} \quad (15)$$

and

$$\left| \frac{\partial R_E}{\partial h_b} \right| \sigma_{h_b} = \frac{R_E}{\sqrt{h_b}(\sqrt{h_c} - \sqrt{h_b})} \sigma_{h_b} \quad (16)$$

so that

$$\sigma_{R_E} = \sqrt{\left| \frac{\partial R_E}{\partial \Delta t} \right|^2 \sigma_{\Delta t}^2 + \left| \frac{\partial R_E}{\partial h_c} \right|^2 \sigma_{h_c}^2 + \left| \frac{\partial R_E}{\partial h_b} \right|^2 \sigma_{h_b}^2} \quad (17)$$



Not to Scale!