DESIGN AND BUILD A PROTOTYPE SHOCK ABSORBER

Objective:
Construct and test a critical damping system for a spring.

Reference:
Halliday, Resnick, and Walter, 5th Edition, chapter 16, special attention to section 16-8 (damped simple harmonic motion).

Discussion:
A shock absorber in a car is designed to damp the oscillations of the suspension springs in the car. Without this damping (for example when shock absorbers get old and lose fluid) after a car passes over a bump, it will bounce (oscillate) up and down many times rather than just once. Damping in shock absorbers is obtained by forcing a piston to move through a liquid-filled cylinder with an appropriate amount of fluid flow through or around the cylinder. This provides a drag force that is approximately proportional to the speed with which the piston moves. With this type of damping, the equation of motion for a mass on a spring becomes

$$m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt} \quad . \tag{1}$$

(See Eq. 16-39.) The solutions of this equation are of the form

$$x = x_0 \exp\left(-\frac{b}{2m}t\right)\exp\left(it\sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}\right) \tag{2}$$

where i = $\equiv \sqrt{-1}$, or in trigonometric notation

$$x = x_0 \exp\left(-\frac{b}{2m}t\right)\cos\left(t\sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}\right). \tag{3}$$

For weak damping, for which

$$\frac{b^2}{4m^2} << \frac{k}{m} \quad , \tag{4a}$$

a displacement from equilibrium would result in oscillations that would continue for many cycles. If the damping coefficient *b* is large, so that

$$\frac{b^2}{4m^2} > \frac{k}{m} \quad , \tag{4b}$$

the solution of equation (2) is purely an exponential decay since $i^2 = -1$, and

$$x = x_0 \exp\left(\left(-\frac{b}{2m} \pm \sqrt{\frac{b^2}{4m^2} - \frac{k}{m}}\right)t\right) \quad . \tag{5}$$

This means that when the mass is suddenly displaced from equilibrium, it will return to its equilibrium position exponentially as a function of time without overshooting or oscillating.

Small values of b would result in a very "soft" suspension for an automobile, so that the car would oscillate terribly after one rode over a bump. In contrast, very large values of *b* would result in a “stiff” suspension for an automobile and would be equivalent to having no springs. A compromise between these extremes is “critical damping”, for which the value of the damping coefficient is chosen so that

$$\frac{b^2}{4m^2} = \frac{k}{m} \tag{6}$$

or

$$b = 2\sqrt{mk} \quad . \tag{7}$$

In this case a displacement returns to zero exponentially in the shortest time. Thus the suspension will be as soft as possible without a disturbing oscillation after a bump.

Realization:
In this experiment you are to take an engineering approach in which you first measure the spring constant, *k*, of a spring. From this you will compute the desired value of *b* to critically damp a mass *m*. You will then set up an experiment to measure *b* for a variety of damping setups, until you are able to adjust it to the desired value. Finally, this damping mechanism will be attached to the spring/mass system and the combination tested to determine if critical damping has been achieved. In order to measure the damping force, $F_{damping} = -b\frac{dx}{dt}$, you will need to measure the velocity of the damping system as a function of the applied force. In the process you will also confirm that the damping force really is proportional to the velocity, as assumed in the above equations.