Proposal

Obtain the mean density of the earth, ρ, by measuring the earth's radius R_E and the gravitational acceleration g at the surface. Attempt to obtain a final accuracy of < 5%.

Newton's gravitation law states

$$F = \frac{GMm}{r^2} \ ,$$

so the acceleration due to gravity at the surface of the earth is

$$g = \frac{F}{m} = \frac{GM}{R_E^2} = \frac{G 4\pi R_E^3 \rho}{3R_E^2} = \frac{4}{3}\pi G\rho R_E \ ,$$

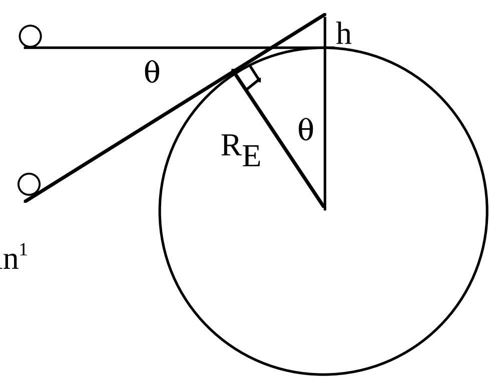
which yields

$$\rho = \frac{3}{4\pi G}\frac{g}{R_E}.$$

We will measure R_E by observing the time of sunset at two elevations: on the beach and on the cliffs at height h. We will synchronize watches, and locate observers on the beach and on top of the cliffs on a clear day at sunset. The times at which the upper edge of the sun passes below the horizon will give a time difference Δt due to altitude. We will measure the cliff height h by triangulation from a known baseline on the beach.

The figure shows (for a simplified geometry)

$$\cos(\theta) = \frac{R_E}{R_E + h},$$



giving (for $h/R_E << 1$)

$$1 - \frac{\theta^2}{2} \approx 1 - \frac{h}{R_E}.$$

and

$$R_E \approx \frac{2h}{\theta^2}.$$

Since $\theta = (2\pi/24\text{h})\Delta t = (2\pi/86{,}400\ \text{s})\Delta t$ we obtain[1]

$$R_E = 2h\left(\frac{86{,}400\ \text{s}}{2\pi \bullet \Delta t}\right)^2.$$

We expect

$$R_E \sim 6000\ \text{km}$$

$$h \sim 50\ \text{m}\ ,$$

which gives

$$\Delta t \sim 56\ \text{s}.$$

The estimated uncertainlies in our measreuments are

sunset time $\Delta t \sim \pm 1\ \text{s} = \pm 2\%$

height $h \sim \pm 1\ \text{m} = \pm 2\%$

[1] Note that a more complete analysis should consider latitude, the angle of the earth's rotation axis, and the height of the measurement on the beach

COMPLETE THE ERROR ANALYSIS FOR R_e (use separate page)

We will measure *g* by constructing a pendulum and measuring its period. Referring to the figure,

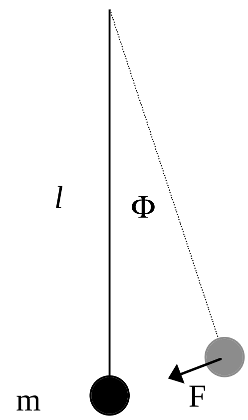
$$F = -mg\sin(\phi) \approx -mg\phi \ .$$

The acceleration

$$a = l d^2\phi / dt^2,$$

leads to

$$-g\phi = l\frac{d^2\phi}{dt^2}$$



The solution is periodic motion, with

$$\phi = \phi_0 \cos\left(\sqrt{\frac{g}{l}}t + \theta_0\right) = \phi_0 \cos\left(\frac{2\pi}{T}t\theta_0\right)$$

so that the period *T* is

$$T = 2\pi\sqrt{\frac{l}{g}} \ .$$

The phase, θ_0, is set by the initial condition and is unimportant for this analysis.

We will make our pendulum using small diameter, single strand nylon fishing line tied to a ring stand at the top end and to a weight at the bottom end. We will tape a pointer to a fixed position just beneath the equilibrium position of the weight to aid in counting the swings of the pendulum. In order to test for a systematic error in the pendulum due to the stiffness of the string, we will measure the period of the pendulum for more than one length.

We expect

$$g \sim 10 \text{ m/s}^2,$$

$$l \sim 1 \text{ m},$$

giving

$$T \sim 2 \text{ s}.$$

We will count 10 swings ≈ 20 s with error $\pm 0.2 \text{ s} = \pm 1\%$. A complementary method is to use the photogate to measure the period.

INSERT THE ERROR ANALYSIS FOR g (use separate page)